

# The Dynamics of Wave Energy

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*Abstract* — This paper examines the challenges of efficiently harnessing wave energy. A variety of energy conversion device types is reviewed and a generic heaving buoy device selected for detailed examination. A number of modelling and control challenges are detailed and a hierarchical control structure is indicated. Both potable water production and electricity generation are included as possible uses of such devices and each presents separate control challenges.

*Keywords* — Wave energy, mathematical modelling, control systems, electricity generation

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## I INTRODUCTION

With the recent sharp increases in the price of oil, issues of security of supply, and pressure to honour greenhouse gas emission limits (e.g. the Kyoto protocol), much attention has turned to renewable energy sources to fulfil future increasing energy needs. Wind energy, now a mature technology, has had considerable proliferation, with other sources, such as biomass, solar and tidal enjoying less deployment. One energy source, which has remained relatively untapped to date, is wave energy. Some comparisons with wind energy are appropriate:

- Both result from solar heating indirectly and are intermittent energy sources;
- wind energy in Ireland (by way of example) currently accounts for approx. 9% (soon to increase to 18%) of total generating capacity (of 6400 MW) and has a total practical annual resource of 6.7 TWh [1, 2], while
- wave energy in Ireland currently accounts for 0% of generating capacity but has a total practical annual resource of 14 TWh [2]

The main reason for this imbalance is that harnessing the (roughly) sinusoidal motion of the sea is not as straightforward as, say, extracting energy from the wind. Wind energy turbine design has, in the main, converged on a generic device form and turbine technology and its associated control systems are well developed. Nevertheless, Ireland enjoys one of the best wave climates in the world [3] (see Fig.1), with up to 70 kW/m of wave crest

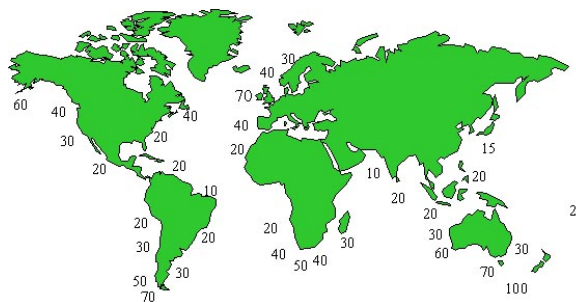


Fig. 1: Outline global wave map [3]

available, comparable with wave energies experienced at the notorious Cape Horn. It is also interesting to note that, as solar energy is subsequently converted into wind and then waves, the power density *increases*. For example, at a latitude of 15 N (Northeast trades), the solar insolation is 0.17 kW/m<sup>2</sup>. However the average wind generated by this solar radiation is about 20 knots (10 m/s), giving a power intensity of 0.58 kW/m<sup>2</sup> which, in turn, has the capability to generate waves with a power intensity of 8.42 kW/m<sup>2</sup> [4].

With both wind and wave technologies being intermittent sources and the majority of waves deriving directly from wind (with the exceptions of waves with movement, tide and seismic origins), one might expect the intermittent availability to be highly correlated and thus compound the problem of guarantee of supply. However, a recent study [5] has shown that an appropriate combination of wind and wave can significantly reduce the overall variability, due to the relatively weak relationship between sea *swell* and local wind conditions. It has

also been shown that considerable benefits can accrue from combinations of tidal and wind energy sources [5].

The current poor state of wave energy technology development is highlighted by the availability of just two commercially available wave energy converters (WECs), the Wave Dragon [6] and the Pelamis [7]. The stark contrast in operational principle of these two devices provides further evidence of the relative immaturity of wave energy technology.

In addition to the relative lack of progress in basic WEC design, there is (understandably) a corresponding ‘fertile field’ in the development of control system technology to optimise the operation of wave energy devices. This paper will attempt to show that the availability of such control technology is not only necessary, but vital, if WECs are to be serious contenders in the renewable energy arena. Ultimately, energy conversion must be performed as efficiently as possible in order to minimise energy cost, while also:

- Maintaining the structural integrity of the device;
- minimising wear on WEC components, and
- operating across a wide variety of sea conditions.

Dynamic analysis and control system technology can impact many aspects of WEC design and operation, including:

- Device sizing and configuration;
- maximisation of energy extraction from waves, and
- optimising the energy conversion in the power take-off (PTO) system.

This paper will consider each of these issues and show how such problems can be addressed, with solution forms focussing on a generic heaving buoy device. In the remainder of the paper, Section II considers the ocean environment itself and examines measures which are appropriate to characterise the energy source. Section III briefly reviews the range of WEC device types available and the principles upon which they are based. In Section IV, a mathematical model for a heaving buoy WEC is introduced, while basic dynamic design considerations are addressed in Section V. A methodology for optimising oscillatory behaviour across a range of sea conditions is also presented. Further control issues and some preliminary designs for the PTO/WEC control system are outlined in Section VI. Finally, conclusions are drawn in Section VII.

## II ENERGY FROM WAVES

The two measurable properties of waves are height and period. With regard to wave height, researchers and mariners found that observed wave heights did not correspond well to the *average* wave height, but more to the average of the one-third highest waves. This statistically averaged measure is termed the *significant wave height* and usually denoted as  $H_{\frac{1}{3}}$ . In addition, real ocean waves do not generally occur at a single frequency. Rather, a distributed amplitude spectrum is used to model ocean waves, with random phases. The use of energy spectra to represent sea states has been enumerated by a number of researchers, including Bretschneider [8] and Pierson, Moskowitz [9] and the spectra resulting from the JONSWAP project [10]. Both the Bretschneider and Pierson-Moskowitz have the general form of:

$$S_T(T) = AT^3 e^{-BT^4} \quad (1)$$

for the *wave spectral density* (or wave spectrum),  $S_T(T)$ , with the coefficients  $A$  and  $B$  for the Pierson-Moskowitz model given as:

$$A = 8.10 \times 10^{-3} \frac{g^2}{(2\pi)^4} \quad (2)$$

$$B = 0.74 \left( \frac{g}{2\pi V} \right)^4 \quad (3)$$

where  $V$  is the wind velocity measured 19.5m above the still water level (SWL),  $g$  is the acceleration due to gravity and  $T$  is the wave period in seconds. Some typical wave spectra generated from the Pierson-Moskowitz model are shown in Fig.2. Note that the available wave energy increases (approximately) exponentially with wave period,  $T$ . It should be noted that not all waves

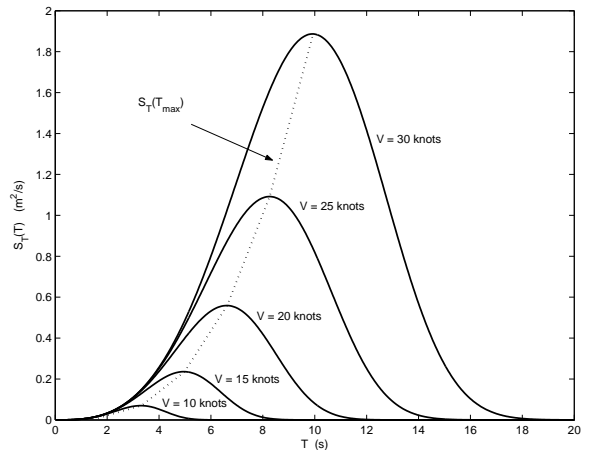


Fig. 2: Typical Pierson-Moskowitz wave spectra

are well represented by the spectral models of the type shown in (1). In some cases, where swell and local wind conditions are relatively uncorrelated

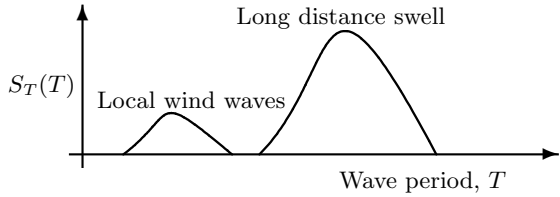


Fig. 3: 'Split' wave spectrum

(which can often be the case, for example, on the West Coast of Ireland [11]), 'split spectra', consisting of spectra containing two distinct peaks, can occur. This type of spectrum is illustrated in Fig.3 and presents a significant challenge to the WEC designer and control engineer alike. Note also that all of these wave spectra models are for *fully developed waves* i.e. the fetch (the distance over which the waves develop) and the duration for which the wind blows are sufficient for the waves to achieve their maximum energy for the given wind speed. Note also that linear wave theory is assumed i.e. waves are well represented by a sinusoidal form. This relies on the following two assumptions:

- There are no energy losses due to friction, turbulence, etc., and
- The wave height,  $H$  is much smaller than the wavelength,  $\lambda$ .

The energy in an ocean wave, consisting of both potential and kinetic energy, is proportional to the square of the wave amplitude [12] as:

$$E_w = E_p + E_k = \frac{\rho g H^2 \lambda b}{8} \quad (4)$$

where  $H$  is the wave height above SWL,  $\lambda$  is the wavelength,  $\rho$  the water density and  $b$  the crest width. In deep water, the energy in a linear wave is equally composed of potential energy (exhibited by the wave height) and kinetic energy (dependent on the motion of the particles) as:

$$E_p = E_k = \frac{\rho g H^2 \lambda b}{16} \quad (5)$$

Also, the *phase velocity* (or *celerity*),  $c$  of a wave is given by:

$$c = \lambda/T \quad (6)$$

Waves, once generated, travel across the ocean with minimal energy loss, providing the region of ocean traversed is deep (i.e. the water depth is much greater than half the wavelength). As waves approach shallow water (i.e. the waves begin to *shoal*), wavelength and phase velocity both decrease. In addition, for waves affected by the seafloor, the wave profile changes to one with a

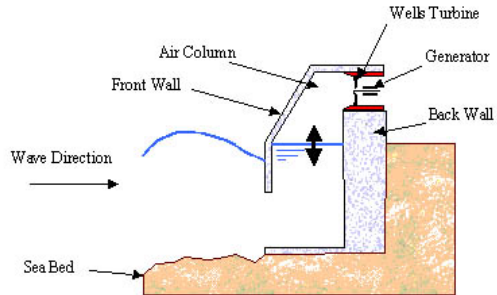


Fig. 4: OWC principle of operation

narrow crest and broad trough and linear wave theory no longer holds. In the limit, waves break with a *breaking height*,  $H_b$ , which may be determined from second order Stokes' theory [4], of:

$$H_b = \frac{16\pi^2 h^2}{3gT^2} \left[ -1 + \sqrt{1 + \frac{3gT^2}{4\pi^2 h}} \right] \quad (7)$$

where  $h$  is the water depth (distance from seabed to SWL). A useful rule of thumb is  $H_b = 2h$ . Breaking waves lose much energy to turbulence and friction and are not of major interest in wave energy conversion, though the *surge* resulting from broken waves can be harnessed by some devices, e.g. [13]. As a result, most wave energy devices are situated in relatively deep water (relative to the condition in Eq. (7)).

### III PRINCIPLES AND PROTOTYPES

The first documented wave energy converter was the prototype conceived by P. Wright, who was granted a patent on March 1st 1898 for a mechanism which utilised a float on the end of a lever driving a hydraulic system. This basic principle is still seen today in many shore-mounted prototypes. Another early device, which trapped a

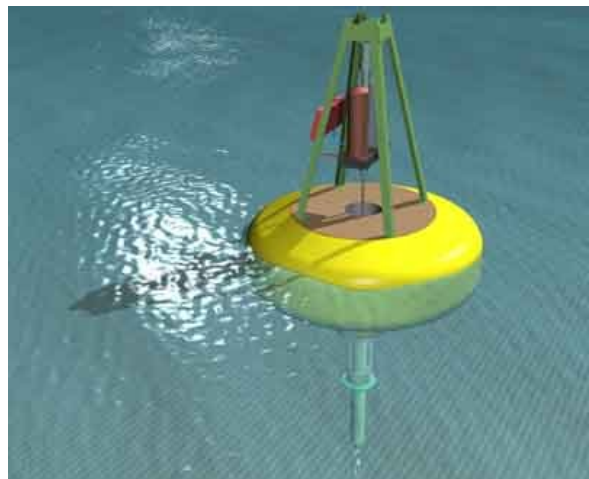


Fig. 5: Heaving buoy device

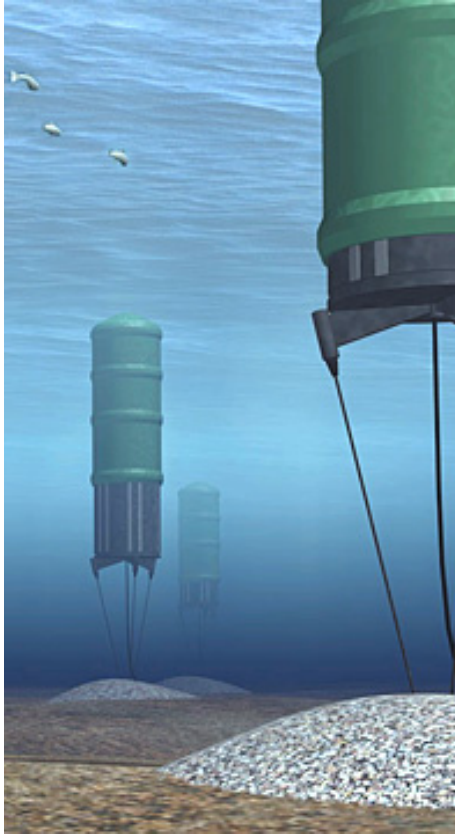


Fig. 6: The Archimedes Wave Swing [19]

column of air above surging waves, was outlined by Palme [14] and provided the inspiration for a range of devices based on the oscillating water column (OWC) principle. The basic principle of the OWC device is demonstrated in Fig.4. A common PTO issue which this device highlights is the reversing (oscillating) direction of movement of the air through the turbine. This oscillating energy flow is a feature common to virtually all WEC devices and is usually dealt with by rectification of some form. In the OWC case, innovative solutions have been found in the form of the Wells [15] and impulse [16] turbines, which provide unidirectional rotational motion of the turbine in spite of reciprocating air flow. OWC devices can be either situated on the shoreline [17] or offshore, with the 'Mighty Whale' being the largest offshore wave energy device built to date [18]. Another popular generic device type is the heaving buoy, the conceptual form of which is shown in Fig.5. In practice, perfect heave motion is difficult to obtain, unless the device is constrained to vertical motion only. For heaving buoys, motion can be obtained relative to a quay wall or the sea bed, with some variants exploiting motion between the buoy itself and an inner inertial mass, such as in the device developed by Wavebob Ltd. [20], a prototype of which has been recently deployed at the



Fig. 7: The 'Pelamis' device [22]

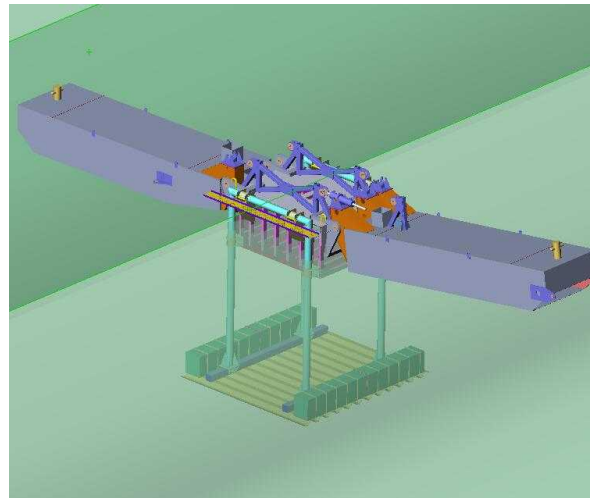


Fig. 8: The McCabe Wave Pump [23]

Marine Institute test site in Galway Bay. A device which works on a somewhat similar principle, but which consists of separate lower (anchored to the sea bed) and upper (rising and falling with passing overhead waves) sections is the Archimedes Wave Swing [21], shown in Fig.6. Two devices which exploit the relative motion of two or more longitudinal structures include the Pelamis [7] and the McCabe Wave Pump [24], shown in Figs. 7 and 8 respectively. The Pelamis (named after a sea snake) has four inter-connected cylindrical sections and can exploit relative yaw and pitch motions between sections for energy capture. The PTO device is hydraulic, with 3 pistons between each 2 sections. The MWP, on the other hand, exploits only pitch motion between two pontoons and a central platform which is restricted in heave motion by an underwater horizontal plate. The original MWP patent [24] targets the production of potable water as the primary application, while the Pelamis device has seen commercial application in the generation of electricity.

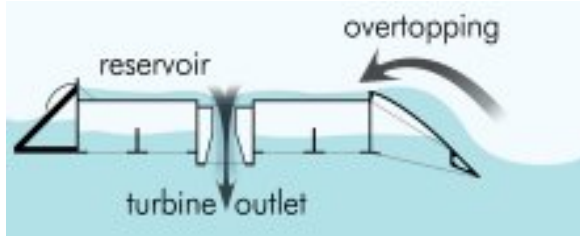


Fig. 9: The principle of overtopping

Another family of devices uses the principle of reservoir ‘overtopping’ to generate a head of hydraulic pressure in order to convert ocean energy. The principle for this is shown in Fig.9, with overtopping achievable by both shore-mounted and floating devices. The advantage of floating devices, such as the Wave Dragon [6], is the insensitivity to tidal height and the Wave Dragon [6] has seen commercial application in electricity generation. Shore-based overtopping devices (see, for example, [25]), however, can have the added bonus of the provision of a breakwater for harbour protection, etc.

One final device worthy of mention, if not least for the alternative energy extraction method, is the Wave Rotor [26], which uses a combination of a near-vertical axis Darrieus turbine with a horizontal Wells turbine to capture wave energy through the particle motion in the waves. This device also has the potential to harness tidal energy in addition to wave energy.

Clearly, there is a wide variety of devices which can be used to harness wave energy and this section has presented but a few in an attempt to show the variety of principles upon which they are based. Indeed, the number and variety of different devices seems to be increasing at an accelerating rate, judging by the significant increase in contributions to the European Wave Energy Conference between its 5<sup>th</sup> and 6<sup>th</sup> events, held in 2003 (Cork, Ireland) and 2005 (Glasgow, Scotland) respectively. This variety, without the emergence of any dominant principle or prototype form, also indicates the lack of maturity in this area compared to wind energy. A further complication is that many (especially off-shore) wave energy devices are designed to be used in farms, where the spatial positioning can be used to further enhance the energy recovered from the incident waves, but a considerable amount of further research is required in this area [27].

Consideration of the wide variety of schemes for wave energy conversion and the number of unsolved problems suggests that there is little for the control engineer to focus on. However, in broad terms, generic problems which exist across the majority of devices include requirements to:

- Position the natural resonance of the device so

that it is co-incident with peak of wave spectrum (see Fig.2);

- ‘rectify’ the oscillatory motion of the waves by some means (e.g. a one-way turbine (Wells or impulse)) in the case of air-powered devices, such as the OWC, by the use of, for example, check valves in the case of hydraulically-driven PTO mechanisms;
- ensure that WEC and PTO work in harmony to achieve maximum conversion efficiency, and
- smooth the rectified oscillatory power using some means in order to produce consistent pressure to reverse osmosis (RO) or electrical turbine mechanisms.

#### IV A GENERIC HEAVING BUOY WEC

In this section, a generic form of heaving buoy WEC will be introduced, in order to provide a target to illustrate the particular control problems arising in wave energy conversion. Several simplifying assumptions will be made, as follows:

- The buoy will be assumed to be constrained to move with heave (vertical) motion only;
- we will assume that the required application is the production of potable water through a reverse osmosis (RO) process (since this results in a reasonably straightforward pressure control requirement), and
- linear waves will be assumed.

A diagrammatic representation of the WEC is shown in Fig.10 [28]. Visible are the following:

- A cylindrical buoy constrained to move in the vertical (heave) direction only, with displacement,  $q$ ;
- a series of (unidirectional) *check valves* which rectify the water pumped by the device, and
- a power take off (PTO) device consisting of a manifold (which can accommodate multiple pumps, if necessary), a particle filter, a reverse osmosis (RO) unit and a throttle valve.

Note the position of an accumulator (capacitor) on the manifold, which helps to smooth variations in the flow resulting from the oscillatory motion of the sea. The main feature of the reverse osmosis unit is a permeable membrane through which water is forced under pressure. The membrane retains a significant fraction of the salt content in the water, which is flushed out of the RO unit via the brine outlet through the throttle valve. Note that the throttle valve is used to maintain an appropriate pressure in the system.

In order for the RO unit to operate effectively:

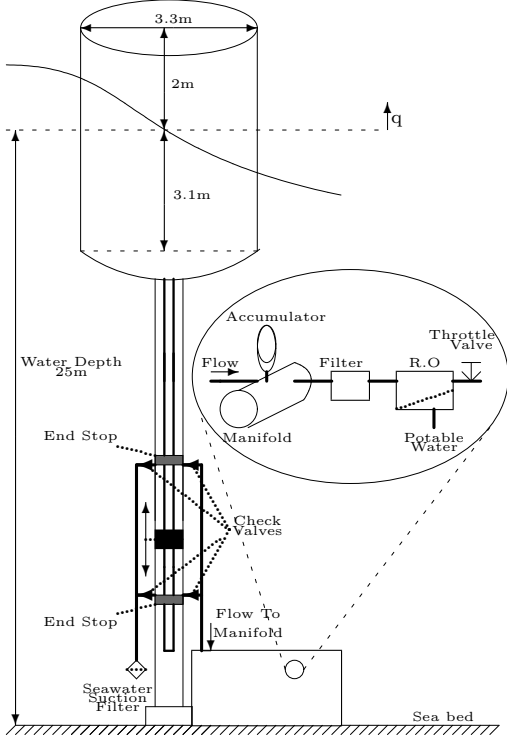


Fig. 10: Conceptual diagram of a heaving buoy WEC

- The upstream pressure (denoted  $P_{RO}$ ) must be maintained at a specific pressure, usually about  $6 \times 10^6$  Pa;
- positive pressure excursions (above the set-point) must be avoided, since these can damage the RO membrane;
- negative pressure excursions result in a loss of RO efficiency, since the RO osmotic pressure,  $R_{osm}$ , must be overcome before any water permeates through the membrane, and
- the flow ratio  $Q_b/Q_p$  must not fall below 1.857, to facilitate adequate flushing of salt from the RO unit, where  $Q_b$  and  $Q_p$  are the flowrates for brine and potable water respectively.

Eidsmoen [29] developed a model for such a heaving buoy as:

$$\ddot{q}(t) = \frac{1}{m_b + m_r(\infty)} \left\{ \int_{-\infty}^{\infty} \eta(\tau) f(t-\tau) d\tau - B^*(t) \dot{q}(t) - \int_{-\infty}^t r(t-\tau) \dot{q}(\tau) d\tau - R_f \dot{q}(t) - Kq(t) + F_b \right\} \quad (8)$$

where  $m_b$  and  $M_r(\infty)$  represent the buoy mass and ‘added mass’ (at infinite freq.) respectively,  $\eta(t)$  is the wave elevation,  $B^*(t)$  is the damping due to the PTO system,  $R_f$  is friction resistance,  $K$  is the hydrostatic stiffness of the buoy and  $F_b$  the net buoyancy force. Any (adjustable) water

ballast is included in  $m_b$ . The impulse response kernels,  $f(t)$  and  $r(t)$ , represent hydrodynamic aspects of the buoy related to wave excitation force and radiation damping, respectively. These quantities are functions of the buoy geometry and can be determined from a (linear) numerical hydrodynamic package such as WAMIT [30], which returns the frequency responses  $P(\omega)$  and  $R(\omega)$ .

Eq. (8) has some familiar terms, but the convolution integrals make straightforward analysis difficult. Using a static approximation for  $r(t)$  [28] (by calculating the area under the kernel) and letting:

$$F(t) = \int_{-\infty}^{\infty} \eta(\tau) f(t-\tau) d\tau \quad (9)$$

we get:

$$M\ddot{q}(t) + (B^* + R_f + R_o)\dot{q}(t) + Kq(t) = F(t) \quad (10)$$

where  $M = m_b + m_r(\infty)$  and  $R_o$  is the static approximation of  $r(t)$ . Note that we do not need to explicitly calculate  $F(t)$ , since it is a (unmeasurable) disturbance to the system. Finally, Eq. (10) can now be recast into the familiar form:

$$M\ddot{q}(t) + B\dot{q}(t) + Kq(t) = F(t) \quad (11)$$

with the obvious representation of  $(B^* + R_f + R_o)$  as  $B$ .

## V DESIGN CONSIDERATIONS

For a floating point absorber, such as is being considered here, Falnes [31] makes a number of important observations:

- Energy conversion is maximised if the device velocity is in phase with the excitation force, and
- The velocity amplitude should equal the wave excitation force divided by twice the device resistance.

Condition (b) above has a number of difficulties:

- It requires that the wave excitation force be measured;
- the device resistance turns out to be a non-causal function, and
- the PTO machinery must *supply* energy during part of the wave cycle in order to achieve the optimum vertical excursion of the device.

The need to supply energy may be considered counter-intuitive, but this can be likened to a person on a swing, who uses body and leg motion to increase the amplitude of the swinging, by appropriate timing of the effort. Nevertheless, the need

to supply energy requires a very complex PTO system and to the best of this author's knowledge, such a system has not yet been realised. However, Condition (a), representing a *passive* requirement, has received considerable attention and several researchers have addressed the problem. In particular, a method used to delay the velocity evolution, called *latching*, has been employed by a variety of researchers, including [32, 33]. Note that Conditions (a) and (b) above can be alternatively formulated in terms of *complex conjugate* [34] (or reactive) control, which considers the complex impedance of the device.

For analysis purposes, Eq.(11) can also be easily recast in transfer function form as:

$$\frac{Q(s)}{F(s)} = G(s) = \frac{1}{Ms^2 + Bs + K} \quad (12)$$

or, in terms of transient response parameters, as:

$$G(s) = \frac{1}{K} \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (13)$$

with

$$\omega_n = \sqrt{\frac{K}{M}}, \quad \zeta = \frac{B}{2} \sqrt{\frac{1}{MK}}$$

Equation (11) can also be conveniently expressed in state-space (companion) form, as:

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{K}{M} & -\frac{B}{M} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} \quad D = [0] \quad (14)$$

with a state vector of:

$$Q(t) = \begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix} \quad (15)$$

#### a) Power and Energy

For a mechanical system, the power ( $P$ ) is the product of force and velocity. In wave energy systems, the PTO device is normally represented by the damper, with the power developed in the damper given as:

$$P_d = \text{force} \times \text{velocity} = B\dot{q} \dot{q} = B\dot{q}^2 \quad (16)$$

The energy developed by the action on the damper over a period of time  $t_1$  is:

$$E_d(t_1) = \int_0^{t_1} P_d dt = \int_0^{t_1} B\dot{q}^2 dt \quad (17)$$

Maximum power is transferred to the damper when Eq. (17) is maximised over a period of the wave force. This results in the condition:

$$\omega_n = \sqrt{\frac{K}{M}} = \omega_w \quad (18)$$

where  $\omega_w$  represents the wave frequency (for a monochromatic sea) or the frequency corresponding to the maximum on the wave spectrum curve (see Fig.2). Under this maximum condition ( $\omega_n = \omega_w$ ), the velocity profile of the device is in phase with the wave force, consistent with Condition 1 specified earlier. Note that some adjustment of the device to achieve (18) may be possible through the use of appropriate quantity and position of water ballast.

The phase of the velocity profile (relative to the force profile) is evaluated as:

$$\angle \frac{G(j\omega)}{s} = \frac{\pi}{2} - \tan^{-1} \left( \frac{\omega B}{K - M\omega^2} \right) \quad (19)$$

Clearly, if  $K = M\omega^2$ , then velocity is in phase with force or, indeed, if  $B = \infty$ . This could, in principle at least, be achieved by using variable water ballast, which would affect both the mass  $M$  and buoyancy  $K$ . One further consideration here is that the force *lags* the velocity when:

$$\omega_w < \omega_n \quad \text{or} \quad M\omega_w < K \quad (20)$$

This places an upper bound on the device mass relative to the buoyancy since, while it may be possible to delay the WEC velocity relative to the excitation force, the converse is in general not possible. In addition, if a device is designed to be optimal for a given wave frequency,  $\omega_w^*$ , the wave force will only lag the velocity when the wave frequency,  $\omega_w$  *decreases* below this value. This has important implications for the possibility of using latching to 'delay' the velocity profile in order to get it in phase with the force profile.

#### b) Latching Basics

Latching can be achieved by means of a mechanical brake (applied at the appropriate latching points) or solenoid valves on the hydraulic lines of the PTO system.

Taking, for example, a case where  $0.5 \omega_n = \omega_w$  (with  $\omega_w = 0.5$ ), a simulated response with latching (with  $M = K = B = 1$  for simplicity) is shown in Fig.11. A number of features can be observed from Fig.11:

- The velocity response, though highly nonlinear, is now in phase with the force profile, and
- the overall energy captured from the system, via the damper, has increased from 1.94 Ws (unlatched) to 4.62 Ws (latched) per period of the incident wave.

Also the position excursions achieved under a latching regime significantly exceed those for the

unlatched case (in many cases almost by a factor of 2). Interestingly, the energy figure in the latching case is even greater than that achieved when  $\omega_n = \omega_w = 1$  (at 3.14 Ws), but this is accounted for by the fact that the wave energy is proportional to wave period (wavelength), as documented in Eq.(4).

### c) Solution to Latching System

A solution route for the latched system can be had by considering Fig.12. One period, or cycle, of the stimulus and response is given by:

$$t_5 - t_1 = \frac{2\pi}{\omega} \quad (21)$$

Given that each of the latching periods occurs consistently for  $T_L$  seconds, this gives the dynamic response period as:

$$t_3 - t_2 = t_5 - t_4 = \frac{\pi}{\omega} - T_L \quad (22)$$

For the (linear) system as given, the solution over the periods  $t_2 \rightarrow t_3$  and  $t_4 \rightarrow t_5$  is equal and opposite (assuming the transient response has died down). Therefore, the solution need only be evaluated over a half period. The solution for  $t_1 \rightarrow t_2$ , using the state-space description in (e:msdss) is:

$$Q(t) = \begin{bmatrix} p \\ 0 \end{bmatrix} \quad (23)$$

The solution for the period  $t_2 \rightarrow t_3$  may be determined from the solution to (14), assuming a reference point of  $t_2 = 0$ , as:

$$Q(t) = e^{At} \begin{bmatrix} p \\ 0 \end{bmatrix} + \int_0^t e^{A(t-\tau)} B a \sin(\omega\tau + \phi) d\tau \quad (24)$$

Though equations (23) and (24) can be used to give an expression for the state (position and velocity) over the entire cycle, there are two unknowns:

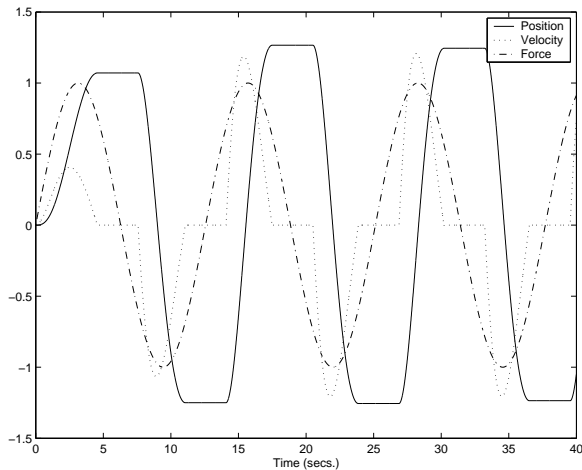


Fig. 11: Simulation of latching action

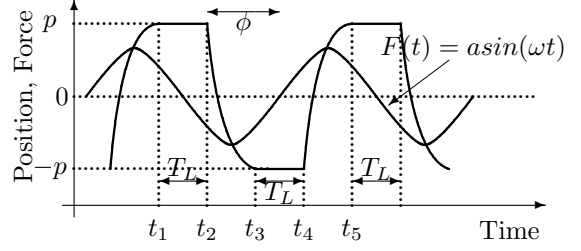


Fig. 12: Latching calculations

$\phi$ , the phase offset between the force,  $F(t)$ , and the position response, and

$p$ , the height of the position response

However, since the response has zero mean, and the transient response has died down, we know that:

$$Q(t_3) = \begin{bmatrix} -p \\ 0 \end{bmatrix} \quad (25)$$

Inserting this in (24) gives:

$$\begin{bmatrix} -p \\ 0 \end{bmatrix} = e^{A(\frac{\pi}{\omega} - T_L)} \begin{bmatrix} p \\ 0 \end{bmatrix} + a \int_0^{\frac{\pi}{\omega} - T_L} e^{A(\frac{\pi}{\omega} - T_L - \tau)} B \sin(\omega\tau + \phi) d\tau$$

Equation (26) represents 2 equations in 2 unknowns and can, conceptually at least, be solved for  $\phi$  and  $p$ . This type of solution procedure is followed in [33], using a transfer function system description.

### d) Latching Results

Figs.13 and 14 summarise the variations in the converted energy and optimal latching period (respectively) for variations in  $B$  and  $\omega_w$ . Some comments are noteworthy:

- Converted energy decreases with increasing  $\omega_w$  at smaller values of  $B$ , while it increases with  $\omega_w$  at larger values of  $B$ .
- There is a clear optimal value for  $B$ , though this does seem to vary a little with  $\omega_w$ .
- At low  $B$  values, the converted energy increases with  $\omega_w$ , as  $\omega_w$  approaches  $\omega_n$ .
- The optimal latching period,  $T_L^{opt}$ , goes to zero as  $\omega_w \rightarrow \omega_n$  (in this case  $\omega_n = 1$ ).
- As stated above, there is little sensitivity of the optimal latching time,  $T_L^{opt}$ , to variations in  $B$ , particularly for the range of  $B$  shown.



- There is a clear  $\frac{1}{\omega_w}$  relation with  $T_L^{opt}$  for all values of  $B$ . Re-plotting  $T_L^{opt}$  against  $\frac{1}{\omega_w}$  (as an example)  $B = 0.1$  shows a linear relationship between  $T_L^{opt}$  and the wave period (slope 0.5065, intercept -3.2022). As might be expected,  $T_L^{opt}$  does not appear as a consistent ‘proportion’ of the wave period,  $T_w$ , but rather is an affine function of  $T_w$ , with an offset of  $2\pi$  in the current example ( $= \omega_n$ ).

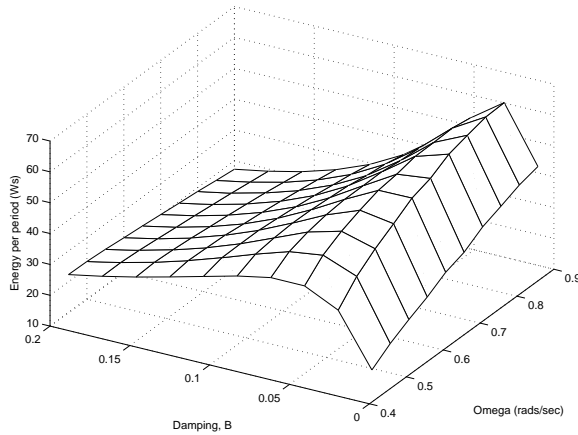


Fig. 13: Variations in  $E_d$  with  $B$  and  $\omega_w$

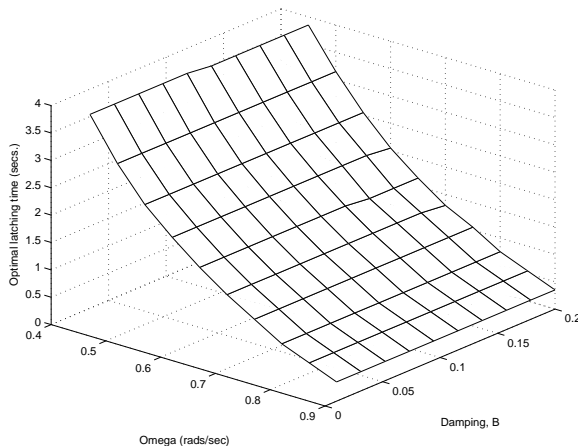


Fig. 14: Variations in  $T_L^{opt}$  with  $B$  and  $\omega_w$

The above analysis focusses on latching as a solution to force the velocity profile to be in phase with the applied wave force. However, since the wave energy is converted in the damping term (see equation (17)) one can conceive of a multitude of nonlinear loading possibilities, where the damping term is varied over the wave cycle, or scheduled with device velocity. Indeed, some researchers have looked at the possibility of having a very low damping ‘load’ at the beginning of the cycle (beginning at a point of zero velocity) and increasing the damping only after a preset velocity is reached. Such a ‘freewheeling’ strategy is in stark contrast

to latching, where the damping is effectively infinite for the latching period i.e. a short period following the point of zero velocity.

However, it is clearly demonstrated in [35, 36] that latching is *the* optimal loading strategy. This was proven via an experiment where the damping was allowed to vary with time. Following optimisation of the system energy capture with respect to the damping profile using a genetic algorithm, a ‘latching’ type profile was returned, where the damping was infinite at the early part of the wave cycle and decreased to a finite value after the ‘latching period’.

A number of further design considerations for a heaving buoy type WEC (and PTO design issues) are considered in [37].

## VI A CONTROLLER FOR AN RO APPLICATION

One of the important conclusions from Section V is that the latching time is independent of the damping,  $B$ . Therefore, both of these variables can, in theory at least, be optimised separately. In the RO application, it is required that the pressure in the RO unit,  $P_{ro}$ , be accurately maintained with the consequence that  $B(t)$  is determined by pressure control requirements and cannot be independently manipulated for maximum energy absorption by the device. The interaction between the WEC mechanics and the PTO can be captured [28] by Fig.15. Since the pressure is (ideally, at

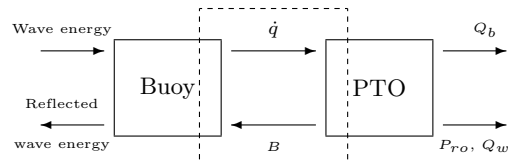


Fig. 15: Coupling between buoy and PTO system

least) held constant, the damping ( $B(t)$ ) that the WEC ‘sees’ varies (approximately) periodically. In particular, when the flow goes to zero (which happens as the device changes direction), the damping effectively becomes infinite, as shown in Fig.16. If latching is performed on the WEC, it is likely that more significant variations in device displacement (and velocity) will occur, which will place extra demands on the smoothing action of the accumulator and pressure controller. However, these variations can be predicted from the model of the system (as depicted in Fig.15, as will be shown presently).

The simplified equations (around the RO pressure operating point or setpoint,  $\overline{P_{ro}}$ ), of the combined WEC and PTO system can be written, with the identification of:

$$x(t) = [x_1 \quad x_2 \quad x_3]^T = [q \quad \dot{q} \quad P_{ro}]^T \quad (26)$$

as [28]:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{S}{M} & -\frac{R_f+R_c}{M} & -\frac{A_p}{M} \\ 0 & \frac{A_p}{C} & -\left(\frac{P_{ro}-P_{osm}}{C P_{ro}}\right) \rho_{ro} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\frac{C_r \sqrt{P_{ro}}}{200C} \end{bmatrix} [\Theta_{tv}^*] + \begin{bmatrix} 0 \\ F(t) \\ 0 \end{bmatrix} \quad (27)$$

where

$$A_p = \begin{cases} 0.009852 & , \dot{q} > 0 \\ -0.009852 & , \dot{q} < 0 \end{cases} \quad (28)$$

where  $A_p$  denotes pump cross sectional area,  $C$  is the capacitance of the accumulator,  $C_r$  is the rated flow coefficient of the valve and  $\rho_{ro}$  is the RO permeability coefficient.  $\Theta_{tv}^*$  is a command signal to a characteristic which linearises the throttle valve. This linearised model has been shown to approximate the full nonlinear system well in the general region of the operating point,  $\overline{P_{ro}}$  [28].

Eq.(27) has the general state-space form of:

$$\dot{x}(t) = \mathcal{A}x(t) + \mathcal{B}u(t) + d(t) \quad (29)$$

However, since  $A_p$  switches between two values (one of the prices of obtaining a linearised description) depending on the direction of motion of the WEC, the system should be correctly regarded as a switched linear system. The issue of switching (and the rationale for discounting its impact) is dealt with in [28].

There are a number of noteworthy items in relation to Eq. (27):

- The interaction from WEC to PTO, depicted in Fig.15, is evident in the state matrix,  $\mathcal{A}$ , particularly in relation to the  $\mathcal{A}_{32}$  term,
- The interaction from WEC to PTO, depicted in Fig.15, is evident in the state matrix,  $\mathcal{A}$ , particularly in relation to the  $\mathcal{A}_{23}$  term,

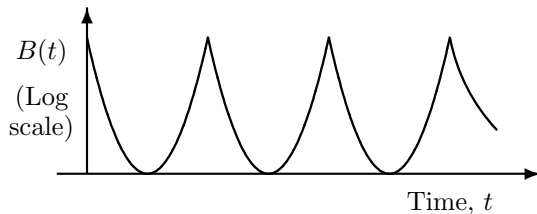


Fig. 16: Variations in damping factor

- The sole control input to the system is the throttle valve position,  $\Theta_{tv}^*$ , and
- The excitation force,  $F(t)$ , is an unmeasurable disturbance to the system.

Since we only wish to control  $x_3$  (and, most importantly, *do not* want to regulate the WEC position and/or velocity), a regulator will be designed for  $x_3$ , with no regulation of  $x_1$  or  $x_2$ . Therefore, a feedback controller employing *partial state feedback* will be employed. Furthermore, the cross-coupling term  $\mathcal{A}_{23}$  will be ignored, since pressure ( $x_3$ ) regulation is of primary importance and we will have to accept that the damping seen by the WEC (via the  $\mathcal{A}_{23}$ ) is non-optimal. However, the interaction term  $\mathcal{A}_{32}$  can be taken into account in the pressure control system. This leads to a feed-forward/feedback structure with  $\mathcal{A}_{23} x_2(t)$  as the measurable disturbance feedforward term, which can help to anticipate disturbances to the pressure control system caused by the oscillatory WEC motion.

The equation for the RO pressure ( $x_3$ ) may be extracted from (27) as:

$$\dot{x}_3 = \mathcal{A}_{33}x_3 + \mathcal{B}_3u + \mathcal{A}_{32}x_2 \quad (30)$$

A state-feedback controller can now be designed using  $\Theta_{tv}^*$  (which is  $u(t)$  in (30) to give:

- Regulation of  $x_3$  around the setpoint pressure of  $6 \times 10^6$  Pa,
- Integral action to ensure zero steady-state error, and
- Suitable transient performance.

Since transient response to setpoint variations is not an issue (the controller is a pure regulator), the system pole can be chosen to optimally offset typical disturbance variations resulting from WEC motion. Therefore, the pole can sensibly be chosen to coincide with the peak of the wave spectrum (see Fig.2).

Finally, it should be noted that the PTO can contain a number of pumps and that the RO unit can be divided into sections. These ‘sectioned’ units provide further freedom to adjust the pumping force and RO resistance appropriate for various sea conditions are dealt with in [28].

## VII CONCLUSIONS

Wave energy is at a relatively early stage of development and presents many exciting challenges. The final efficiency achieved by wave energy devices will not only be a function of the basic WEC configuration, but also of the control system(s) used to ensure its effective operation. Since wave

energy devices will have to compete with other renewable (and conventional) sources of energy, it is imperative that efficiency is maximised if the price of wave generated energy is to be competitive. Only if this is achieved, will the great wave energy resource be harnessed and contribute to our ever-increasing energy needs. The possibilities offered by wave energy are now being closely examined in a number of jurisdictions where good wave energies are available e.g in the U.K. [38].

The control problem for a generic heaving-buoy type WEC has been articulated in some detail. It is clear that the control problem is multi-faceted and there is a hierarchy which can be identified as follows:

1. Ensure that mass and buoyancy are optimal for predominant wave conditions (water ballast control),
2. Ensure that velocity of WEC is in phase with excitation force (phase control - can be achieved with latching),
3. Ensure that pressure is well regulated for a potable water application, to ensure that maximum RO efficiency, without RO damage, is achieved (pressure control), and
4. Ensure that the damping presented to the WEC by the PTO allows maximum energy capture.

In the case of potable water production, 4. is obviated by the requirement (in 3.) to focus on pressure regulation. In the alternative application of electricity production, more freedom may exist, depending on the type of generator employed. This is discussed in further detail in [37], but employment of a variable speed generator with AC-DC-AC conversion, combined with a pelton wheel (which converts fluid flow energy to rotational energy) gives an extra degree of freedom which may be used to improve the damping presented to the WEC. The development of such a control system is the subject of further research.

#### VIII ACKNOWLEDGEMENT

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