



Non-relativistic spectrum of two-color QCD at non-zero baryon density

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ABSTRACT

The heavy quarkonium spectrum of Two Color QCD (QC₂D) at non-zero quark chemical potential μ and temperature T with $\mu/T \gg 1$ has been calculated in both S - and P -wave channels using a lattice non-relativistic formulation of QC₂D. As μ is varied, the quarkonium spectra reveal three separate regions, corroborating previous findings that there are three distinct physical regimes of QC₂D at low temperature and high baryon density: hadronic matter, quark/quarkyonic matter, and deconfined matter. The results are interpreted in terms of the formation of heavy-light Qq states in the two-color baryonic medium.

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1. Introduction

Cold dense baryonic matter in Quantum ChromoDynamics (QCD) is difficult to study theoretically because the lattice gauge theory method allowing a first principles investigation of non-perturbative physics does not work due to the “complex action problem”: the introduction of a quark chemical potential $\mu \neq 0$ in the Euclidean formulation of QCD makes its action complex, and importance sampling used for Monte-Carlo evaluation of the partition function is difficult if not impossible to implement.

In previous work [1–3] we presented non-perturbative results at non-zero density for QC₂D, a QCD-like theory with two colors, using orthodox lattice simulations. Since the gauge group is SU(2) and the quark representation pseudoreal, the functional measure $\det^{N_f} \mathcal{M}(\mu)$ remains real even once $\mu \neq 0$, and positivity can be ensured by insisting that the number of flavors N_f is even. In QC₂D baryons are bosonic, and hadron multiplets, including Goldstone boson states associated with the breaking of global symmetries, contain both $q\bar{q}$ mesons and qq baryons. Despite these clear differences from physical QCD, QC₂D has an unexpectedly rich structure as μ is increased, and we expect many lessons

learned here about issues such as deconfinement, chiral symmetry restoration, and exotic ground states, may be more widely applicable [4].

In brief, we have found three regimes with distinct behaviours beyond the onset at $\mu = \mu_0 \equiv m_\pi/2$ where baryonic matter is first induced into the ground state at zero temperature. For $\mu_0 \leq \mu \leq \mu_Q$ the matter consists of tightly bound qq scalars, which form a Bose–Einstein Condensate (BEC). Since the scalar diquarks are Goldstone modes associated with the spontaneous breaking of global baryon number conservation leading to superfluidity, thermodynamics in this regime is well described by an effective approach based on chiral perturbation theory [5]. However, at the larger densities found in the range $\mu_Q \leq \mu \leq \mu_D$, we find the thermodynamic properties of the system to be more like those of a degenerate system of quarks having a well-defined Fermi surface; superfluidity persists, but is now understood as resulting from a Bardeen–Cooper–Schrieffer (BCS) mechanism involving the condensation of weakly-bound diquark pairs at opposite points on the surface. Finally, for the largest densities $\mu \geq \mu_D$ we find color deconfinement via the classic signal of the non-vanishing expectation of the Polyakov loop. With the simulation parameters outlined in Section 2 below we estimate $\mu_0 \simeq 360$ MeV, $\mu_Q \simeq 530$ MeV, and $\mu_D \simeq 850$ MeV [2]. The regime $\mu_Q \leq \mu \leq \mu_D$ is particularly interesting, since it resembles “quark matter” while remaining color-confined; as such it is reminiscent of the “quarkyonic” phase first discussed in [6].

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In this work, we study QC₂D with both $\mu \neq 0$ and temperature $T > 0$ using correlation functions associated with heavy quarkonium $Q\bar{Q}$ states, which offer a sophisticated probe of medium effects. The behaviour of quarkonia for $T > 0$ in QCD is suggested as one of the signatures for quark-gluon plasma formation [7]. Since the energy required to create a $Q - \bar{Q}$ pair is much larger than both Λ_{QCD} and the average thermal energy, the $Q - \bar{Q}$ production rate is dominated by short distance physics which is T -independent. Heavy quarkonium is a low energy $Q\bar{Q}$ bound state, and the thermal medium found in, say, heavy-ion collisions can influence quarkonium formation. In [7], the authors modelled the thermal medium using a screening potential $V_{Q\bar{Q}}$ and showed the “melting” of heavy quarkonium (i.e., non-existence of $Q\bar{Q}$ bound states above a certain temperature) by solving the resulting Schrödinger equation. Evidence for suppression of excited bottomonium states at $T > T_c$ in lattice simulations is given in [8,9].

It is important to stress that in this context the heavy quarks are regarded as test particles and are not in thermal equilibrium. In the current study this implies that Q, \bar{Q} do *not* couple directly to the chemical potential μ . To treat the heavy quarks we will use an effective approach known as non-relativistic QCD (NRQCD); in QCD this is usually applied to the $b\bar{b}$ system (the Υ and χ_b meson families) where there is a well-defined hierarchy of scales. For heavy quark mass M and velocity v , for NRQCD to be applicable we require [10]

$$M \gg p \sim r^{-1} \sim Mv \gg \Delta E \sim Mv^2, \quad (1)$$

where ΔE is the quarkonium “binding energy” and r is the typical distance between the quark and the anti-quark. For Υ systems $v^2 \sim 0.1$. In Section 2 we briefly review the lattice approach to QC₂D with $\mu \neq 0$ and outline the formulation of NRQC₂D. Our main results, for spin-singlet and spin-triplet states in both S - and P -waves are presented in Section 3, and a discussion of the observed μ - and T -variation follows in Section 4.

2. Formulation

We investigate the heavy quarkonium spectrum at non-zero temperature and baryon density by calculating $\mathcal{O}(v^4)$ non-relativistic QC₂D correlators using background lattice gauge field configurations generated on $16^3 \times 12$, $12^3 \times 16$, and $12^3 \times 24$ lattices, at $\beta = 1.9$, $\kappa = 0.168$ with two dynamical flavors of Wilson quark (see [1–3]). These parameters correspond to lattice spacing $a = 0.186(8)$ fm ($= 1/1.060(45)$ GeV⁻¹), $m_\pi a = 0.68(1)$ and $m_\pi/m_\rho = 0.80(1)$, where the scale is set by the string tension ($(440 \text{ MeV})^2$ at $\mu = 0$). The corresponding temperatures are $T = 44, 66$ and 88 MeV. The range of chemical potential studied is $0 \leq \mu a \leq 1.1$; in [2] at $\mu = 0.8a^{-1} \approx 850$ MeV a quark density $n_q = 16\text{--}32 \text{ fm}^{-3}$ was reported, corresponding to between 35 and 70 times matter density, where the uncertainty is due to discretisation artifacts.

A standard Hybrid Monte-Carlo algorithm was used to generate lattice configurations, where the action

$$S = \sum_{x,i} \bar{\psi}_i(x) \mathcal{M}_{x,y}(\mu) \psi_i(y) + \kappa j \sum_x [\psi_2^{\text{tr}}(x) (C\gamma_5)\tau_2\psi_1(x) - h.c.] \quad (2)$$

with

$$\mathcal{M}_{x,y} = \delta_{x,y} - \kappa \sum_\nu [(1 - \gamma_\nu) e^{\mu\delta_{\nu,0}} U_\nu(x) \delta_{y,x+\nu} + (1 + \gamma_\nu) e^{-\mu\delta_{\nu,0}} U_\nu^\dagger(y) \delta_{y,x-\nu}]. \quad (3)$$

The diquark source term proportional to j mitigates large infrared fluctuations in a superfluid phase with $\langle \psi_2^{\text{tr}}(C\gamma_5)\tau_2\psi_1 \rangle \neq 0$, and also helps ergodicity by enabling real eigenvalues of \mathcal{M} to traverse the origin. To assess the effect of the diquark source, configurations generated with two different magnitudes $j = 0.02$ and 0.04 were used. Details of the simulation algorithm and previous analyses of various bulk thermodynamic quantities are given in [1–3] and the temperature dependence of μ_D is discussed in [11,12].

We used the following $\mathcal{O}(v^4)$ Euclidean NRQC₂D lagrangian density for the heavy quark with mass M :

$$\mathcal{L}_Q = \mathcal{L}_0 + \delta\mathcal{L}, \quad (4)$$

with

$$\mathcal{L}_0 = \phi^\dagger \left(D_\tau - \frac{\mathbf{D}^2}{2M} \right) \phi + \chi^\dagger \left(D_\tau + \frac{\mathbf{D}^2}{2M} \right) \chi, \quad (5)$$

and

$$\begin{aligned} \delta\mathcal{L} = & -\frac{c_1}{8M^3} [\phi^\dagger (\mathbf{D}^2)^2 \phi - \chi^\dagger (\mathbf{D}^2)^2 \chi] \\ & + c_2 \frac{ig}{8M^2} [\phi^\dagger (\mathbf{D} \cdot \mathbf{E} - \mathbf{E} \cdot \mathbf{D}) \phi + \chi^\dagger (\mathbf{D} \cdot \mathbf{E} - \mathbf{E} \cdot \mathbf{D}) \chi] \\ & - c_3 \frac{g}{8M^2} [\phi^\dagger \boldsymbol{\sigma} \cdot (\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D}) \phi \\ & + \chi^\dagger \boldsymbol{\sigma} \cdot (\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D}) \chi] \\ & - c_4 \frac{g}{2M} [\phi^\dagger \boldsymbol{\sigma} \cdot \mathbf{B} \phi - \chi^\dagger \boldsymbol{\sigma} \cdot \mathbf{B} \chi]. \end{aligned} \quad (6)$$

It is similar to NRQCD [13] with the only difference that D_τ and \mathbf{D} are now gauge covariant temporal and spatial derivatives for SU(2) gauge theory. Here ϕ and χ are two-color two-spinor fields for the heavy quark and anti-quark, and the tree-level value for the c_i is 1.

We use the following discretised Green function of (4) to calculate the heavy quark Green function:

$$\begin{aligned} G(\mathbf{x}, \tau = 0) &= S(\mathbf{x}), \\ G(\mathbf{x}, \tau = a_\tau) &= \left(1 - \frac{H_0}{2n}\right)^n U_4^\dagger(\mathbf{x}, 0) \left(1 - \frac{H_0}{2n}\right)^n G(\mathbf{x}, 0), \\ G(\mathbf{x}, \tau + a_\tau) &= \left(1 - \frac{H_0}{2n}\right)^n U_4^\dagger(\mathbf{x}, \tau) \\ &\quad \times \left(1 - \frac{H_0}{2n}\right)^n (1 - \delta H_n) G(\mathbf{x}, \tau), \end{aligned} \quad (7)$$

where $S(\mathbf{x})$ is the source and the lowest-order hamiltonian reads (see (10) below)

$$H_0 = -\frac{\Delta^{(2)}}{2M}, \quad (8)$$

and

$$\begin{aligned} \delta H_n = & -\frac{(\Delta^{(2)})^2}{8M^3} + \frac{ig}{8M^2} (\boldsymbol{\Delta} \cdot \mathbf{E} - \mathbf{E} \cdot \boldsymbol{\Delta}) \\ & - \frac{g}{8M^2} \boldsymbol{\sigma} \cdot (\boldsymbol{\Delta} \times \mathbf{E} - \mathbf{E} \times \boldsymbol{\Delta}) \\ & - \frac{g}{2M} \boldsymbol{\sigma} \cdot \mathbf{B} + \frac{a^2 \Delta^{(4)}}{24M} - \frac{a(\Delta^{(2)})^2}{16nM^2}. \end{aligned} \quad (9)$$

This approach is used for heavy quarkonium spectroscopy in QCD at $T = 0$ in [14–16] and recently in QCD at $T > 0$ [8]. The integer n controls the high-momentum behaviour of the evolution equation. Since in QC₂D we do not have any phenomenologically compelling choice for the heavy quark mass, we choose $Ma \geq 3$ and $n = 1$, to ensure compatibility with (1). The last two terms in

δH are corrections to the kinetic energy term at non-zero lattice spacing [17]. The lattice covariant derivatives are defined by

$$\begin{aligned}\Delta_i \phi &= \frac{1}{2a} [U_i(x) \phi(x + \hat{i}) - U_i^\dagger(x - \hat{i}) \phi(x - \hat{i})], \\ \Delta^{(2)} \phi &= \sum_i \Delta_i^{(2)} \phi \\ &= \sum_i \frac{1}{a^2} [U_i(x) \phi(x + \hat{i}) - 2\phi(x) + U_i^\dagger(x - \hat{i}) \phi(x - \hat{i})], \\ \Delta^{(4)} \phi &= \sum_i (\Delta_i^{(2)})^2 \phi,\end{aligned}\quad (10)$$

and \mathbf{E} and \mathbf{B} in Eq. (9) are lattice cloverleaf definitions of the SU(2) chromoelectric and chromomagnetic fields. To mitigate quantum corrections we use tadpole improvement [18], replacing $U_\mu(x) \rightarrow u_0^{-1} U_\mu(x)$, where u_0 is the average link determined from the plaquette expectation value, and setting the coefficients c_i of (6) to 1. Note that in SU(2) gauge theory, heavy quarkonium ($Q\bar{Q}$) states are equivalent to heavy baryon states (QQ) [19]. Since the P -wave excitation needs an extended source, Coulomb gauge fixing is performed on SU(2) gauge fields prior to calculation of $G(\mathbf{x}, \tau)$. By combining non-relativistic heavy quark correlators, 1S_0 , 3S_1 , 1P_0 and 3P_1 heavy quarkonium states could be studied. The expressions for the interpolating operators for all the states are listed in [20].

3. Results

3.1. S -wave states

We found that correlators for the S -wave states 1S_0 and 3S_1 could be fitted with an exponential decay $\propto e^{-\Delta E_n \tau}$ corresponding to a simple pole even for large μ , as shown in Fig. 1. Even at the largest simulated value of $\mu a (= 1.10)$ we see no deviation from the simple exponential form which would indicate a melting of the ground state. Moreover the fits are quite stable over large ranges of τ . This suggests that S -wave quarkonium bound states persist throughout the region $0 \leq \mu a \leq 1.1$ and $\frac{1}{24} \leq Ta \leq \frac{1}{12}$.

In non-relativistic QCD, $M_n = 2(Z_M M - E_0) + \Delta E_n$ for the state n , where Z_M is the heavy quark mass renormalisation, E_0 is a state-independent additive renormalisation, and ΔE_n is the fitted energy of the state [17]. Usually, the experimental value for one of the heavy quarkonium masses is chosen to fix E_0 which is independent of n . This cannot be done in QC₂D due to the lack of experimental spectrum data. However, since introducing $\mu \neq 0$, $T > 0$ does not induce any new UV divergences, the change of the S -wave state energy from that at $\mu = 0$ or $T = 0$, which must reflect underlying physics, can be measured.

Fig. 2 shows the T - and μ -dependences of the 1S_0 state energy ΔE . The absolute values have unquantified contributions $E_0(M)$, which in principle could be subtracted by matching $\Delta E(\mu = 0)$, and as such contain little useful information. As argued above, however, the variation with μ is physical. As Ma increases, and hence higher order effects in v^2 become less important, Fig. 2 suggests three distinct regimes as μ is varied: initially the 1S_0 state energy decreases from that at $\mu = 0$, but once μ reaches the region $\mu_1 (\simeq 0.5) \leq \mu a \leq \mu_2 (\simeq 0.85)$, the 1S_0 state energy stays roughly constant. For $\mu > \mu_2$, the 1S_0 state energy starts increasing again. The variation with μ becomes more marked as the heavy quark mass M is decreased; this may possibly be associated with the increasing size of the quarkonium state. The existence of three distinct μ -regions in which the S -wave quarkonium state energies show markedly different behaviour and the agreement of

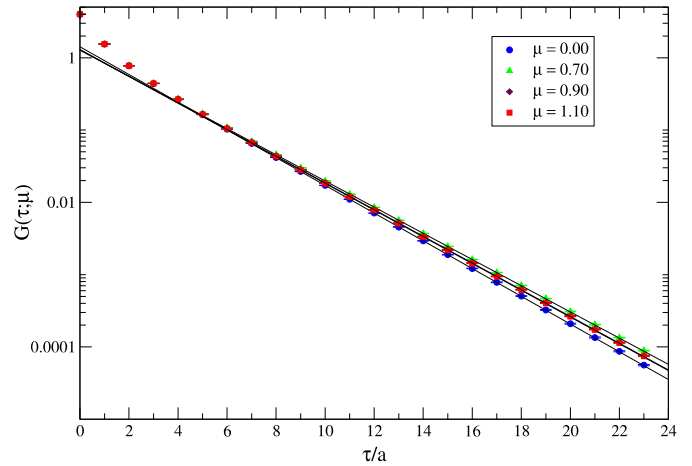


Fig. 1. Correlators for the 1S_0 state with heavy quark mass $Ma = 5.0$ and $j = 0.02$ on $12^3 \times 24$ lattice together with single exponential fit.

$\mu_{1,2}$ with the values μ_Q and μ_D found in [2] strengthens the argument for the existence of three different regimes described in Section 1 as the BEC phase, the BCS/quarkyonic phase and the deconfined phase.

The three temperatures shown in Fig. 2 all lie below the estimated deconfining transition temperature $T_{ca} \sim \frac{1}{6}$ at $\mu = 0$. As μ is increased, there is a deconfinement transition, signalled by the Polyakov loop increasing from zero, at $\mu_D(T)a \approx 0.3, 0.55, 0.75$ for $Ta = \frac{1}{12}, \frac{1}{16}, \frac{1}{24}$ respectively [11,12]. In accordance with this, we see that the 1S_0 energy starts increasing roughly at the deconfinement transition at $Ta = \frac{1}{24}, \frac{1}{16}$. However, no such behaviour is observed for the highest temperature. Interestingly, at $\mu a = 0.6$, $\Delta Ea = 0.4157(6), 0.4226(2), 0.4235(2)$ as Ta rises from $\frac{1}{24}$ to $\frac{1}{12}$. This positive shift is similar to that observed in the thermal mass of heavy quarkonium with increasing temperature in hot QCD with $\mu = 0$ [21,9].

The overall qualitative behaviour of the 3S_1 state energy is quite similar to that of the 1S_0 state energy. The 3S_1 state energy also shows the same three chemical potential regimes as the 1S_0 state energy: the 3S_1 state energy decreases until μ reaches μ_1 and then stays roughly constant until μ reaches μ_2 and then increases for $\mu \geq \mu_2$. Thus, instead of the absolute 3S_1 state energy, the hyperfine splitting $\Delta E_{3S_1} - \Delta E_{1S_0}$ is shown as a function of μ in Fig. 3. Only a weak μ -dependence in the splitting for three explored Ms is observed, at most roughly 10% of the magnitude of the effect seen in Fig. 2. This may not be surprising since even in NRQCD at $T = \mu = 0$, the hyperfine splitting is strongly affected by light quark dynamics and renormalisation effects [22], and the light quark mass in our simulation is relatively heavy ($m_\pi a = 0.68(1)$). Clearly, much further study is needed to isolate chemical potential/temperature effects in the $^3S_1 - ^1S_0$ splitting.

The influence of the diquark source term on the heavy quarkonium spectrum and its effect in the three distinct μ -regimes we identified need to be studied. In general, increasing j induces a larger superfluid condensation [1]. How this affects the heavy quarkonium spectrum is a highly non-trivial question. Thus, the spectrum calculation has been repeated with two different source magnitudes $j = 0.02$ and 0.04 . Fig. 4 shows the j -dependence of ΔE_{1S_0} and ΔE_{3S_1} . For both $j = 0.02$ and 0.04 , the S -wave state energies continue to manifest three separate regimes, but while for $\mu < \mu_1$, the S -wave state energies show little j -dependence, for $\mu \geq \mu_1$, the S -wave state energies for $j = 0.04$ are mostly larger than those for $j = 0.02$, signifying a larger diquark condensate effect in the quarkyonic and deconfined regions.

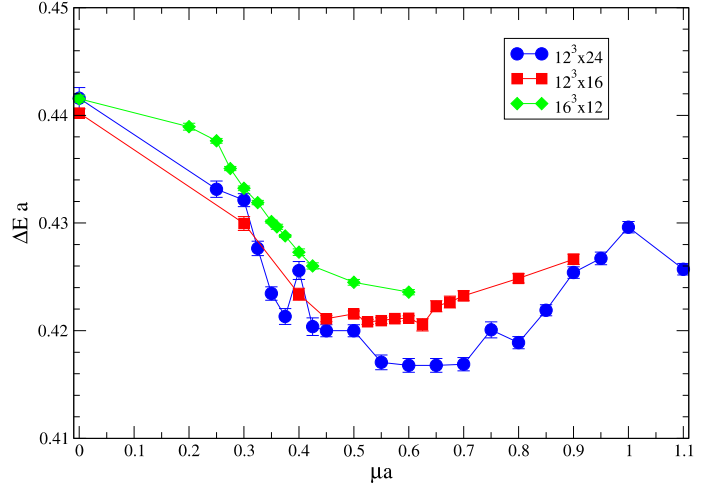
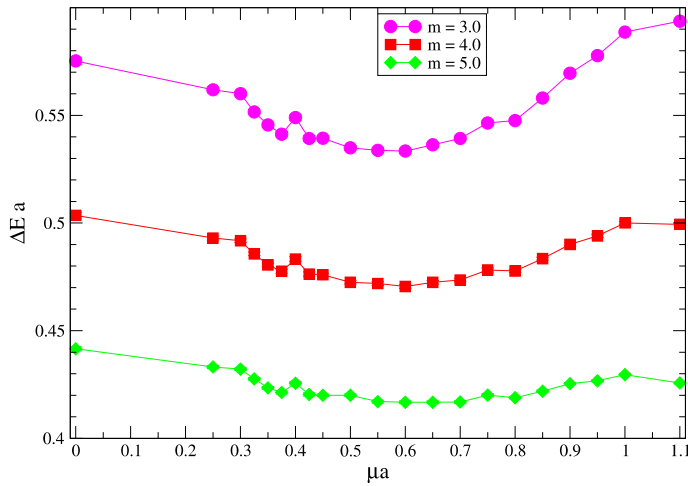


Fig. 2. (Left) Energy of the 1S_0 state vs. quark chemical potential μ for heavy quark mass $Ma = 3.0, 4.0$ and 5.0 with $j = 0.02$ on $12^3 \times 24$ lattice. (Right) Temperature dependence of the 1S_0 state energy vs. μ for $Ma = 5.0$ with $j = 0.04$.

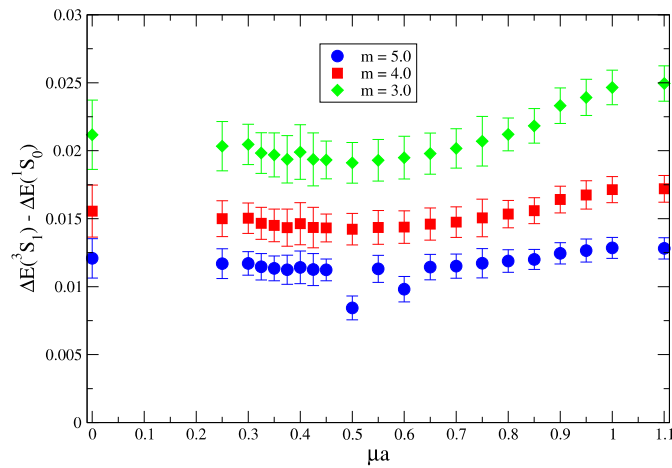


Fig. 3. The splitting between the 3S_1 state energy and 1S_0 state energy for three different M on $12^3 \times 24$.

3.2. P-wave states

In contrast to the S -wave states, it is difficult to find stable exponential fits to the P -wave correlators with the current Monte-Carlo data before statistical noise sets in, except for the case $\mu a \leq 0.25$. We therefore use a more primitive quantity: the ratios of the 1P_0 state correlators at several values $\mu \neq 0$ (chosen within the three different regimes) to the correlator at $\mu = 0$ are compared in Fig. 5. The other P -wave states (e.g., 3P_1 state) show similar behaviours. The corresponding ratios of the 1S_0 state correlators are shown to the right. Note that any effect we observe is entirely due to the dense medium.

The S -wave correlators may be represented as a sum of exponential functions $\sum_i A_i e^{-\Delta E_i \tau}$ with $\Delta E_1 < \Delta E_2 < \Delta E_3 \dots$ so that the large- τ behaviour is dominated by the lowest energy state ΔE_1 . The S -wave correlator ratios in Fig. 5 confirm this expectation. At large τ the ratio will be simply $R(\tau; \mu) = e^{-\delta(\mu)\tau}$ where $\delta(\mu) = \Delta E_1(\mu) - \Delta E_1(\mu = 0)$. In the BEC region ($\mu \leq \mu_1$) and the deconfined region ($\mu \geq \mu_2$), the ratio is $\sim 20\%$ at $\tau/a = 23$ and may be approximated as a straight line, $R(\tau; \mu) \approx 1 - \delta(\mu)\tau$, consistent with the small, negative 1S_0 state energy difference $\delta(\mu)$ that was previously observed. For the BCS region ($\mu_1 \geq \mu a \geq \mu_2$), the ratio is $\sim 60\%$ at $\tau/a = 23$ and may be approximated as an exponential function, which is consistent with the large energy

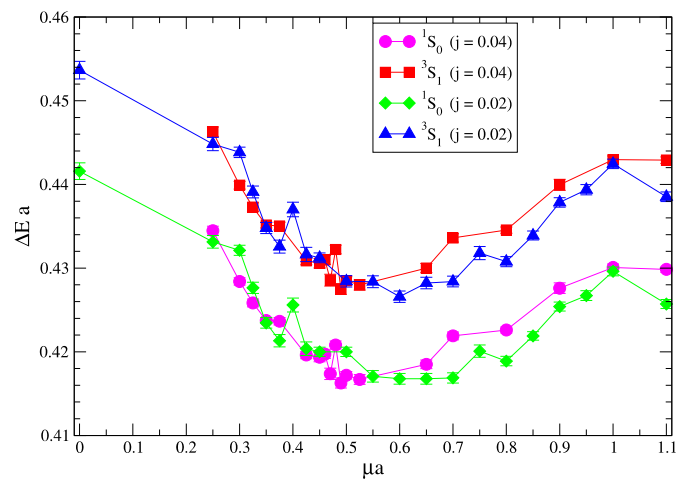


Fig. 4. diquark source dependence of the 1S_0 and the 3S_1 state energies vs. μ with $Ma = 5.0$.

difference in this region. In either case the ratio increases monotonically with τ .

Unlike the simple behaviour seen for the S -wave, the P -wave correlator ratios show an interesting τ -dependence. In the BEC and BCS regions ($\mu \leq \mu_2$), the P -wave correlator ratios behave similar to the S -wave, but in the deconfined region ($\mu \geq \mu_2$), the P -wave correlator ratios are non-monotonic, initially decreasing with τ before turning to rise above unity for $\tau a \sim 4$. On the other hand, the ratios of P -wave state correlators on $12^3 \times 16$ and $16^3 \times 12$ lattices show monotonic behaviour as the S -wave correlator ratios do, which suggests a subtle interplay of density and temperature effects on the P -wave states.

4. Discussion

We have mapped out the variation of heavy quarkonium S -wave states in a two-color baryon-rich medium with $\mu/T \gg 1$, as both μ and T are varied. The behaviour is unexpectedly complex; the state energies initially decrease, then plateau, and finally rise again to become comparable or even exceed the vacuum value. The medium effect increases as the heavy quark mass M is decreased. Using the string tension to set the scale we find a downwards energy shift $\Delta E \sim 40$ MeV for $Ma = 3.0$ and $\mu_1 < \mu < \mu_2$.

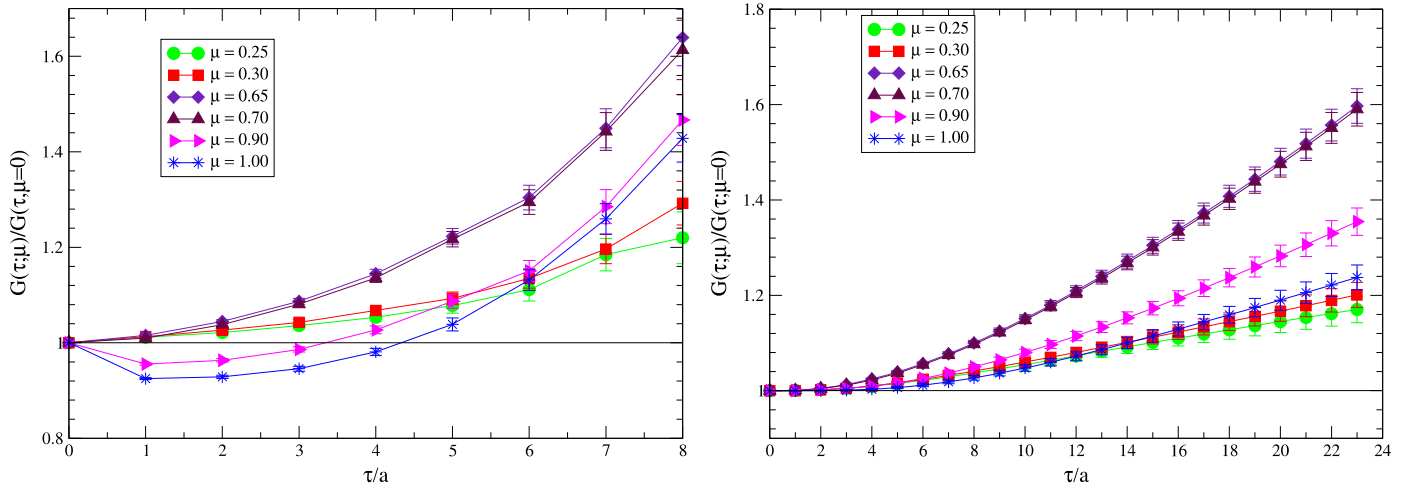


Fig. 5. (Left) The ratio $\sum_{\mathbf{x}} G(\mathbf{x}, \tau; \mu) / \sum_{\mathbf{x}} G(\mathbf{x}, \tau; 0)$ for 1P_0 correlators on $12^3 \times 24$ with $Ma = 5.0$. Due to the noisiness of the P -wave data, only a limited τ range is shown. (Right) The corresponding ratio for 1S_0 correlators for comparison.

It is natural to seek an explanation in terms of the various ground states set out in [1,2]. Initially assume $T \approx 0$; $\Delta E(\mu)$ must then arise solely from interactions between the heavy quark pair $Q\bar{Q}$ (or equivalently $Q\bar{Q}$) and light quarks q in the medium. An obvious possibility is the formation of two Qq states, or conceivably even a tetraquark $Qq\bar{q}Q$. In vacuum the quarkonium state usually lies $\mathcal{O}(\Lambda_{\text{QCD}})$ below the threshold for this to occur [23]. In a baryonic medium (i.e. above onset $\mu > \mu_0$) the qs are already present and no longer need to be excited from the vacuum, so that the $Q\bar{Q}$ energy may now be above threshold. A naive energy budget must take into account both the breaking apart of the gauge singlet $Q\bar{Q}$ and $q\bar{q}$ states, and the subsequent formation of two Qq states. We assume that only $q\bar{q}$ breaking has any significant μ -dependence, since it is the properties of the q -medium that evolve with μ .

Henceforth assume that μ_1 coincides with the transition from BEC to quarkyonic phase at μ_Q identified in [2], and μ_2 with the deconfining transition at μ_D . For $\mu_0 < \mu < \mu_1$ the medium thus consists of tightly-bound diquark states, which are also Goldstone bosons associated with superfluidity, with mass proportional to \sqrt{j} [5,24]. We deduce that the energy required to break such bound pairs falls as the quark density n_q rises and, since the effect increases as M falls, with the ultimate separation of the resulting $q\bar{q}$ system. Both these factors suggest an in-medium screening of the interaction between light quarks, or equivalently a non-trivial μ -dependence of the Goldstone decay constant $F_\pi(\mu)$, whose detailed mechanism remains unclear.

For $\mu_1 < \mu < \mu_2$ the system is hypothesised to be in a quarkyonic phase, which we take to be a state in which quarks, though still confined, form a degenerate system with well-defined Fermi energy $E_F \sim \mu$, and in which superfluidity arises through BCS condensation of weakly-bound and spatially delocalised Cooper pairs. To excite light qs capable of forming Qq states now thus requires an energy of $\mathcal{O}(\Delta)$, the superfluid gap, believed to be approximately μ -independent in this regime [2]. Forming a Qq state at rest also requires the heavy quark to have kinetic energy $\mathcal{O}(\mu^2/M)$, which is a small, perhaps negligible, correction. This accounts for the approximate μ -independence of ΔE observed in this regime. Since the qs are no longer bound within Goldstone bosons, we expect their excitation energy in this regime to vary linearly with j , which is thus responsible for the mild increase of ΔE with j seen in Fig. 4.

Finally, for $\mu > \mu_2$ the system is deconfined, as signalled by the non-vanishing expectation of the Polyakov loop. In this regime the

physical states could in principle be isolated heavy quarks dressed by a cloud of both light quarks and now gluons, although we should not at this stage rule out the persistence of bound states – indeed, the contrast between S - and P -wave states, which are more spatially extended, in this region in Fig. 5 suggests an interesting story remains to be told. In either case the light constituents are now expected to have a Debye mass $m_D \propto g\mu$ generated via quantum loop corrections; hence ΔE now rises with increasing μ , as confirmed by Fig. 2. Thermal mass generation $m_D \propto gT$ may also be responsible for the systematic increase of ΔE with T seen in Fig. 2; it is notable in this case that the thermal effect appears to be equally manifest in all three μ -regimes.

The reader will no doubt agree that these are speculative ideas, inevitably constrained by theoretical pictures which can only become accurate in limits of vanishingly small or asymptotically high densities. It may well turn out that our identification of three different μ -regimes is over-elaborate and masks a more unified explanation. Nonetheless, in view of the interesting heavy quarkonia physics in the non-zero temperature environment [8,9], we expect from this exploratory study that heavy quarkonia can yield important insights into the nature of dense baryonic matter.

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