A non-linear potential model to predict large-amplitudes-motions: application to a multi-body wave energy converter.

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Abstract
The paper is dedicated to the study of non-linear waves acting on three dimensional surface-piercing structures. A time-domain non-linear potential-flow model has been developed to predict the large-amplitude motions of devices in a prescribed seaway. While earlier models have been developed for single wave energy devices, this paper addresses multi-body devices, with particular application to the WAVEBOB wave energy device.

Keywords: Wave Energy; Point Absorber; Hydrodynamics; Non-Linear; Time-Domain

1. Introduction

Environmental loads on wave energy converters arise essentially from waves, current and wind. In most cases the operational and extreme loads are due to waves. Most general theoretical formulations developed for applications within seakeeping of ships and offshore structures and within coastal engineering problems may be useful for analyzing wave energy devices. The problem of wave energy converters (WEC) in waves have been extensively studied in the frequency domain. This approach is meaningful only if the oscillatory response is linear. However, wave energy converters are floating bodies which may have large amplitude motions and nonlinear responses. Time-domain formulations are more general, and are able to deal with motions.

The present paper deals with a non-linear time model. In a first section, the resolution of the equations of motion is covered. The body orientation has been described by using quaternions and the non-linear equations of motions are solved in the time-domain by the fourth-order Runge-Kutta technique. A second section deals with the hydrodynamic modelling. The pressure of the incident wave train is integrated over the instantaneous wetted surface to obtain the Froude-Krylov forces. The first-order diffraction-radiation forces are computed by a linear potential flow formulation [1] and second-order terms are added. The quadratic term of Bernoulli's equation is taken into account and the first-order force is expanded to the second-order via Taylor series expansions.

This approach has already been used to predict large amplitude motions of a single wave energy device [3] and is adapted for multi-body devices. An application to the WAVEBOB device is presented in this paper. This device is a wave energy point absorber, composed of two concentric circular cylinders. This WEC is as two separate bodies but its geometry implies that the only relative motion between them is in the vertical direction. Therefore the device could be also seen as a single body with seven degrees of freedom corresponding to three rotations (roll, pitch, yaw) and two translations (surge, sway) for the whole system, plus two translations representing the heave motions of each body.

2. Methodology

2.1. *Resolution of the Newton's law*

Under the latter hypothesis, the whole system is considered as a single floating body and the two body approach is just used for the heave case.

In a first step we define an initial inertial frame of reference R_0 linked to the physical space, assimilated to a Galilean referential. The origin O of this referential is fixed to the center of mass of the body at the initial time. A rigid motion moving R_0 to a new referential R_b is then carried out to place the body in space.

The Newton's second law leads to the two Eq.(1) and Eq. (2) where G is the center of gravity of the system, f_G^b the total force acting on the bodies and m_G^b the torque.

$$m_b\left(\dot{v}_G^b + \dot{\Omega}_{0b}^b \times v_G^b\right) = f_G^b \qquad (1)$$

$$I_G\dot{\Omega}_{Ob}^b + \Omega_{Ob}^b \times I_G \Omega_{Ob}^b = m_G^b \qquad (2)$$

Eq. (1) corresponds to the translation motion of the centre of gravity in body-fixed coordinates and Eq. (2) describes the attitude dynamics of the whole body in the body-fixed frame.

The bodies orientation has been described by using quaternions. This alternative method, in comparison to the classical Euler or Cardan angle representation, has been used to avoid singular configurations which can appear for large amplitude motions. This technique has already been used by McDonald and Whitfield [6] and Leroyer and Visoneau [5]. One can show that for any rotation around the unit vector $\vec{u}$ with an angle θ is associated to only one quaternion $Q = \cos\left(\frac{\theta}{2}\right)\mathbf{e} + \vec{u}\sin\left(\frac{\theta}{2}\right)$ in the quaternion-space basis (**e**, **i**, **j**, **k**) and that the time-derivative can be related to the instantaneous rotation vector $\vec{\Omega}_{Ob}^b$.

In vectorial settings Eqs. (1) and (2) may be expressed as

$$\mathbf{Mv} + \boldsymbol{\tau}_{Coriolis} = \boldsymbol{\tau} \qquad (3)$$

Where

- $\mathbf{v} = \left[v_G^b, \Omega_{Ob}^b\right] = \left[u, v, w_T, w_C, p, q, r\right]^T$ is the generalized velocity vector decomposed in the body-fixed frame where w_T and w_C correspond respectively to the Torus and the cylinder vertical velocity.
- **M** the inertia matrix.
- $\boldsymbol{\tau}_{\text{Coriolis}}$ the Coriolis forces.
- $\boldsymbol{\tau} = \left[f_G^b, m_G^b\right] = \left[X, Y, Z_T, Z_C, K, M, N\right]^T$ is the generalised vector of external forces and moments where Z_T and Z_C correspond to the vertical external forces applied, respectively, to the Torus and the cylinder. $\boldsymbol{\tau}$ is composed by the pressure forces due to the fluid-structure interactions, the power-take-off (PTO) loads modeled here as a linear damper and the mooring loads.

Therefore, using Eqs. (1) and (2) the motion decomposition leads to the following coupled system

$$\begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_T & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_C & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{xx} & I_{xy} & I_{xz} \\ 0 & 0 & 0 & 0 & I_{yx} & I_{yy} & I_{yz} \\ 0 & 0 & 0 & 0 & I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \frac{d}{dt}\begin{bmatrix} u \\ v \\ w_T \\ w_C \\ p \\ q \\ r \end{bmatrix} + \begin{bmatrix} m_{tot}\left(-vr + w_T q + w_C q\right) \\ m_{tot}\left(-w_T q - w_C q + ur\right) \\ m_T\left(-uq + ur\right) \\ m_C\left(-uq + ur\right) \\ \left(I_{zz} - I_{yy}\right)qr \\ \left(I_{xx} - I_{zz}\right)rp \\ \left(I_{yy} - I_{xx}\right)pq \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z_T \\ Z_C \\ K \\ M \\ N \end{bmatrix} \qquad (4)$$

$$\frac{d}{dt}\begin{bmatrix} x \\ y \\ z_T \\ z_C \end{bmatrix} = \begin{bmatrix} u \\ v \\ w_T \\ w_C \end{bmatrix} \qquad (5)$$

$$\frac{d}{dt}\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix}\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \qquad (6)$$

$$q_0^{\,2} + q_1^{\,2} + q_2^{\,2} + q_3^{\,2} = 1 \qquad (7)$$

Eq. (4) corresponds to the Newton's second law Eqs. (1) and (2), giving the generalised velocity components of the body. Then, Eq. (5) gives the translation displacement and Eq. (6) the rotational motion. Eq. (7) is redundant because a quaternion rotation has always a unit length. However, the numerical resolution of Eq. (6)

does not maintain precisely this length and a renormalization is then necessary.

2.2. Fluid-structure interactions

The fluid is considered homogeneous, uncompressible, inviscid and with an irrotational flow. Surface tension is not taken into account and the depth is considered infinite and a linearized free surface and body boundary conditions are used. The moon pool effects between the two bodies are also not taken into account in the present work.

The fluid forces acting on the two bodies can then be non-linear with respect to certain motion variables, e.g. the quadratic component of the Bernoulli's equation, the nonlinear incident potential flow. Froude-Krylov forces contain "geometric" non-linearities as the forces are computed by integrating over the exact instantaneous position and wetted surface. In addition, the incident wave field applied on the bodies is given by a higher-order method allowing the simulation of highly non-linear waves [7].

- Froude-Krylov forces

The Froude-Krylov forces are the loads introduced by the unsteady pressure field generated by undisturbed waves. In this model, the Froude-Krylov forces are completely nonlinear. The pressure of the incident wave-train is integrated on the instantaneous wetted-surface *S(t)* defined by the intersection between the non-disturbed incident free-surface and the shifted floating bodies, as follow

$$F_{FK}(t) = \iint_{S(t)} \left(P_{stat}(t) + P_{dyn}(t) \right) n(t) dS \qquad (8)$$

with

$$\begin{cases} P_{stat} = -\rho g z \\ P_{dyn} = -\rho \dfrac{\partial \phi_I}{\partial t} - \rho \dfrac{\nabla |\phi_I|^2}{2} \end{cases} \qquad (9)$$

The dynamic pressure ϕ_I is derived from the nonlinear incident potential of a higher-order method.

A robust geometry processing is essential for large amplitude motions. Consequently, an automatic remeshing routine is used for this task. At each time step, the underwater geometry is represented by a number of panels (Fig. 1). As the bodies move, their new locations and orientations are updated in the global coordinate system and the new waterline is found from the intersection with the instantaneous free surface. The underwater portion of each panel is then repanelized using the transfinite method [4].

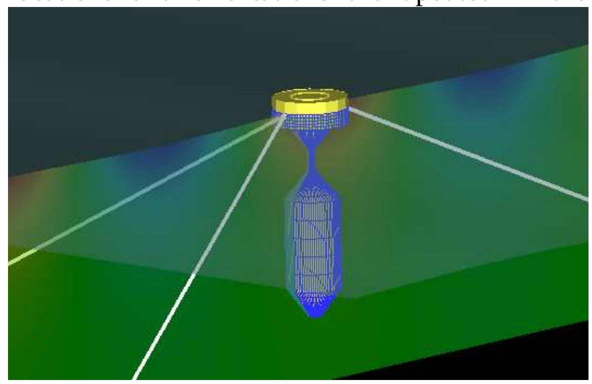

Figure 1: Screen capture of simulation

- Radiation forces

The radiation forces are the hydrodynamic forces associated with the motion of the floating bodies. The linear radiation forces have been expressed as a convolution product according to the well-known Cummins' decomposition

$$\tau_{rad}(t) = -\boldsymbol{\mu}_{\infty} v(t) - \int_0^t \mathbf{K}(t-\tau)\mathbf{v}(\tau) d\tau \qquad (10)$$

where $\boldsymbol{\mu}_{\infty}$ is the added masses matrix and **K** the impulse response function for the radiation forces which are previously computed by the commercial software ACHIL3D [1].

It can be first transformed in order to remove the convolution product by using Prony's method. This method has been developed by Clément [2] for the computation of impulse response of radiation forces. This method computes couples of variables (α_i, β_i) defining the following approximation of the real function *K* of the Eq. (10)

$$K_{ij}(t) \approx \sum_{m=1}^{N_{ij}} \alpha_{ijm} \exp^{\beta_{ijm} t} \qquad (11)$$

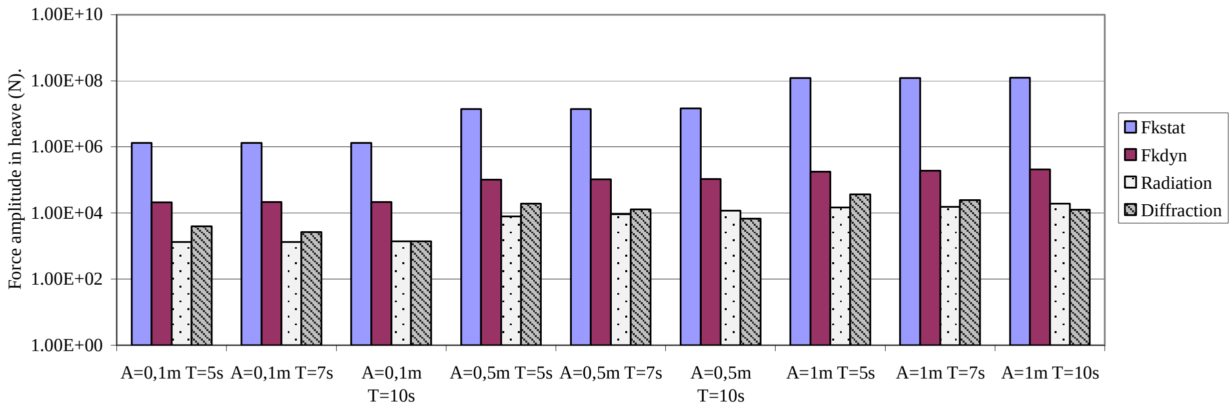

Figure 2: Comparison of the mean amplitudes of radiation forces, diffraction forces, and of static and dynamic part of Froude-Krylov forces (Fkstat, Fkdyn) in heave.

K being a real function, either (α_i β_i) are real, either they are complex and systematically associated with their complex conjugates. So, if

$$I_{ij}(t) \approx \int_0^t K_{ij}(t-\tau)\mathbf{v}(\tau)\,d\tau \qquad (12)$$

the computation of the convolution product in the equation gives the following result

$$\begin{cases} \tau_{rad}(t) = -\boldsymbol{\mu}_\infty v(t) - \mathbf{I} \\ \dot{\mathbf{I}} = \beta \mathbf{I} + \alpha v \end{cases} \qquad (13)$$

- Diffraction forces

The diffraction forces are associated with the disturbance introduced into the wave system by the presence of the floating bodies. Like the radiation forces, the diffraction forces are based here on linear time-domain theory. The diffracted wave forces are computed as

$$\tau_{diff}(t) = \int_{-\infty}^{+\infty} \mathbf{K}_7(t-\tau)\eta_I(\tau)\,d\tau \qquad (14)$$

where η_I is the free-surface elevation of the incident-wave train at a given reference point and $\mathbf{K_7}$ the impulse response function for the diffraction forces.

- Expansion to the second-order

The expansion to the second order is performed in two steps. In the first step, the linear hydrodynamic force, computed by ACHIL3D, is developed up to the second-order. This development can be done by using two approaches. The first approach consists to expand to the second-order the forcing terms around the mean wetted surface. The other approach is to expand directly the equation of the hydrodynamic force

The latter solution is used here. A Taylor series expansion of the time derivative of the total potential and of the normal to the wetted surface is performed around the mean position of the body to obtain the following force

$$\tau_h^* = \iint_{S_0} \left[\partial x' \frac{\partial \phi_t}{\partial x'} + \partial y' \frac{\partial \phi_t}{\partial y'} + \partial z' \frac{\partial \phi_t}{\partial z'} \right] n\left(M_0^{'}\right) dM_0^{'} + \iint_{S_0} \left[\frac{\partial \phi}{\partial t} \right] n\left(M_0^{'}\right) dM_0^{'} \qquad (15)$$

In the second step, the quadratic term of the Bernoulli's equation is added, the considered hydrodynamic force is then as follow

$$\begin{aligned} \tau_h^{**}(t) = & -\frac{\rho}{2}\iint_{S_0} \left|\nabla \phi_{\delta,p} \otimes V_p\right|^2 n_0 dS - \frac{\rho}{2}\iint_{S_0} \left|\nabla \phi_{\delta,p} \otimes V_I\right|^2 n_0 dS \\ & -\rho \iint_{S_0} \left|\nabla \phi_{\delta,p} \otimes \nabla \phi_I\right| \left|\nabla \phi_{\delta,p} \otimes V_p\right| n_0 dS \\ & -\rho \iint_{S_0} \nabla \phi_I \left|\nabla \phi_{\delta,p} \otimes V_p\right| n_0 dS \\ & -\rho \iint_{S_0} \left|\nabla \phi_{\delta,p} \otimes V_p\right| \left|\nabla \phi_{\delta,p} \otimes \nabla \phi_I\right| n_0 dS \end{aligned} \qquad (16)$$

where $\nabla \phi_{\delta,p}$ corresponds to the gradient vector of the velocity potential computed by ACHIL3D. The potential gradients $\nabla \phi_{rad}$ and

$\nabla\phi_{diff}$ are then computed by convolution between $\nabla\phi_{\delta,p}$ and the bodies velocity for the radiation forces and with the incident waves velocity for the diffraction forces.

3. Results

Simulations with different regular waves have been performed in order to assess the interest of such numerical model for a wave energy device like WAVEBOB. The wave field is considered regular and no control was applied on the relative motion. Nine different regular waves have been considered with a period range between 5 to 10 s and with a wave amplitude range between 0,1 to 1 m.

Fig. 2 shows the mean amplitudes of the radiation, diffraction and Froude-Krylov forces for the different regular wave trains. We can see that the Froude-Krylov forces are predominant for all of the cases. Indeed, the hydrostatic restoring force is the main component and the force associated with the dynamic component of the Froude-Krylov forces is also significant.

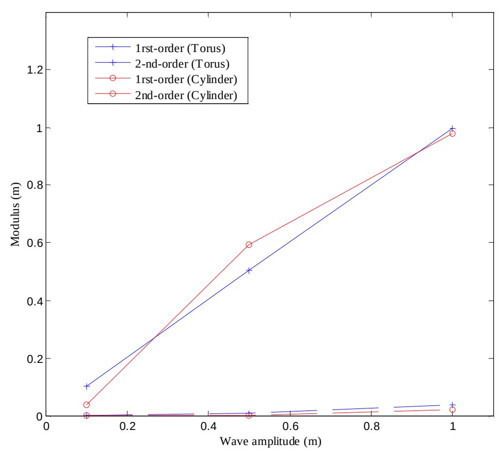

Figure 3: Fourier components of heave motion for the Torus and the cylindrer for a wave period of 5 s and 3 different wave amplitudes (0.1m, 0.5m and 1m).

Figs. 4, 5 and 6 show the variations of the first two harmonic components of the heave motion. For each body, components have been computed using a Fourier decomposition of the time history of the heave motion in a moving window of one wave period long. We can see that 2nd-order terms are insignificant in comparison to the first-order terms for all wave conditions. Their influence increase as the amplitude and the period increase but stay insignificant.

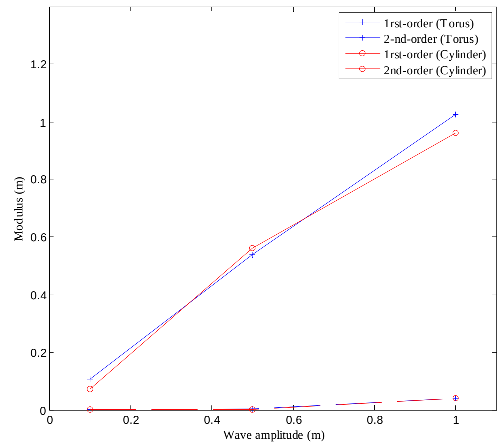

Figure 4: Fourier components of heave motion for the Torus and the cylinder for a wave period of 7 s and 3 different wave amplitudes (0.1m, 0.5m and 1m).

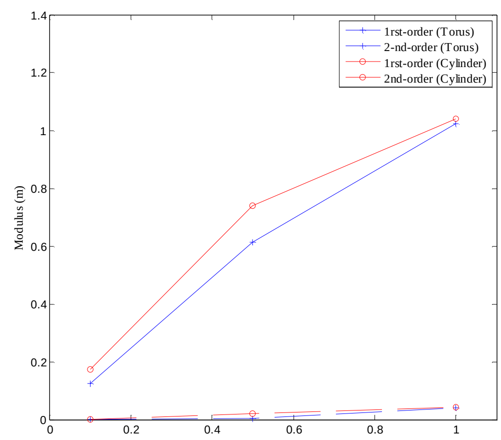

Figure 5: Fourier components of heave motion for the Torus and the cylinder for a wave period of 10 s and 3 different wave amplitudes (0.1m, 0.5m and 1m).

4. Conclusion

In this work a numerical model has been presented to determine the large amplitude motions of a floating wave energy converter subjected to incoming regular waves and composed by two bodies. The body orientation is modeled with quaternions and the non-linear equations of motion are solved by using the fourth-order Runge-Kutta method. Regarding the hydrodynamic model, the theory is based on the linear potential program ACHIL3D for determining the first-order diffraction-radiation forces. The Froude-Krylov forces are obtained by integrating the wave pressure over the instantaneous wetted surface, taking into account the large amplitude motion of the floating body. Then, the first-order diffraction-radiation forces are expanded to the second-order by using a Taylor's development and the quadratic term of the Bernoulli's equation is taken into account by convolution.

The results show that the Froude-Krylov forces are dominant in all wave conditions. Thus, the approach seems to be relevant for this kind of wave energy converter. Concerning the body motions, the results show that the second-order terms are insignificant in all wave conditions.

In terms of perspectives, validations with the WAVEBOB device have to be done. Wave tank tests are planned in this way and for quantifying moon pool and viscosity effects in order to be included in the numerical model.

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