SPLIT-SIDEBAND SYNTHESIS

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ABSTRACT

This article introduces a new non-linear distortion synthesis technique, Split-Sideband Synthesis (SpSB). The technique is based on a variation of three well known distortion techniques, Waveshaping, Single-sideband modulation and Frequency Modulation. The basic technique is outlined and the relationship between these three techniques and SpSB is discussed. A reference implementation is presented, as a Csound5 user-defined opcode, followed by illustrated examples. The article closes with a discussion of enhancements to the basic technique with the use of multiple SpSB processors.

1. INTRODUCTION

Distortion techniques have had a long history of development, since the introduction of the audio Frequency Modulation (FM), synthesis by John Chowning in his 1973 article[2]. Over the years, not only that technique was further developed[9][10], but also several others were explored, including Phase Modulation (PM)[1], Waveshaping[7], Summation Formulae[8] and Phase Distortion[4] methods. Recently, some of these techniques have attracted renewed interest, with the development of Adaptive FM (AdFM)[5] and similar approaches[11].

Most of these techniques are correlate, as they work on the principle of distorting a simple waveshape in some way to generate complex spectra. An advantage of many of these techniques is that they require few components and are relatively inexpensive in computational terms.

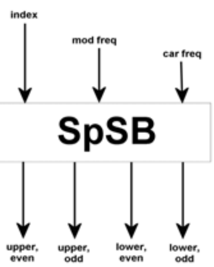

Figure 1. SpSB synthesis inputs and outputs

In this article, we propose a non-linear distortion technique which has strong links with several of the above synthesis methods, but is novel in its formulation. We have named it Split-Sideband (SpSB) synthesis, to denote its major feature, that of separate sideband control. The SpSB method uses familiar distortion synthesis parameters such as modulation/carrier frequencies and modulation index. It produces four independent outputs containing the resulting complex spectra in separate sideband groups: lower-odd, lower-even, upper-odd and upper-even (fig.1). These signals can be then mixed down in a variety of combinations and at different levels to produce different spectra, or they can be further processed, spatialised, etc..

2. SPSB SYNTHESIS

The synthetic signal produced by SpSB synthesis is defined by the following equations (with $H\{.\}$ denoting the Hilbert transform of; ω_m and ω_c, the modulation and carrier frequencies, respectively; and I the index of modulation):

$$s_{upper,even}(t) = \cos(I\sin(\omega_m t))\sin(\omega_c t) + H\{\cos(I\sin(\omega_m t))\}\cos(\omega_c t) = \quad (1)$$
$$= J_0(I)\sin(\omega_c t) + 2\sum_{n=1}^{\infty} J_{2n}(I)\sin(\omega_c + 2n\omega_m t)$$

$$s_{upper,odd}(t) = \sin(I\sin(\omega_m t))\sin(\omega_c t) + H\{\sin(I\sin(\omega_m t))\}\cos(\omega_c t) = \quad (2)$$
$$= 2\sum_{n=1}^{\infty} J_{2n-1}(I)\cos(\omega_c + [2n-1]\omega_m t)$$

$$s_{lower,even}(t) = \cos(I\sin(\omega_m t))\sin(\omega_c t) - H\{\cos(I\sin(\omega_m t))\}\cos(\omega_c t) = \quad (3)$$
$$= J_0(I)\sin(\omega_c t) + 2\sum_{n=1}^{\infty} J_{2n}(I)\sin(\omega_c - 2n\omega_m t)$$

$$s_{lower,odd}(t) = \sin(I\sin(\omega_m t))\sin(\omega_c t) - H\{\sin(I\sin(\omega_m t))\}\cos(\omega_c t) = \quad (4)$$
$$= 2\sum_{n=1}^{\infty} - J_{2n-1}(I)\cos(\omega_c - [2n-1]\omega_m t)$$

The different outputs are produced by heterodyned waveshaped signals in different combinations. The carrier frequency splits the upper and lower sideband groups. Independent even and odd sums/differences of the carrier and integer multiples of the modulator frequencies are produced. Components are scaled by Bessel functions of the 1st kind $J_n(I)$ of different orders, shown plotted in Fig.2.

2.1. SpSB and other distortion techniques

SpSB synthesis is related to several distortion techniques. It is in essence an amplitude modulation method, using single-sideband (SSB) principles[12], which also includes non-linear waveshaping of sinusoidal inputs. In addition, it is a reformulation of the usual FM or PM synthesis equations. It actually produces an exact spectral match of an FM signal if the four signals are mixed together. This section discusses the relationship of SpSB to these three techniques.

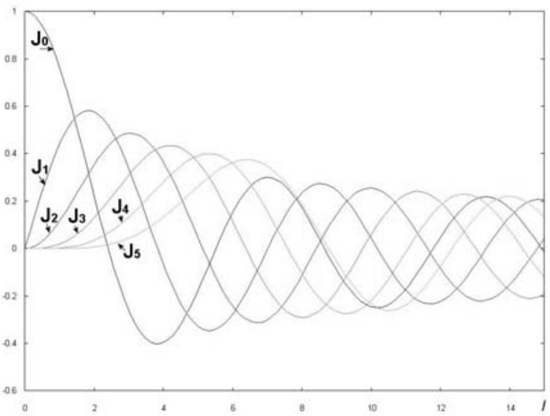

Figure 2. Bessel functions, of orders 0 through 5, plotted against the index of modulation *I*

The technique of SSB is simply an extension of the ring-modulation principle and its formulation is a matter of applying trigonometric identities to obtain the correct formulae. The upper sideband is the real part of the product of a complex sinusoid and a complex signal, which contain only positive frequencies. Conversely, the negative sideband (difference frequency) can be obtained by multiplying the same complex sinusoid by a signal with only negative frequencies and taking its real part. SSB is normally implemented by using a Hilbert transform filter to produce the required complex signal.

As mentioned above, SpSB synthesis is based on the principle of non-linear waveshaping, a type of amplitude distortion, where a signal is mapped according to a certain transfer function. At the heart of the technique, we have sinewaves being waveshaped by cosine or sine functions. The distortion basis for the resulting SpSB spectrum is shown by the following mix of signals derived from a Taylor's series expansion and the definition of a Bessel function of the 1st kind [13]:

$$
\begin{aligned}
&\cos(I\sin(\omega t)) + \sin(I\sin(\omega t)) = \\
&= \sum_{n=0}^{\infty} (-1)^n \frac{[I\sin(\omega t)]^{2n}}{2n!} + (-1)^n \frac{[I\sin(\omega t)]^{2n+1}}{(2n+1)!} = \\
&= J_0(I) + \\
&2\sum_{n=1}^{\infty} J_{2n}(I)\cos(2n\omega t) + J_{2n-1}(I)\sin([2n-1]\omega t)
\end{aligned} \tag{5}
$$

The resulting spectrum, and therefore the polynomial expansion of the sinusoidal transfer functions, is dependent on *I*. This means that, in fact, SpSB actually implements a form of dynamic waveshaping.

We can also see in Eq.5 the origin of the scaling functions $J_n(I)$ in Eqs.1-4, which also feature in FM and PM synthesis. In fact, it is easy to demonstrate that by multiplying this equation by a sinusoid, we have another form of the usual FM (more exactly PM), which has been described as heterodyne FM[6]:

$$
\begin{aligned}
y(t) &= \sin(\omega_c t)\cos(I\sin(\omega_m t) \\
&\quad + \sin(\omega_c t)\sin(I\sin(\omega_m t)) = \\
&= 0.5[\sin(\omega_c t + I\sin(\omega_m t)) \\
&+ \sin(\omega_c t - I\sin(\omega_m t)) \\
&- \cos(\omega_c t + I\sin(\omega_m t)) \\
&+ \cos(\omega_c t - I\sin(\omega_m t))] = \\
&= \sum_{n=-\infty}^{\infty} J_{2n}(I)\sin(\omega_c t + 2n\omega_m t) \\
&+ \sum_{n=-\infty}^{\infty} J_{2n-1}(I)\cos(\omega_c t + [2n-1]\omega_m t)
\end{aligned} \tag{6}
$$

This is only slightly different from the original FM formulation by Chowning in that the even and odd sidebands have different phase offsets. In its full form, SpSB is basically a single-sideband form of FM synthesis.

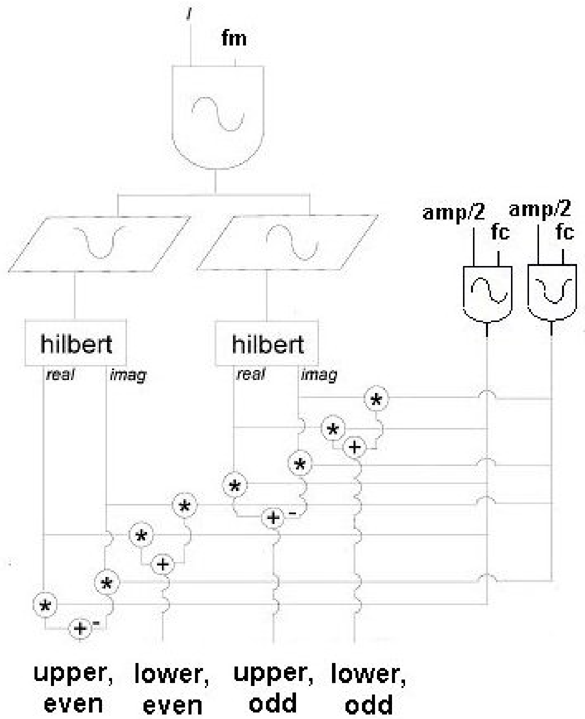

Figure 3. SpSB synthesis

3. IMPLEMENTATION

A reference implementation in Csound5[3] is presented here. It uses the hilbert[1] opcode, which implements a pair of allpass filters providing a 90 degree phase shift between their outputs. A signal flowchart describing the SpSB design is shown on fig.3.

```
/* SpSB opcode
a1,a2,a3,a4 SpSB kamp, kfc, kfm, kndx, ifn

a1,a2,a3,a4 - upper/even, upper/odd,
              lower/even, lower/odd outputs
kamp – amplitude
kfc  – carrier frequency
kfm  – modulator frequency
kndx – index of modulation
ifn  – sinewave function table number
*/

opcode SpSB,aaaa,kkkki

ka,kc,km,kndx,ifn  xin

a1 oscili kndx/(2*$M_PI),km,ifn ; sine
modulator
a2 tablei a1,ifn,1,0.25,1      ; cosine
function
a3 tablei a1,ifn,1,0,1         ; sine function

; complex modulators
aae, abe hilbert a2  ; cos(sin()): even sb
aao, abo hilbert a3  ; sin(sin()): odd sb

; complex sine carrier:
; 0.5(sin(ifc) - jcos(ifc))
ac oscili  ka/2,kc,ifn
ad oscili  ka/2,kc,ifn,0.25

;  even and odd sidebands, lower/upper sides
aeu = aae*ac + abe*ad
aou = aao*ac + abo*ad
ael = aae*ac - abe*ad
aol = aao*ac - abo*ad

  xout  aeu,aou,ael,aol

endop
```

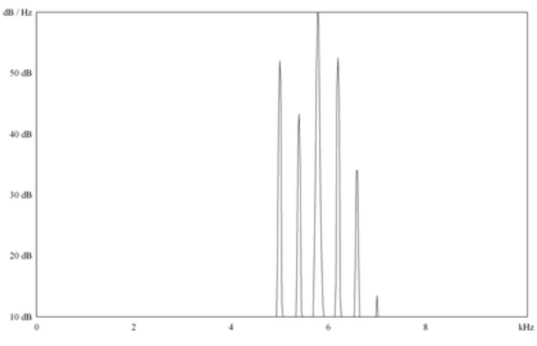

Figure 4. Upper even SpSB sidebands

4. EXAMPLES

The spectral plots of the four different SpSB outputs are shown on figs.4 – 7. These were produced with f_c=5000, f_m=200 and I =5. Figure 4 shows the resulting upper even sidebands (f_c:+2n f_m): 5000, 5400, 5800…, whereas in fig.5 we see the complementary upper odd sidebands at 5200, 5600, 6000, etc.

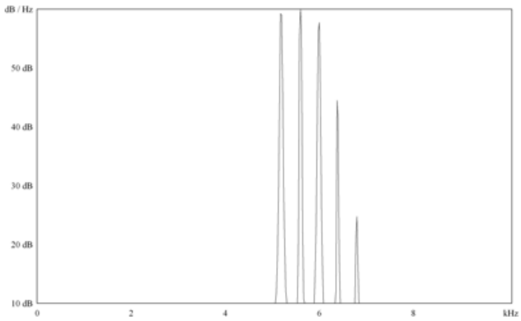

Figure 5. Upper odd SpSB sidebands

The lower side, all frequencies below the carrier, is shown in the next two figures. Even sidebands, 5000, 4600, 4200, 3800, 3400 and 3000 are seen in fig.6. Their complement, 4800, 4400, 4000, 3600 and 3200, are shown in fig.7.

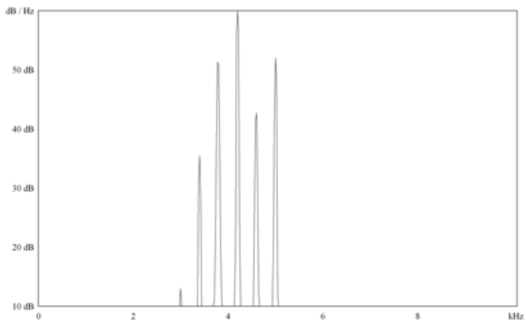

Figure 6. Lower even SpSB sidebands

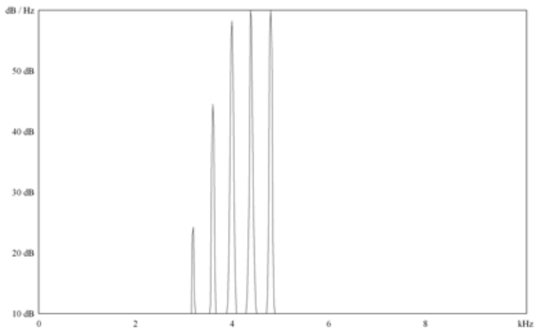

Figure 7. Lower odd SpSB sidebands

5. FURTHER ENHANCEMENTS

The SpSB synthesis technique has been presented here in its simplest form. It is possible to develop it further with a number of variations. For instance, we can use two SpSB processors in a stacked arrangement, whereby one of the outputs from the 1st processor is fed into the modulation input of the second, generating different combinations of sidebands. One very interesting result is obtained by the following structure: SpSB1 upper even sideband output feeding into SpSB2 modulator

[1] Prior to version 5.08, this opcode had its outputs reversed (im,re instead of re,im). The code presented here was written for the corrected version.

frequency input (fig.8). In this case, the lowest sideband will be the carrier frequency of the second processor.

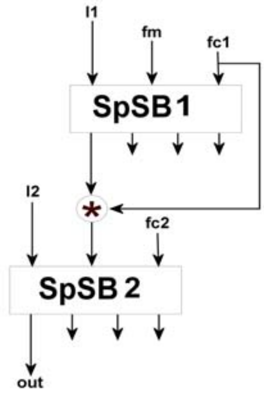

Figure 8. Complex-modulation SpSB

With this design we will obtain a spectrum with formant regions. The spacing of components around these regions will be controlled by f_m and the first formant region centre frequency by a certain choice of f_{c1}:

$$f_{c1} = (f_a - f_{c2} - 2f_m)/2 \tag{7}$$

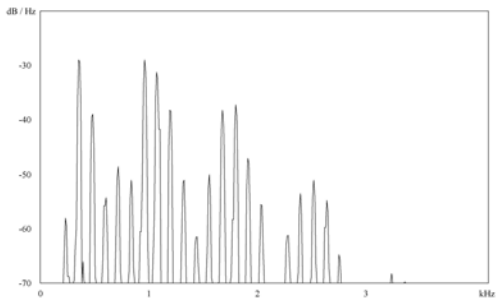

Figure 9. Complex-modulated SpSB output

The f_a frequency is made an integer multiple of the fundamental f_0, the closest to a target formant frequency f_f.

$$f_a = \mathrm{int}(\frac{f_f}{f_0} + 0.5) f_0 \tag{8}$$

The lowest component will always be f_{c2}, but the perceived fundamental frequency will depend on the ratio $f_{c2}:2f_m$.

6. CONCLUSION

In this article, we have introduced a synthesis method based on a non-linear distortion process. The technique is capable of generating a variety of timbres, producing independent even/odd and lower/upper sideband groups. As with other similar methods, it employs the typical parameters of modulator and carrier frequencies as well as modulation index. In addition to the use of a single SpSB processor, it is also possible to create other arrangements of multiple such units. Considering that each unit provides four different outputs, by combining them we can create a variety of instrument designs.

7. REFERENCES

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