Theory and Practice of Modified Frequency Modulation Synthesis*

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The theory and applications of a variant of the well-known synthesis method of frequency modulation, modified frequency modulation (ModFM), is discussed. The technique addresses some of the shortcomings of classic FM and provides a more smoothly evolving spectrum with respect to variations in the modulation index. A complete description of the method is provided, discussing its characteristics and practical considerations of instrument design. A phase synchronous version of ModFM is presented and its applications on resonant and formant synthesis are explored. Extensions to the technique are introduced, providing means of changing spectral envelope symmetry. Finally its applications as an adaptive effect are discussed. Sound examples for the various applications of the technique are offered online.

0 INTRODUCTION

The elegance and efficiency of closed-form formulas, which arise in various distortion synthesis algorithms, represent a resource for instrument design that is explored too rarely. Among the many techniques in this group there are frequency modulation synthesis [1], summation formula method [2], nonlinear waveshaping [3], [4], and phase distortion [5]. Some of these techniques remain largely forgotten, even when many useful applications for them can still be found. Modified frequency modulation (ModFM) synthesis is one such technique. Some of the elements that make up this method were identified by Schafer [6] and Hutchins [7] with regard to analog signals. In Moorer [8] the ModFM equation is presented as one of the more practical of various well-known closed-form formulas [9] for digital sound synthesis. In Palamin et al. [10] we find it linked to one of their two asymmetric FM algorithms. It was not until [11] that this method was investigated again as an alternative means for the synthesis of band-limited classical analog waveforms. Recently new applications for distortion techniques have been demonstrated to range from source-modifier synthesis without filters [12] to the emulation of formants [13] and adaptive effects [14]–[16].

In this paper we will try to sketch a complete theory of ModFM synthesis, highlighting its main characteristics, and contrasting it with other related techniques. In addi-

tion we will examine further applications in complement to the ones presented in [11]. We will also look at extensions to the basic technique as well as its use as an adaptive effect.

1 SIMPLE MODFM SYNTHESIS

ModFM synthesis represents an interesting alternative to the well-known FM [1] (or phase-modulation) synthesis. It is almost as efficient in computational terms, but avoids some of the more awkward features of simple FM. We preface our exploration of ModFM by a discussion of these issues.

1.1 Problems of FM Synthesis

The major feature of the FM spectrum is represented by the scaling functions that arise in its expansion, namely, the Bessel functions of the first kind of various orders, denoted by $J_n(k)$,

$$\cos[\omega_c t + k\sin(\omega_m t)] = \sum_{n=-\infty}^{\infty} J_n(k)\cos(\omega_c t + n\omega_m t) \quad (1)$$

where k is the index of modulation, $\omega_c = 2\pi f_c$, and $\omega_m = 2\pi f_m$; f_c and f_m are the carrier and modulator frequencies.

The presence of Bessel functions on the right-hand side of Eq. (1) is what sets the perceptual quality of FM sounds apart from other discrete summation schemes and distortion methods. What was once thought of as an aurally interesting feature became one of the most undesirable

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side effects of the technique, the oscillating pattern of Bessel functions (Fig. 1). These give the characteristic FM shimmer, as the sidebands vary in amplitude and phase with the index of modulation k, making it difficult to create natural sounding dynamic spectra. Of course, for applications where synthetic sounding tones are intended, this is not too much of an issue, and for some uses the Bessel function spectral characteristics may be desirable. (For instance, they allow the realization of formants with a single-carrier arrangement.) However, even in these circumstances the predictable nature of its spectral evolution makes it hard to avoid typical FM clichés in sound design.

The major problems, however, occur when trying to use FM synthesis for the modeling of real instrument tones. Many of these require a particular spectral evolution, rather than a steady-state spectrum. This is generally low pass in nature and does not normally involve great oscillations in partial amplitudes. Given these conditions, in many cases the simple FM equation provides a poor match. In fact, at times it might not even be possible to achieve the correct steady-state spectrum.

Complex FM schemes address some of these issues by not relying on high values of k, thus featuring spectra where only low-order Bessel functions (in their rising phase) are significant [17]. However, this leads to much more complicated expansions and harder to estimate spectra, as well as to comparatively higher computational cost. These problems have compromised the continued application of FM to instrument design.

However, on the positive side, FM synthesis is a well-known and well researched technique. Many of its features, such as design simplicity, reduced number of parameters, and flexibility to create both harmonic and inharmonic spectra, are very useful. If the less desirable effects of Bessel function scaling can be avoided, we will have an interesting alternative to the classic FM synthesis technique.

1.2 Theory of ModFM Synthesis

The issues described in Section 1.1 are addressed by proposing a variation to classic FM, ModFM. This is a technique that can be derived from that method by some small modifications to its algorithm. We begin with a less well-known version of the FM formula by casting Eq. (1) with a cosine modulator (with $\omega_c = 2\pi f_c$ and $\omega_m = 2\pi f_m$),

$$s(t) = \cos[\omega_c t + z \cos(\omega_m t)] = \Re\{e^{i\omega_c t + iz\cos(\omega_m t)}\}. \quad (2)$$

Now we effect a change of variable, $z = -ik$, which will yield a variant of the FM synthesis equation employing a purely imaginary index of modulation,

$$s(t) = \Re\{e^{i\omega_c t + k\cos(\omega_m t)}\} = e^{k\cos(\omega_m t)}\cos(\omega_c t). \quad (3)$$

The significance of this simple operation is only appreciated if we consider that we now have a different set of scaling functions for the FM spectrum, shown in the expansion of Eq (3),

$$s(t) = I_0(k)\cos(\omega_c t) + \sum_{n=1}^{\infty} I_n(k)[\cos(\omega_c t - n\omega_m t)$$
$$+ \cos(\omega_c t + n\omega_m t)] \quad (4)$$

with

$$I_n(k) = i^{-n}J_n(ik) \quad (5)$$

where $I_n(k)$ stands for the modified Bessel function of order n. Eq. (4) has been discussed in Moorer [8] and is also a special case of one of the asymmetric FM equations in [10]. These are purely imaginary-argument versions of the original Bessel functions. Their Taylor series expansion is [18]

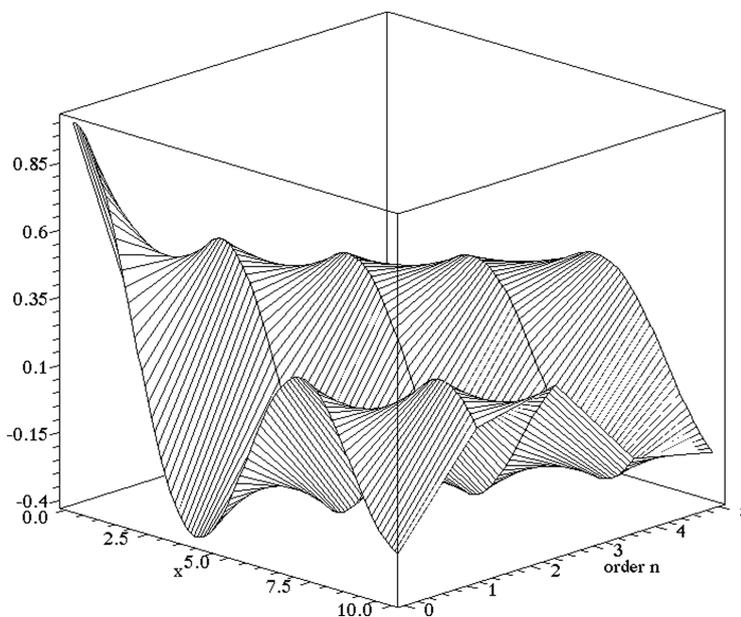$$I_n(k) = \sum_{m=0}^{\infty} \frac{(k/2)^{n+2m}}{m!(m+n)!}. \quad (6)$$

Fig. 1. Bessel functions of the first kind $J_n(x)$ of orders $n = 0, \ldots, 5$.

It is possible to see that these functions might offer a solution to the issues discussed in Section 2.1. If we contrast Eq. (6) with the expansion of Bessel functions of the first kind,

$$J_n(k) = \sum_{m=0}^{\infty} (-1)^m \frac{(k/2)^{n+2m}}{m!(m+n)!} \qquad (7)$$

we can see that the modified Bessel functions differ only by the absence of the oscillating factor $(-1)^m$, making it always nonnegative. In fact, they will eliminate all undesirable features of the Bessel functions, as we shall examine later. Furthermore we have the useful property

$$I_{-n}(k) = I_n(k) \qquad (8)$$

for integer orders n, which simplifies the effects of the reflection of lower sidebands around 0 Hz (compared to the awkward relationship $J_{-n}(k) = (-1)^n J_n(k)$, which flips the phase of odd-order sidebands).

Another important difference between Eqs. (6) and (7) is that whereas the latter decays with k, the former does not. This means that for practical applications of the modified FM algorithm of Eqs. (3) and (4) we will need to apply some means of normalization, whereas no such thing is required for classic FM. Luckily we can easily find a normalization expression as a function of the index of modulation k in

$$A(k) = e^{-k}. \qquad (9)$$

This normalization can be justified by the following asymptotic approximation of $I_n(k)$[18] for $k >> n$,

$$I_n(k) \approx \frac{e^k}{\sqrt{2\pi k}}. \qquad (10)$$

Now we have all the pieces to define the synthesis expression for ModFM synthesis,

$$\begin{aligned} s(t) = Ae^{k\cos(\omega_m t)} &= -k\cos(\omega_c t) \\ &= \frac{A}{e^k} \sum_{n=-\infty}^{\infty} I_n(k)\cos(\omega_c t + n\omega_m t) \end{aligned} \qquad (11)$$

where A is the signal amplitude, k the index of modulation, $\omega_c = 2\pi f_c$, and $\omega_m = 2\pi f_m$.

The implications of this modification of the classic FM synthesis algorithm are demonstrated by the plot of normalized modified Bessel functions in Fig. 2. If we compare it with Fig.1, we can see how they effectively fix the issues raised in Section 2.1. A comparison between ModFM and classic FM spectra for the same parameters of $k = 5$ and f_c: $f_m = 1$ is shown in Fig. 3. It can be seen that the major differences are in relation to the monotonically decreasing characteristic of the ModFM spectrum, which sets it apart from classic FM. In addition the absence of phase-reversed partials will allow a more predictable result when combining several ModFM instruments.

In particular we can summarize the following characteristics of normalized modified Bessel functions, which are very welcome:

- Unipolar, positive only
- $I_{n+1}(k) < I_n(k)$.

The last item in the preceding list is worth discussing further. The implications are that higher sidebands will never have more weight than lower ones. It is the case that, for higher values of the index of modulation, the energy in all components will tend to converge to a mean value. However, there will always be some spectral decay toward
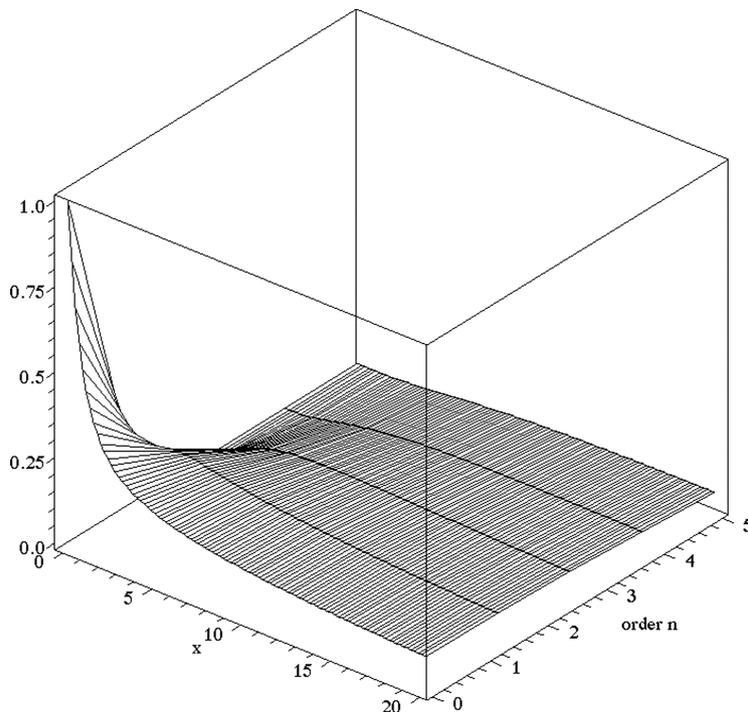

Fig. 2. Normalized modified Bessel functions $e^{-x}I_n(x)$ of orders $n = 0, \dots, 5$.

the higher end of the spectrum (which might be very small). This will make the design of instruments with low-pass and bandpass spectral envelopes easier. The other listed features of modified Bessel functions will also facilitate the synthesis of more natural-sounding dynamic spectra.

Finally it is worth noting that in the implementation of ModFM [Eq. (11)] the frequency is not modulated directly. However, the effect is similar to that of Eq. (3), where we can clearly see frequency (or, to be specific, phase) modulation. However, given that the index of modulation (the modulator amplitude) is a purely imaginary complex number, in practice it is simpler to implement ModFM using exponential waveshaping and ring modulation. In fact classic FM can also be implemented using waveshaping and ring modulation, as noted in Le Brun [3], even though that form is normally employed only where it is required [19].

1.3 Instrument Design

ModFM synthesis instrument design will take advantage of the various concepts and ideas associated with classic FM synthesis. In particular we will observe the following principles [1]:

1) The carrier-to-modulator frequency ratio $f_c{:}f_m$ will determine the harmonicity of the spectrum and its fundamental frequency, if any.

2) If $f_c{:}f_m$ is represented with no common factors as $N_1{:}N_2$, for small integer ratios we have $f_0 = f_c/N_1 = f_m/N_2$. For other cases, when $N_1{:}N_2$ is neither rational nor small, the spectrum will be inharmonic.

3) If $N_2 = 1$, the spectrum will contain all harmonics; with integral $N_2 > 1$, every N_2th harmonic will be missing.

4) For integral N_1 and N_2 the carrier frequency f_c will always be the N_1th harmonic in the spectrum.

The amount of significant components in ModFM is determined by the ratio of the loudest component I_0 and the sideband component in question. In general, for the same modulation index, ModFM will feature more energy spread and components. This will need to be controlled carefully to avoid aliasing. Taking the safe threshold of -60 dB (1/1000 of maximum amplitude), we can set a limit for the modulation index k as

$$\max_{k}\left\{20\log_{10}\frac{I_n(k)}{I_0(k)}\right\} \leq 60 \text{ dB}, \quad n = \left(\frac{f_{sr}}{2}-f_c\right)f_m \quad (12)$$

with f_{sr} being the sampling rate.

In practice, instrument designers will only need to be concerned with avoiding aliasing on high combined values of f_c and f_m, such as with high fundamental frequencies. However, in some applications, such as oscillators for band-limited classic waveforms, maximizing k is desirable [11]. A plot of the threshold suggested by Eq. (12) is shown in Fig. 4 for $f_c{:}f_m = 1$ and a selected range of fundamental frequencies.

1.4 Some Comparisons with Simple FM

Implementing Chowning's classic FM recipes [1] with ModFM allows us to evaluate the advantages of the latter. The basic ModFM design is shown in Fig. 5. It employs two oscillators and an exponential waveshaper. The ModFM synthesis of brassy tones provides a much more natural attack, whereas the steady state has a similar overall timbral aspect as classic FM. Similarly we noted that woodwind tones improved significantly where their
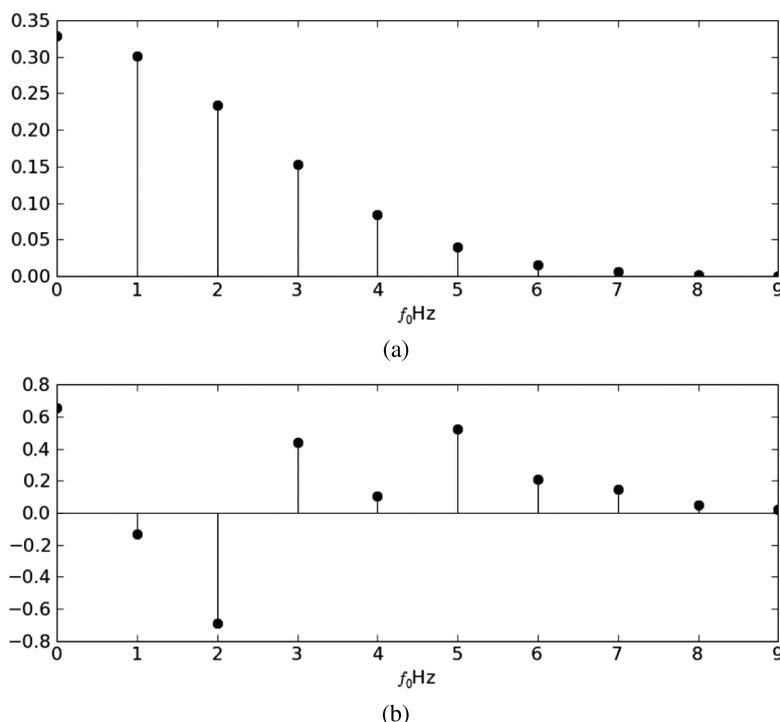

Fig. 3. Comparison of ModFM and classic FM spectra for $k = 5$ and $f_c{:}f_m = 1$ ($f_0 = f_c = f_m$). Negative values indicate inverted cosine phase. (a) ModFM, Eq. (11). (b) Classic FM, Eq. (1).

spectra change. In particular, ModFM emulation of a bassoon tone using Chowning's parameters is quite successful. In addition it was found that we will require a ModFM distortion index that is 50% higher than the one used for classic FM to match its steady-state brightness. This is due to the sharper rolloff observed in the modified Bessel-scaled spectral envelope.

Nonetheless it is in the bell tone where we find the most dramatic improvement. Classic FM generates a very synthetic sounding result, mainly due to the oscillation in the amplitude of its partials during the decay period. This is clearly seen in the spectrogram plots of Fig. 6. ModFM performs much better, providing a natural and orderly decay in spectral energy. These simple examples demonstrate the technique's potential for instrument design. Sound examples demonstrating the comparisons discussed here can be found online at http://music.nuim.ie/synthesis#modfm.

2 PHASE-SYNCHRONOUS MODFM

For certain applications, such as formant [13] and resonance synthesis, it is useful to implement the ModFM in a phase-synchronous manner. In this case we will lock the carrier and modulator phases by employing strictly integral f_c:f_m ratios (and using a single-phase generator for table lookup). This will have the effect of synchronizing the phases of the carrier and modulator waveforms.

The relationship between the carrier, modulator, and formant frequencies is expressed by

$$f_c = nf_m = \text{int}\left(\frac{f_f}{f_m}\right)f_m \tag{13}$$

where f_f is the center of the resonance (or formant) region we want to create. The ModFM spectrum will now be strictly harmonic. It is, however, possible to reintroduce

inharmonicity by means of a frequency shift, as discussed in [20]. This allows us to keep f_c:f_m locked to an integral ratio, facilitating the control of formants, while at the same time providing the possibility of inharmonic spectra. In addition, if we want to modify the resonance frequency smoothly, it will be necessary to use two carriers. This is because f_f is only equal to f_c as a special case. The second carrier, tuned to an adjacent harmonic (of f_m), will allow the formant to be centered anywhere between the frequencies of these two carriers, as their output is cross-faded for this effect.

The phase synchronous ModFM expression is thus defined as

$$\begin{aligned}
s(t) &= e^{k\cos(\omega_0 t)-k}\{(1-a)\cos(n\omega_0 t + \omega_s t) \\
&\quad + a\cos[(n+1)\omega_0 t + \omega_s t]\} \\
&= \frac{1}{e^k}\sum_{n=-\infty}^{\infty} I_m(k)\{(1-a)\cos[(n+m)\omega_0 t + \omega_s t] \\
&\quad + a\cos[(n+1+m)\omega_0 t + \omega_s t]\}
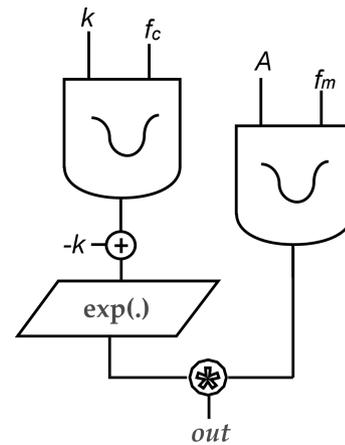\end{aligned} \tag{14}$$

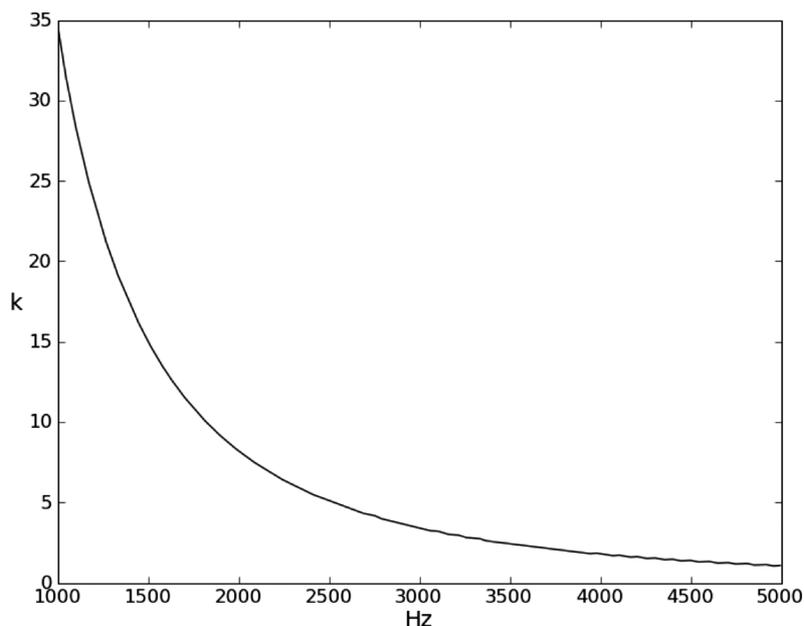Fig. 5. Basic ModFM synthesis flowchart.

Fig. 4. Maximum values of index of modulation k to avoid aliasing, as defined by Eq. (12) for $f_c = f_m$ in the range of 1–5 kHz; sampling rate $f_{sr} = 44\ 100$ Hz.

with $n = \mathrm{int}(f_f/f_m)$, $f_0 = f_m$, $\omega_0 = 2\pi f_0$, and $\omega_s = 2\pi f_s$, the shift frequency, and where

$$a = \frac{f_f}{f_m} - n.$$

The cross-fade factor a will place the center of resonance between the two carrier frequencies, as it is defined as the fractional part of the $f_f{:}f_m$ ratio. If this is zero, then the carrier frequency (disregarding the frequency shift) is an integer multiple of f_m and only one carrier is used.
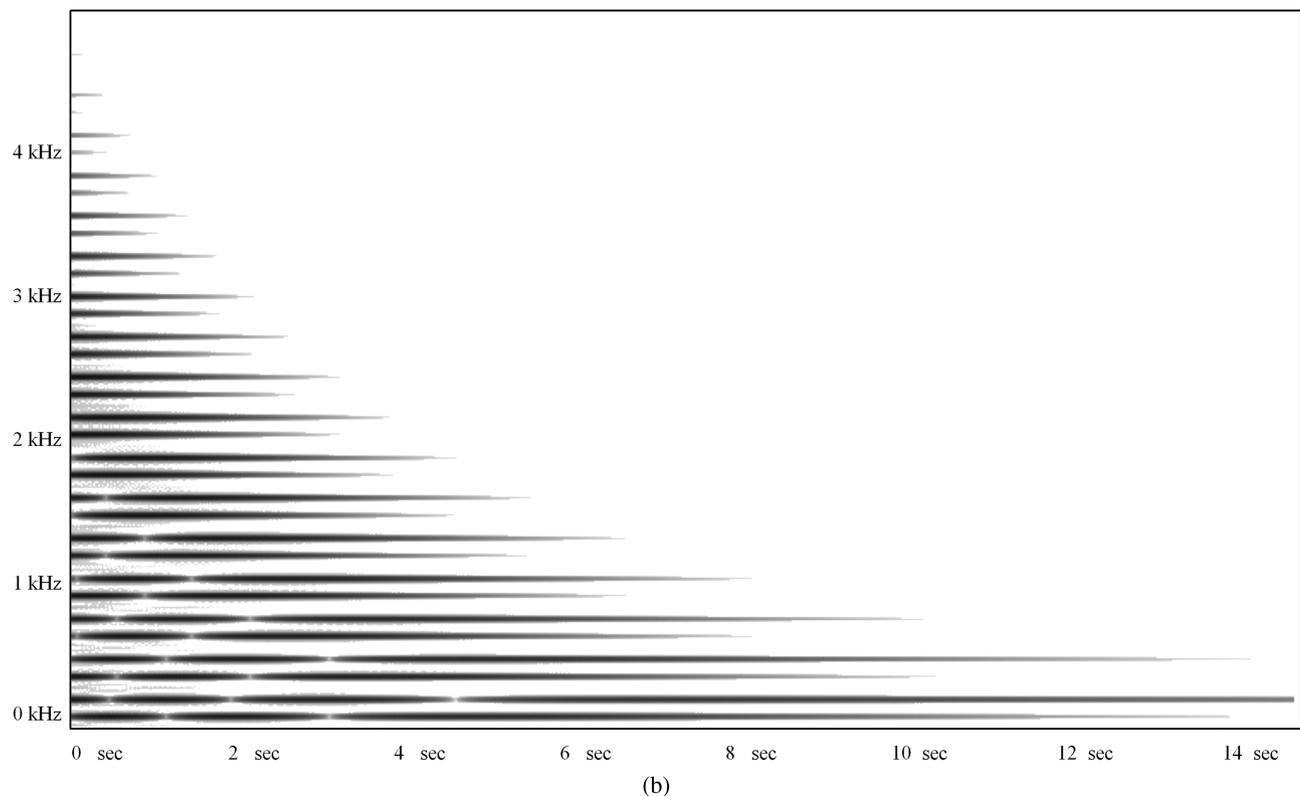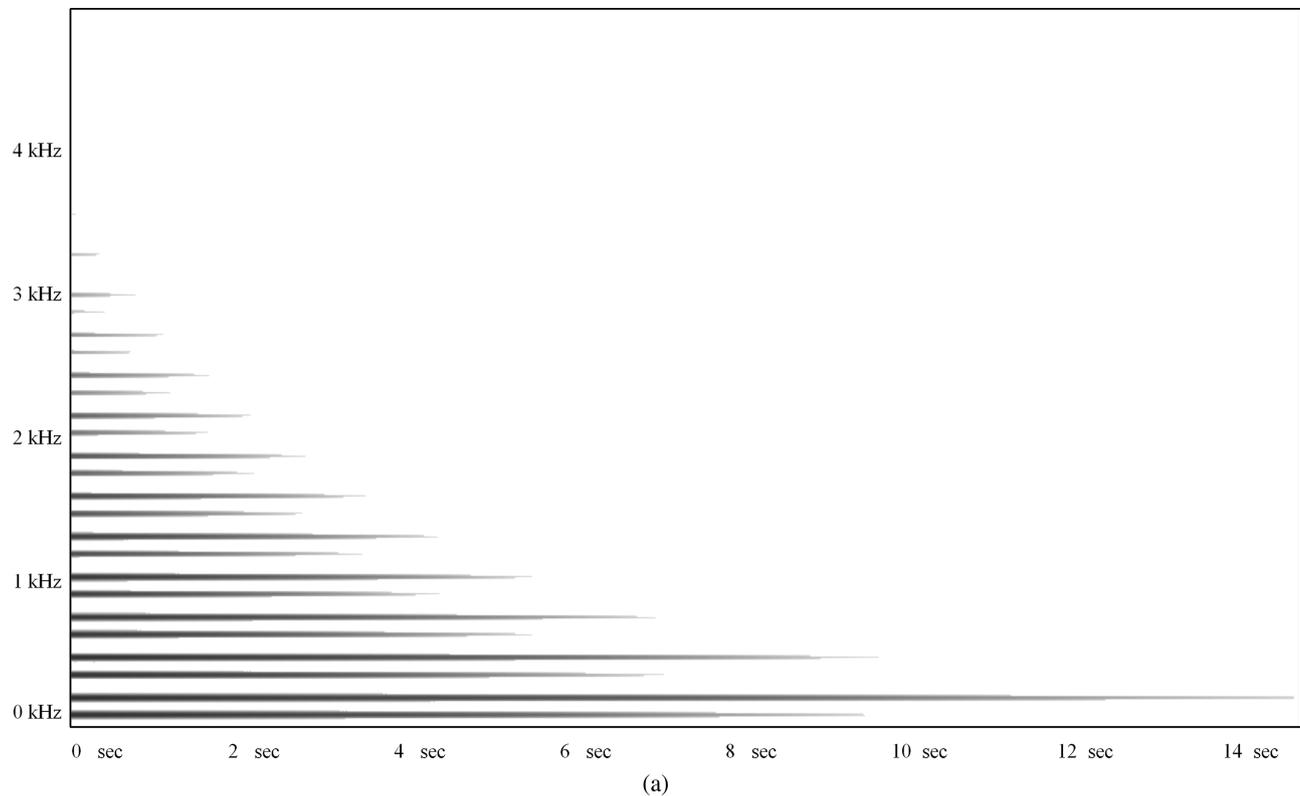

Fig. 6. Spectrograms of bell tones using Chowning's recipe. (a) ModFM. (b) Classic FM. The latter shows oscillating-amplitude partials (note discontinuous lines), whereas the former has perfectly decaying ones.

Otherwise we will mix some of the second carrier signal, which will shift the formant frequency upward toward it.

2.1 Bandwidth Control

An important concern in the synthesis of resonance is bandwidth control. In ModFM this is determined by the index of modulation k, which regulates spectral richness. In order to approximate the -3-dB bandwidth, we first observe some similarities between ModFM and phase-aligned formant (PAF) synthesis [20] for low distortion indexes (using $(1 + x^2)^{-1} \approx e^{(-x^2)}$, for $x \ll 1$, and the identity $\sin^2(\theta) = 2^{-1}[1 - \cos(2\theta)]$),

$$\frac{1}{1 + \frac{4g}{(1-g)^2}\sin^2\left(\frac{\omega_m t}{2}\right)} \approx \exp\left\{\frac{2g}{(1-g)^2}[\cos(\omega_m t) - 1]\right\} \quad (15)$$

where g is a PAF parameter dependent on -3-dB bandwidth B and fundamental frequency f_0 $(= f_m)$,

$$g = 2^{-f_0/B}. \quad (16)$$

The right-hand side of Eq. (15) shows the PAF modulator signal, which can be approximated to an exponential expression characterizing a ModFM modulator signal. Thus we can approximate k as

$$k \approx \frac{2g}{(1-g)^2} \quad (17)$$

Now with Eq. (16) we have a means of finding an approximate value for k, according to B and f_0, by heuristically adapting Eq. (17). Experiments have indicated that the value of g in Eq. (16) can be set to

$$g = 2^{-f_0/0.29B}. \quad (18)$$

Fig. 7 shows a comparative plot of the spectral envelopes of ModFM and PAF, whose parameters where calculated using eqs. (18) and (17), respectively. It is clear from this plot that the -20-dB bandwidth of PAF is wider than ModFM. The narrower and more selective nature of the latter can be an advantage when trying to model well-defined resonance regions of target spectra.

The phase synchronization is effected by employing a single-modulo counter (or phasor) to look up sinusoidal function tables for both carriers and modulator. The signal flowchart for the ModFM formant operator is shown in Fig. 8. Examples of phase-synchronous sounds for formant

synthesis can be found online at http://music.nuim.ie/synthesis#formant.

3 EXTENSIONS

The basic ModFM expression can be extended to allow some variations in the symmetry of its output spectrum. By incorporating an extra frequency (phase) modulation term and two extra parameters r and s (with ranges $0 <= r <= 1$ and $-1 <= s <= 1$), we have, excluding the normalization factor e^{-rk},

$$x(t) = e^{rk\cos(\omega_m t)}\cos[\omega_c t + sk\sin(\omega_m t)]$$

$$= \sum_{a=-\infty}^{\infty} I_a(rk)\cos(a\omega_m t)\sum_{b=-\infty}^{\infty} J_b(sk)\cos(\omega_c t + b\omega_m t)$$

$$= \frac{1}{2}\sum_{a=-\infty}^{\infty}\sum_{b=-\infty}^{\infty} I_a(rk)J_b(sk)\{\cos[\omega_c t + (a+b)\omega_m t]$$

$$+ \cos[\omega_c t - (a-b)\omega_m t]\}. \quad (19)$$

This equation appears at first to have a complicated expansion, involving both ordinary and modified Bessel
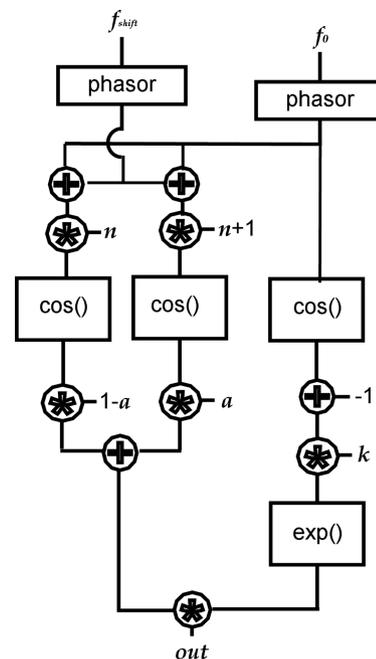
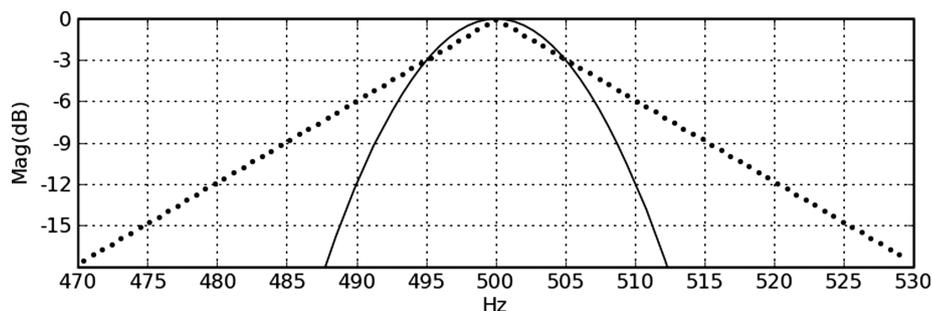Fig. 8. Phase-synchronous ModFM flowchart using two carriers and frequency shift.

Fig. 7. Comparative plots of spectral envelopes of ModFM (——) and PAF (. . .), with a 500-Hz formant center frequency and a 10-Hz, -3-dB bandwidth.

functions. However, in the extreme ranges of its r and s parameters it is very easily defined. With $s = 0$ and $r = 1$ we have the original ModFM expression [see Eq. (11)]. If $r = 0$ and $s \neq 0$, we have classic FM synthesis. If $r = s = 1$, the expression becomes

$$e^{k \cos(\omega_m t)} \cos[\omega_c t + k \sin(\omega_m t)]$$

$$= e^{k \cos(\omega_m t)} \{ \cos[k \sin(\omega_m t)] \cos(\omega_c t)$$

$$- \sin[k \sin(\omega_m t)] \sin(\omega_c t) \}$$

$$= \frac{e^{k(\cos \omega_m t + i \sin \omega_m t)} + e^{k(\cos \omega_m t - i \sin \omega_m t)}}{2} \cos(\omega_c t)$$

$$- \frac{e^{k(\cos \omega_m t + i \sin \omega_m t)} - e^{k(\cos \omega_m t - i \sin \omega_m t)}}{2i} \sin(\omega_c t)$$

$$= \sum_{n=0}^{\infty} \frac{k^n}{2n!} \{ [(\cos \omega_m t + i \sin \omega_m t)^n$$

$$+ (\cos \omega_m t - i \sin \omega_m t)^n] \cos(\omega_c t)$$

$$- i^{-1} [(\cos \omega_m t + i \sin \omega_m t)^n$$

$$- (\cos \omega_m t - i \sin \omega_m t)^n] \sin(\omega_c t) \}$$

$$= \sum_{n=0}^{\infty} \frac{k^n}{n!} [\cos(n\omega_m t) \cos(\omega_c t) - \sin(n\omega_m t) \sin(\omega_c t)]$$

$$= \sum_{n=0}^{\infty} \frac{k^n}{n!} \cos(\omega_c t + n\omega_m t). \tag{20}$$

This is actually an interesting case, first noted by Moorer [8] and then by Le Brun [3] (although neither provided a full proof or derivation), where we have a single-sided spectrum that is not directly scaled by Bessel functions of any form. The resulting spectrum for $f_c = 5$ kHz, $f_m = 1$ kHz, and $k = 5$ is shown in Fig. 9. No sidebands are produced below the carrier frequency.

It is also worth noting that here the spectral envelope peaks at sideband $n = k$, but its shape is close to that of the original ModFM ($s = 0$ and $r = 1$). The scaling functions of Eq. (18) approximate very closely modified Bessels
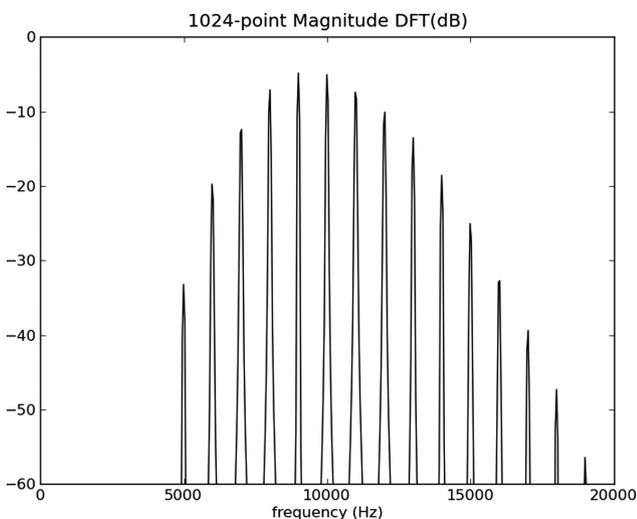
shifted in frequency. Fig. 10 demonstrates this similarity by plotting the spectral envelopes obtained with $I_{n-k}(k)$, that is, $I_n(k)$ shifted to peak at $n = k$ for integral values of n, and $k^n(n!)^{-1}$ in the extended ModFM formula (with $k = 20$). This actually gives us the useful result

$$I_n(k) \approx \frac{k^{n+k}}{\Gamma(n + k - 1)!} \tag{21}$$

which reintroduces modified Bessel functions into the extended ModFM equation. $\Gamma(\cdot)$ is the Gamma function.

The error in this approximation, which was observed experimentally, is inversely proportional to k, so it improves with higher indexes (namely, more partials in the spectrum). The error also decreases with the Bessel function order n (Fig. 11).

The final case, $s = -r = -1$, provides also a similarly single-sided spectrum, now at the lower side of the carrier frequency. Transitions between these four different spectral shapes can be effected by changing the values of s and
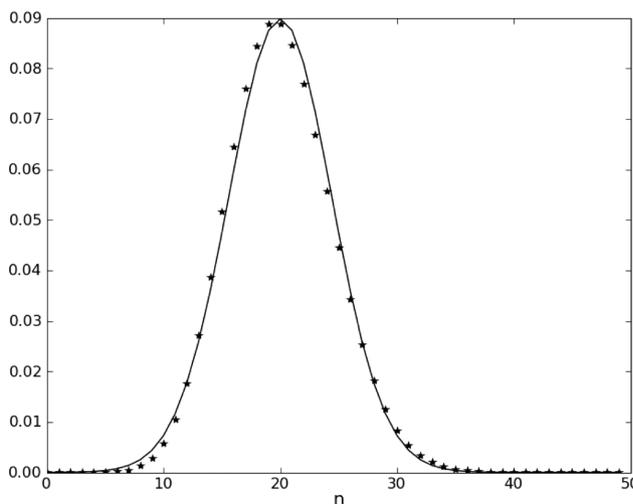

Fig. 10. Comparison of extended ModFM spectral envelopes for sideband numbers $n = 0$ through 50, $I_{n-k}(k)$ (——), and $k^n(n!)^{-1}$ (*), with $k = 20$. Note that in this case f_c corresponds to $n = 0$.
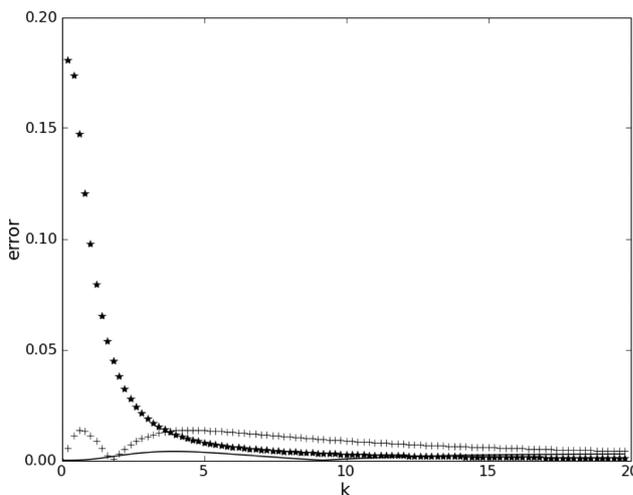

Fig. 9. Spectrum realized by means of Eq. (19), normalized using e^{-k}, with $s = r = 1$, $f_c = 5$ kHz, $f_m = 1$ kHz, and $k = 5$.

Fig. 11. Approximation error in Eq. (21) for $n = 0$ (*), 2 (+), and 5 (——) plotted against k.

r within their assigned ranges (Fig. 12). Fig. 13 shows eight different spectra for intermediary values of r and s. In the spectrogram of Fig. 14 we can see the effect of varying s, while keeping r constant, as changes in the spectral envelope peak, much like a bandpass filter whose frequency is swept.

Finally Palamin et al. [10] have also discussed a different method of altering the symmetry of the FM spectrum. Their second algorithm is effectively another extension to ModFM, with a single parameter ρ controlling the spectral shape. The formula, cast here using a cosine carrier to keep in line with our definition of ModFM, is

$$x(t) = e^{0.5(\rho+1/\rho)k\cos(\omega_m)} \cos\left[\omega_c + 0.5\left(\rho - \frac{1}{\rho}\right)k\sin(\omega_m)\right]$$

$$= \sum_{n=-\infty}^{\infty} \rho^n I_n(k) \cos(\omega_c + k\omega_m). \qquad (22)$$

Their method does not allow a direct transition from ModFM to FM or a completely single-sided spectrum. Nevertheless it constitutes another interesting extension to the basic algorithm. Equating $r = 0.5(\rho + 1/\rho)$ and $s = 0.5(\rho - 1/\rho)$ can also provide a means of describing the spectrum for some of the intermediary values of r and s that might lead to an otherwise complicated expansion. However, it is important to point out that in this description of r and s these variables are no longer independent. It is not possible in this case to keep $r = 1$ while s is varied from -1 to 1.

The extensions to the basic technique, in particular Eq. (19), add some very useful features to it. This is achieved at a minimal cost, namely, the use of another oscillator (or simply an extra table lookup, as we could use the same phase generator for both modulators) and two extra multiplications. It is therefore worth considering it as an integral part of a full ModFM implementation.

4 ADAPTIVE MODFM

By virtue of its heterodyne structure, the ModFM algorithm can be easily implemented for adaptive applications, as discussed for other forms of FM and distortion
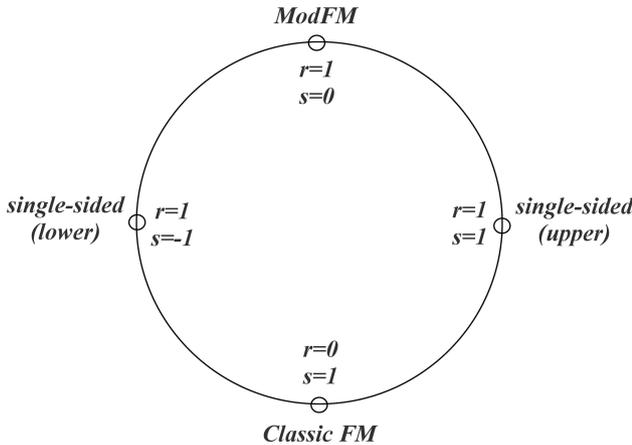
Fig. 12. Parameters r and s of extended ModFM synthesis and transition between its four spectral shapes.
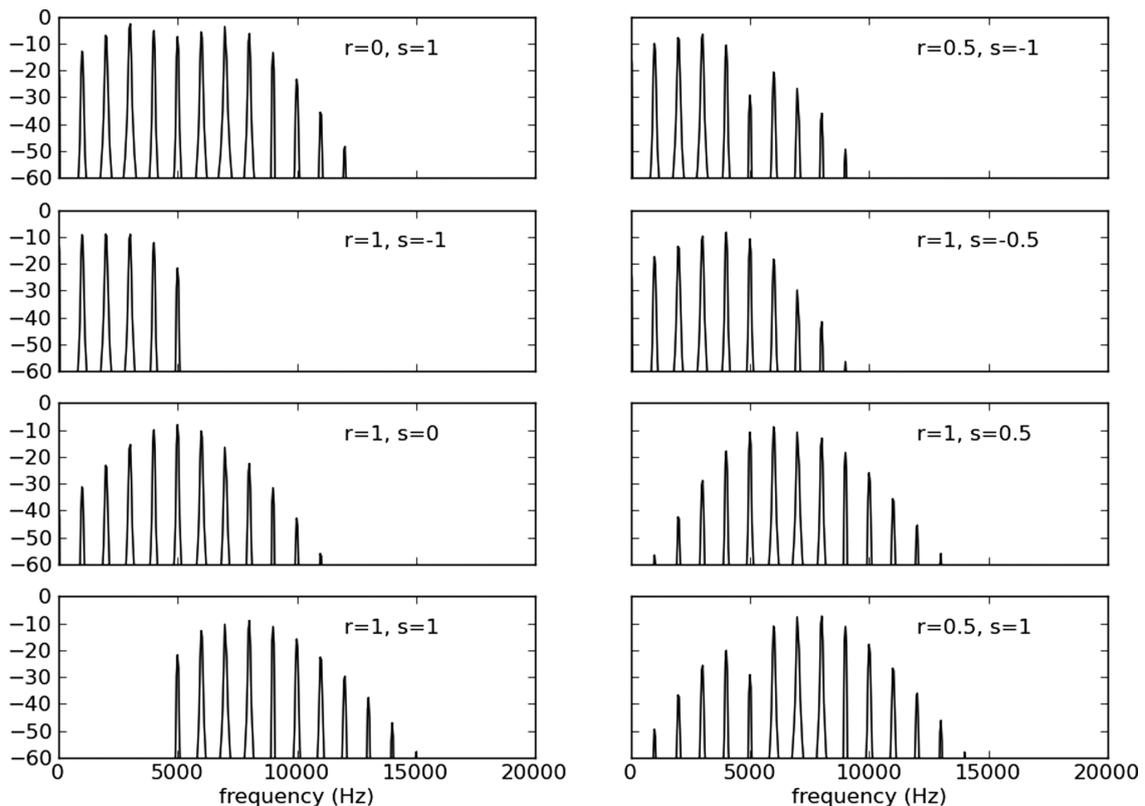
Fig. 13. Eight different spectra with various values of r and s, demonstrating different types of extended ModFM output. $f_c = 5$ kHz, $f_m = 1$ kHz, and $k = 3$.

synthesis in [14]–[16], [21]. Specifically, by adaptive it is meant that extracted features of an input audio signal are used to drive parameters that will be acting to transform this input signal [22].

The design of an adaptive ModFM instrument involves the substitution of the cosine wave carrier oscillator term $\cos(\omega_c t)$ in Eq. (11) by an arbitrary input signal whose fundamental frequency is tracked and used as the carrier frequency f_c to control the modulation frequency ω_m ($=2\pi f_m$) according to the required $f_c{:}f_m$,

$$s_{\text{output}}(t) = A e^{k\cos(\omega_m t)-k} s_{\text{input}}(t). \tag{23}$$

The timbral character of this effect is related to other adaptive FM implementations, with some special characteristics due to the spectral qualities of ModFM discussed in Section 1. Pitch tracking can be realized using any good-quality method, such as the one used in adaptive FM (AdFM)[14], [15].

In particular we observed that integral values of $f_c{:}f_m$ above 1 offer some good-quality octave transposition effects. What we observe in the output spectrum is that each harmonic will generate a small formant region around it. The resulting sound will have a distinct timbral signature. As an example, in Fig. 15 we plot the spectra of
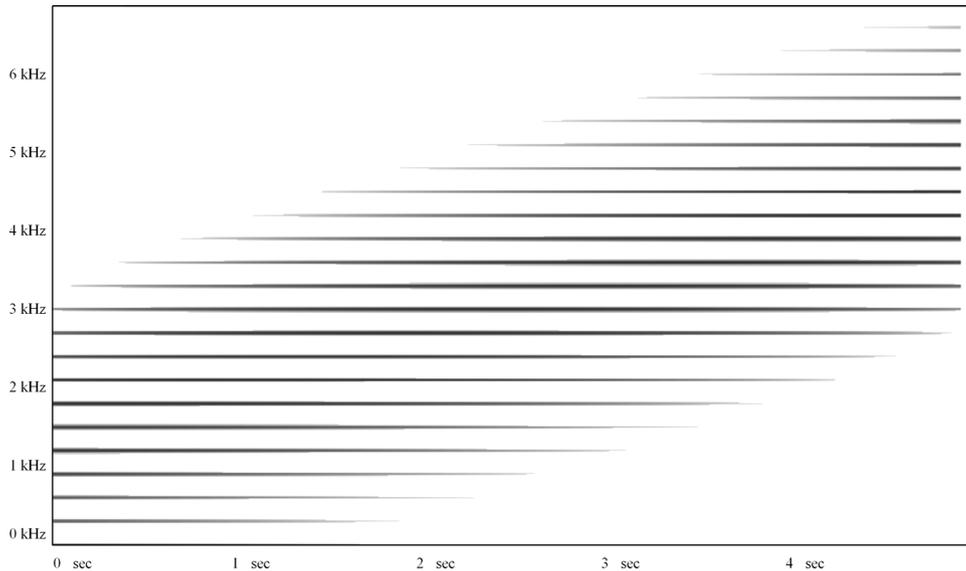

Fig. 14. Spectrogram of extended ModFM with Symmetry parameter s varying from -1 to 1. $f_c = 3$ kHz, $f_m = 300$ Hz, $k = 5$, and $r = 1$.
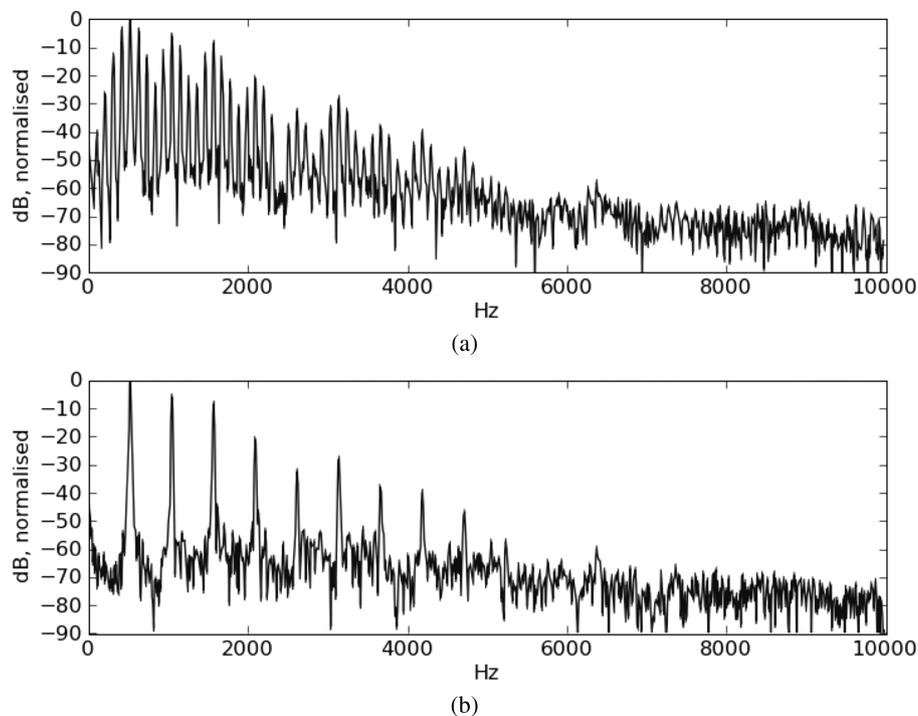

Fig. 15. Steady-state spectra of adaptive ModFM processing. (a) 440-Hz flute tone. (b) Original signal. Synthesis parameters were $f_c{:}f_m = 5$, $k = 2$, $f_c = 440$ Hz, $f_m = 88$ Hz.

flute sound and its transformation using adaptive ModFM (f_c:f_m = 5, k = 2).

Finally the implementation of the extended algorithm [derived from Eq. (19)] should provide some interesting effects. Here the phase modulation term $\cos[\omega_c t + sk \sin(\omega_m t)]$ is substituted by a modulated variable-delay line fed with an arbitrary input signal. The delay line modulator is a sinusoid, tuned to f_m, which provides the equivalent phase modulation effect, as discussed in [14], [15]. A flowchart for this algorithm is shown in
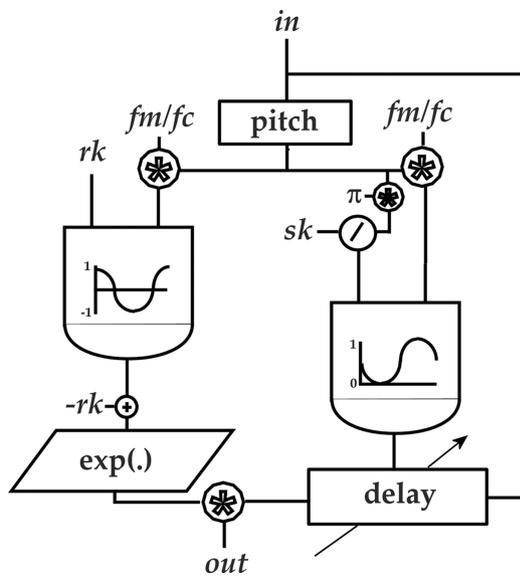

Fig. 16. Extended adaptive ModFM synthesis flowchart.

Fig. 16. The phases of the two modulators have to be controlled strictly. Otherwise the implementation will not meet the conditions of Eq. (19). The exponential waveshaper is driven by a cosine wave, whereas the variable-delay line needs to be modulated by a raised inverted sine wave.

Since with this algorithm we can move from the original AdFM [14] to adaptive ModFM and single-sided FM [8], this is probably the most definitive form of the technique. In particular the use of the symmetry parameter s should prove very useful, as by modulating it we could implement some interesting filterlike effects. The implementation of this algorithm follows the form discussed in [14], where a variable-delay line is used to achieve the desired frequency modulation effect.

Fig. 17 shows the spectrogram of a flute tone processed with the extended algorithm, whose s parameter [Eq. (19) and Fig. 16] is modulated by a sine wave at 1 Hz with a peak current of 1 A. The flanger-like effect is clearly seen as the distribution of energy in the spectrum varies. Sound examples of adaptive ModFM can be found online at http://music.nuim.ie/synthesis#adaptive.

5 CONCLUSIONS AND FURTHER PROSPECTS

In this paper we have sketched the theoretical basis for ModFM synthesis. We have contrasted it with classic FM and examined its main characteristics. Some general points of instrument design were made and we have provided a description of its basic algorithm. While ModFM is not thought to be a substitute for the various
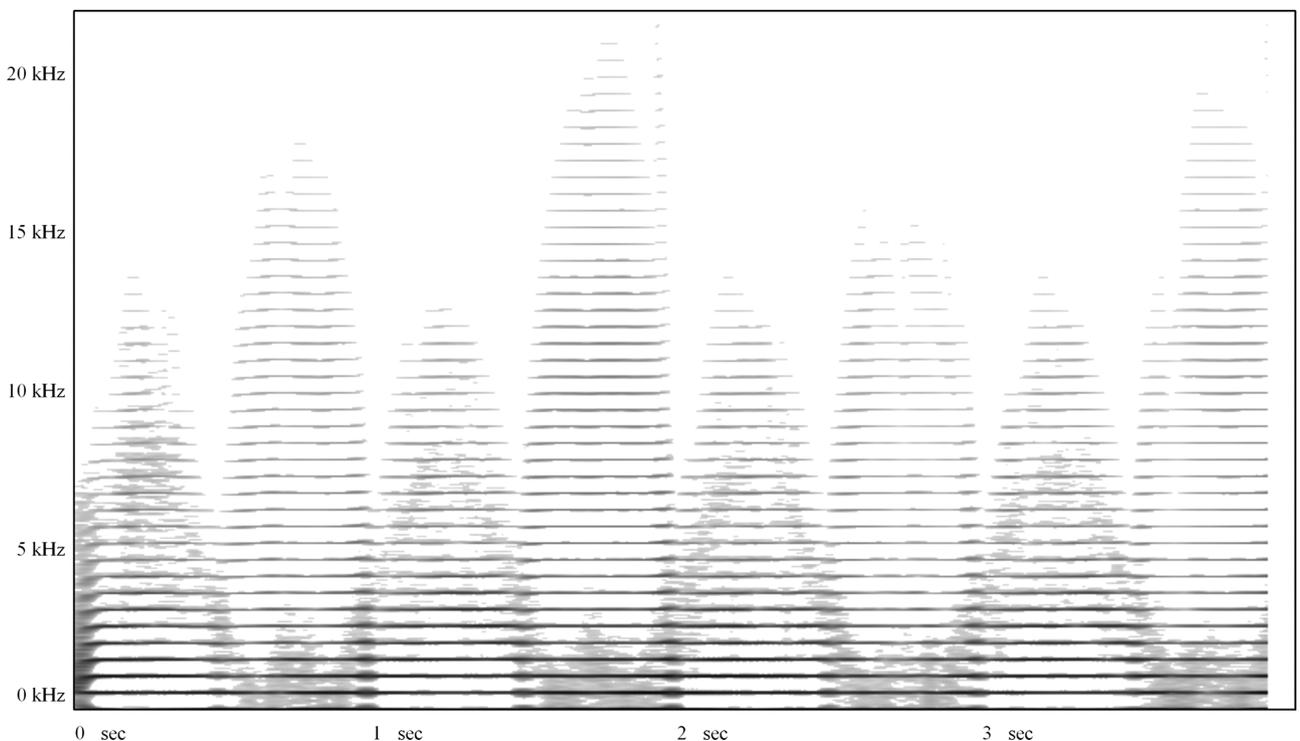

Fig. 17. Spectrogram of 440-Hz flute tone processed with extended adaptive ModFM. Symmetry parameter s is fully modulated by a 1-Hz sine wave. f_c = f_m = 440 Hz, k = 2, and r = 1.

applications of FM, we have shown that it can offer an alternative in some cases where FM does not provide the desired effect.

Variations on the original formula were also discussed, starting with a phase-synchronous version and including extensions to the method. The former allows for the synthesis of formants and resonant tones, whereas the latter form a complete set of FM-related synthesis methods, described by a single equation. The extended ModFM principles provide means for synthesizing transitions between various spectral configurations that can have a number of musical applications.

Complementing the discussion, we have looked at some adaptive ModFM applications. A version of the extended ModFM algorithm was provided for the processing of arbitrary input signals, which works in a similar manner as other methods discussed in the literature, such as adaptive FM.

Further prospects include the exploration of parametric analysis–synthesis models such as the one in [13], where an analysis–synthesis method was developed. In addition we observe the possibility for the application of spectral matching principles[17], [23], originally developed for classic FM synthesis. Given the characteristics of the ModFM spectrum outlined in Section 1, the method appears to offer good potential for these applications.

6 ACKNOWLEDGMENT

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