RANGE-BASED RISK ESTIMATION IN EURO AREA COUNTRIES

by

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Summary

This dissertation considers a range of topics on the use of range-based risk estimators for financial markets (with the exception of Chapter 5 discussed below). Chapter 1 provides an introduction to the existing literature and the research objectives of the dissertation.

Chapter 2 uses time series of daily high-low ranges of national equity market indices to analyse daily volatility dynamics and volatility spillover across four European markets. Chapter 2 is based on the joint research with Gregory Connor. We develop a dynamic linear model of expected daily range which is a variant of Chou's conditional autoregressive range model. We find significant, but not uniform, range-based volatility spillovers. During the crisis period (after July 2007) we find significant increases in daily range, increases in contemporaneous correlation, and increases in the influence of previous-day US market range on the conditional expected range of these European markets. A gamma-distribution-based model of realized daily range fits more closely than one based upon a Feller distribution, but it sacrifices the link to a specific distribution for underlying returns.

In Chapter 3 we use information on the daily opening, close, high, and low prices of individual stocks to estimate range-based correlation and to construct a new estimator of market betas. We create a measure called "range-beta", which is based on the daily range-based volatility and covariance estimators of Rogers and Zhou (2008). These range-based betas reflect the current day's intra-day price movements. They avoid a weakness of return based betas, which typically are based on close-to-close returns. Our approach yields competitive estimates compared with traditional methodologies, and outperforms other methodologies when analysing highly liquid assets.

Chapter 4 studies the relationship between options-implied and realized-range-based volatility estimates for Euro area countries. When both implied volatility and historical range-based volatility are used to forecast realized range-based volatility, we find that implied volatility outperforms historical range-based volatility. We also find that the stochastic volatility is priced with a negative market

price of risk. The volatility implied from option prices is higher than the realized range-based volatility under the objective measure due to investor risk aversion.

Chapter 5 considers financial market risk from a different perspective. Chapter 5 analyses the tone and information content of the two external policy reports of the Internal Monetary Fund (IMF), the IMF Article IV Staff Reports and Executive Board Assessments, for Euro area countries. In particular, we create a tone measure denoted *WARNING*, based on the existing DICTION 5.0 Hardship dictionary. We find that in the run-up to the current credit crises, average *WARNING* tone levels of Staff Reports for Slovenia, Luxembourg, Greece, and Malta are one standard deviation above the EMU sample mean; and for Spain and Belgium, they are one standard deviation below the mean value. Furthermore, on average for Staff Reports over the period 2005-2007, there are insignificant differences between the EMU sample mean and Staff Reports' yearly averages. We also find the presence of a significantly increased level of *WARNING* tone in 2006 for the IMF Article IV Staff Reports. There is also a systematic bias of *WARNING* scores for Executive Board Assessments versus *WARNING* scores for the Staff Reports.

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Conferences and Publications

I have presented Chapter 2 titled “*Range-based Analysis of Volatility Spillovers in European Finanacial” Markets* at the Irish Society of New Economists Conference (August 2011), at the 26th Annual Irish Economic Association Conference (April 2012), and at the 44th Annual Money, Macro and Finance Conference (September 2012). This paper has been also submitted to the Journal of International Money and Finance.

I have presented Chapter 3 titled “*Measuring Equity Risk Exposures with Range-based Correlations”* at the 27th Annual Irish Economic Association Conference (May 2013), at the 3rd International Conference of the Financial Engineering and Banking Society (June 2013), and at the 6th International Risk Management Conference (June 2013). This paper is included in the Conference Proceedings of the 6th International Risk Management Conference.

I have presented Chapter 6 titled “*Content Analysis of the IMF Article IV Staff Reports for Euro Area Countries”* at the FMC2/MASCI Colloquim, at the 25th Annual Irish Economic Association Conference (April 2011), and at the DICTION Conference in University of Texas at Austin (February 2013). This paper was accepted as a chapter to the *Handbook of Research on Institutional Language* (*Forthcoming* June 2014).

Chapter 1: Introduction

1.1 Range-based Volatility

In finance, volatility is a measure of the price variation of a financial instrument over time. Volatility plays an important role in financial economics and is a fundamental concept in several subjects including asset allocation, market timing, portfolio risk management and the pricing of assets and derivatives.

Historical volatility is computed as the standard deviation of daily returns within a certain period, say two months. One implicitly assumes that the volatility is a constant within two months. However, it is unrealistic to assume that the volatility of asset return remains constant during a long period. Therefore, the volatility estimated with the classical estimator is essentially a measure of the average true volatility over the specified period.

Besides estimating volatility using asset returns, it is also possible to use the range based approach as a measure of return volatility. The daily high-low range is defined as the log of the ratio of the intradaily high and low prices of the national market index.

In an early application, Mandelbrot (1971) employed the range to test the existence of long-term dependence in asset prices. The widespread application of the range in the context of financial volatility and in particular to the estimation of volatilities started from the early 1980s, e.g., Garman and Klass (1980), Beckers (1983), Rogers and Satchell (1991). Parkinson (1980) notes that the log price range over an interval potentially gives more information regarding volatility than the log difference between two preselected points such as the beginning and end prices. This is due to the max – min operator implicit in its definition (see Equation (2.1) in Chapter 2) which embodies information from the full set of realized daily prices. For more extensive discussion on the properties of the range see Alizadeh et al. (2002).

Recent studies have shown that the range-based measure of volatility is often superior to traditional volatility estimators, e.g., Brunetti and Lildholdt

(2002), Andersen et al. (2003b), McAleer and Medeiros (2008). Suppose, as is true for many European indices, the econometrician only has data on the daily open, close, high and low. The daily return (log difference between today's and yesterday's close) uses information contained in two prices, while the high-low range implicitly uses information from all trade prices during the day. Thus, a daily return is often less informative about what happened during the day than the range. As noted by Chou (2005), Chou et al. (2009), on a turbulent day with intraday drops and recoveries, the daily return may be near zero, while the daily price range will reflect the high intraday price fluctuations. Shu and Zhang (2003) provide relative performance of different range-based volatility estimators and find that range estimators perform very well when asset prices follow a continuous Brownian motion. Parkinson (1980) observes a theoretical relative efficiency gain (ratio of estimation variances) from using sample average daily range to estimate return variance (rather than using daily sample return variance) of approximately 5. Garman and Klass (1980) report that their range-based variance estimator has a relative efficiency of 7:4 compared to daily sample variance. Andersen and Bollerslev (1998) find that the daily range has approximately the same information content as sampling intradaily returns every four hours. Engle and Gallo (2006) have shown that the daily range has good explanatory power in predicting future values of realized variance.

Daily range can be interpreted as the maximum loss, that is, the negative of the minimum possible realized log return, on a one unit intradaily trade. If the high price occurs before the low price during the day, then the trade is sell-buy rather than buy-sell; this is interpreted as the maximum loss on a unit short-sale established and closed during the day. Maximum intraday loss is quite important in a trading environment, hence daily range has direct relevance for portfolio risk management, in addition to its usefulness as an indirect measure of intradaily volatility.

1.2 Volatility Spillover

Recent developments in financial markets such as for instance the bursting of the IT bubble, the US subprime mortgage crisis and Europe's ongoing sovereign debt crisis have shifted focus on the interdependence level of financial markets, and volatility spillovers.

The empirical literature studying volatility spillover is extensive, typically based on daily close-to-close returns, e.g., Yang and Doong (2004), Lee (2006), Koulakiotis et al. (2009), Diebold and Yilmaz (2009), McMillan and Speight (2010). Koutmos and Booth (1995) examine the spillover effects among the New York, Tokyo and London stock markets and show that the transmission of volatility is asymmetric and is more pronounced when the news is bad and coming from either the US or UK market. Kanas (1998) examines volatility transmission across the London, Paris and Frankfurt stock markets and concludes that returns and innovations spillovers are higher during the post-crash time. Billio and Pelizzon (2003) obtain evidence that volatility spillovers from the world index to European equity indices increased after the introduction of European Monetary Union. Baele (2005) and Christiansen (2007) investigate volatility spillover from the US and aggregate European asset markets into European national asset markets, incorporating bond markets into analysis. They find evidence of volatility spillover from the aggregate European and US markets to local European markets.

The research literature studying volatility spillover using the range volatility measure is limited. Chou et al. (2010) document that the volatility spillover exists between the European markets over the period 2004-2010, whereas the countries are independent over the post-subprime period.

1.3 Return-based, Range-based, and Options-implied Volatility Estimates

Merton (1980) notes that the variance of the returns on an asset over an extended period of time can be estimated with high precision if during that period a sufficient number of sub-period returns is available. Because the squared mean return converges to zero as the sampling frequency increases, the variance of the returns over an extended period can be calculated by summing the squared sub-period returns and ignoring the mean return. This is what today is called the concept of realized volatility and this term is interchangeably used with realized variance. In the context of high frequency data, estimating the realized volatility is complicated by the microstructure effects such as the bid-ask bounce which can significantly bias the estimator upward (Alizadeh et al., 2002). Second, we should expect that the estimates made will not show much intertemporal stability (in view of the well-known profile of intraday trading activity). Indeed, the work of Barndorff-Nielsen et al. (2009) confirms this, showing estimates of volatility which vary very substantially from day to day. Third, we have to handle a huge amount of data; while this is not in itself a problem, it is reasonable to ask whether the effort (human and computer) is worth the goal and, indeed, whether the additional effort will actually help toward the goal. The intradaily range-based volatility measure is also considered as a proxy of the realized volatility. As it was suggested by Brandt and Diebold (2006) the range is not affected by market microstructure noise. The estimator requires the knowledge of prices within a day and therefore, is formally high frequency estimator.

The volatility implied by option prices is the option market's forecast of future return volatility over the remaining life of this option. Under a rational expectations assumption, the market uses all the information available to form its expectations about future volatility, and hence the market option price reveals the market's true volatility estimate. Furthermore, if the market is efficient, the market's estimate, the implied volatility, is the best possible forecast given the currently available information. That is, all information necessary to explain future realized volatility generated by all other explanatory variables in the market information set should be subsumed in the implied volatility. The hypothesis that implied volatility is an efficient forecast of the subsequently realized volatility has been the subject of many empirical studies.

Early papers studying the relative performance of options-implied and the future realized volatility find that the volatility inferred from the option markets is a biased predictor of stock return volatility. To illustrate, Canina and Figlewski (1993) found that the implied volatility from S&P 100 index options is a poor forecast for the subsequent realized volatility of the underlying index. In contrast, Day and Lewis (1992), Lamoureux and Lastrapes (1993), Jorion (1995) and Fleming (1998) report evidence supporting the hypothesis that implied volatility has predictive power for future volatility. They also find that implied volatility is a biased forecast for future realized volatility.

Christensen and Prabhala (1998) and Christensen and Strunk (2002) first note that ex-ante implied volatility in fact is an unbiased and efficient forecast of ex-post volatility after the 1987 stock market crash, while they point to large bias before the 1987 crash. Authors also refuted their results by showing that the weakness of the options-implied volatility in future volatility prediction is mainly resulted from the methodological issues like overlapping sample and mismatched maturities (options with longer expiration are used to predict day/week ahead realized volatilities).

However, early research on the information content of options-implied volatility focuses on the Black-Scholes implied volatility, and fails to incorporate information contained in other options. In addition, tests based on the Black-Scholes implied volatility are joint tests of market efficiency and the Black-Scholes model. The results are thus potentially contaminated with additional measurement errors due to model misspecification.

A strikingly simple method to extract volatilities from options across all strike prices, model-free implied volatility was introduced by Demeterfi et al. (1999). The model-free implied volatility measure can be derived directly from a comprehensive cross-section of European put and call options with strikes spanning the full range of possible values for the underlying asset at option expiry. Recent research has confirmed that this pricing relationship is robust and remains approximately valid for a broad class of relevant return generating processes, including jump-diffusive semimartingales models. Unlike the traditional concept, the model-free implied volatilities are computed from option prices without the use of any particular option-pricing model and it is derived from no-arbitrage conditions and the martingale measure (Demeterfi et al., 1999; Jiang and Tian, 2004; Lynch and Panigirtzoglou, 2003). Informational content of option implied volatility in the subsequent research is analysed using the model-free measure. For instance, paper by Jiang and Tian (2004).

From the theoretical point of view, the model-free implied volatility aims to measure the expected integrated variance, or, more generally, return variation, over the coming month, evaluated under the so-called risk-neutral, or pricing (Q), measure. Since volatility is stochastic, the model-free implied volatility is not a pure volatility forecast for the underlying asset but rather bundles this forecast with market pricing of the uncertainty surrounding the forecast. This implies that, in general, implied volatilities will include premia compensating for the systematic risk associated with the exposure to equity-index volatility. In addition, the volatility index will rise in response to a perceived increase in future volatility and vice versa, all else equal. As a result, the model-free implied volatility index should be strongly correlated with future realized volatility.

1.4 Range-based Covariance

The covariance of assets is important for the computation of the prices of derivatives written on many underlying products. The traditional method of estimating the covariance between different assets assumes that the daily log-returns are i.i.d. multivariate Gaussian variables and produces an unbiased estimator of the covariance matrix. Estimating the covariance between different assets using the range-based methodology is quite a new concept. For instance, Brandt and Diebold (2006), Brunetti and Lildholdt (2002) work with foreign exchange data, where the availability of data on the cross rates means that one is able to observe highs and lows of linear combinations of the log asset prices, allowing one to reduce to existing univariate methodology by polarization. However, such an approach would be impossible if assets were equities, since we do not have information on the highs and lows of linear combinations of the log asset prices (unless full tick data is available).

In Chapter 3 we develop the range-based covariance measure that can be applied to equities. We employ Rogers and Zhou (2008) approach of estimating the covariance of linear combination of the two log prices based on the daily opening, closing, high, and low prices of each. The daily range-based covariance estimator has attractive properties such as the relatively low variance of the range-based covariance estimator. Realized covariances are unaffected by bid-ask bounce under the assumption that bid and ask transactions occur independently across assets.

1.5 Range-based Beta

The capital asset pricing model (CAPM) due to Sharpe (1964) and Lintner (1965) relates the expected return on an asset to its systematic market risk or beta. This beta is the sensitivity of the asset return to the return on the market portfolio. It is defined as the covariance of an asset's returns with the market's returns, divided by the variation of the market returns. Specifically, beta measures the portion of an asset's statistical variance that cannot be mitigated by the diversification of a portfolio composed of many risky assets, or the market portfolio. Beta is used by financial economists and practitioners to identify mispricings of a stock, to calculate the cost of capital and to evaluate the performance of managers.

A number of empirical studies (e.g., Fama and French, 1992, 1993, 1996; Choudhry, 2002, 2004) have suggested that a constant-beta CAPM is unable to satisfactory explain the cross-section of average returns on equities and the market to capture dynamics in volatility. By constant, it is meant that betas are calculated on a set period-by-period basis, as oppose to a continuous evolution. Specifically, Adrian and Franzoni (2009) argue that model without time-evolving betas fail to capture investor characteristics and may lead to inaccurate estimates of the true underlying beta. Following this criticism, multiple time-varying beta models were proposed (e.g., Campbell and Voulteenaho, 2004; Andersen et al., 2005; Petkova and Zhang, 2005; Lewellen and Nagel, 2006; and Ang and Chen, 2007). Some of these studies use a parametric approach proposed by Shanken (1990), in which the variation in betas is modelled as a linear function of conditioning variables. Early parametric approaches include the multivariate GARCH framework (Bollerslev et al., 1988) and the instrumental variables or "conditioned down" betas (Harvey, 1989). Recent parametric models suggest treating conditional betas as latent variables: Adrian and Franzoni (2009) suggest using the Kalman filter while Ang and Chen (2007) apply Markov chain Monte Carlo and Gibbs sampling to obtain time varying betas.

An alternative, non-parametric approach to model risk dynamics was first implemented by Fama and MacBeth (1973). The non-parametric approach is based on purely data-driven filters, including short-window regressions (e.g., Lewellen and Nagel, 2006) and rolling regressions (e.g, Fama and French).

The parametric specification is appealing from a theoretical perspective because it explicitly links time variation in betas to macroeconomic state variables and firm characteristics (e.g., Gomes et al., 2003; Santos and Veronesi, 2004). However, the main drawback of this approach is that the true investor's set of conditioning information is unobservable. Ghysels (1998) shows that misspecifying beta risk may result in serious pricing errors that might even be larger than those produced by an unconditional asset pricing model. In addition, this method can produce jumps in betas due to sudden spikes in the macroeconomic variables that are often used as instruments. Finally, many parameters need to be estimated when a large number of conditioning variables is included, which leads to noisy estimates when applied to stocks with a limited number of time series observations. An important advantage of non-parametric approaches is that they preclude the need to specify conditioning variables, which makes them more robust to misspecification. However, the time series of betas produced by a data-driven approach will always lag the true variation in beta, because using a window of past returns to estimate the beta at a given point in time gives an estimate of the average beta during this time period. Although reducing the length of the window results in timelier betas, the estimation precision of these betas will also decrease.

In Chapter 3 we use information extracted from the daily opening, closing, high, and low prices of the stocks to improve the estimation of the current betas and the predictions of the future betas. We create a new time-varying beta measure called "range-based beta", which is based on the daily range-based volatility and covariance estimators of Rogers and Zhou (2008) for estimating market beta. Within this context, the range-based beta is the ratio of the range-based covariance of stock and market to the range-based market variance. We improve the specification of betas by combining the parametric and non-parametric approaches to modelling time variation in betas. Since the main strengths of each approach are the most important weaknesses of the other, we show that a combination of the two methods leads to more accurate betas than those obtained from each of the two methods separately.

1.6 Macroeconomic Risk

Finally, the evidence suggests that the financial markets volatility is affected by the communication of the intergovernmental agencies such as the IMF, the ECB, the Federal Reserve, and other. In Chapter 6 we evaluate the effectiveness of the IMF external surveillance in the run-up to the current credit crisis. In contrast to previous studies, this study is the first to apply content-analysis methodology to analysing the IMF Reports.

Content analysis is defined as the systemic, objective, quantitative analysis of message characteristics (Neuendorf, 2002). It is a highly structured and systemic way for analysing qualitative text from a researcher's perspective. It provides a well-developed set of procedures to make sense of the multiple sources of qualitative data. There is extensive research in accounting, finance, and other social science fields that analyses the content of textual documents using computer algorithms. Within this literature, there is extensive research on the information content of corporate earnings releases (Davis et al., 2006; Rogers et al., 2009), accounting policy disclosures (Levine and Smith, 2006), financial news (Core et al., 2008), Internet stock message board, and multiple sources of financial text (Kothari et al., 2008). However, most of the existing studies are closely related to the firm-level characteristics, and very little are dealing with country-level reports.

There exist a range of computerized content analysis algorithms that analyse the thematic character of the text. For instance, the DICTION 5.0 (Hart, 2001) is a dictionary-based program that counts types of words most frequently encountered in contemporary American public discourse and is designed to capture the linguistic style (i.e. verbal tone) of narratives (Hart, 1984). DICTION 5.0 uses a lexicon of 10,000 words to divide a text into five semantic features: Activity, Optimism, Certainty, Realism and Commonality. These five features are composed of combinations of 35 sub-features (Pennebaker et al., 2003). DICTION 5.0 analyses texts in 500 word blocks. The resulting DICTION score represents the number of times each word (per 500 word text length) from 1 of the 35 sub-features appears in the text. These sub-feature scores are then aggregated to form the five major thematic categories. The aggregation process is simply the sum of various sets of the sub-features. DICTION's Report Files produce both raw scores and standardized scores for each of the standard dictionaries.

There are potential strengths and weaknesses in using DICTION 5.0 computerized content analysis software. In terms of strengths, DICTION performs textual analysis based on pre-existing search rules and algorithms, and is systemic and thus free from criticisms of researcher subjectivity and potential bias. Moreover, computer-based system can examine multiple phenomena simultaneously and can report on combinations of word usages that the researcher could hardly conceive of, never mind calculate, without machine assistance. Finally, content analysis software facilitates the efficient analysis of a large number of texts and a partial correction for the context. The principal weakness of DICTION is that it is based on the assumption that higher frequency usages of a word or phrase mean the concept is more meaningful or important than infrequently utilized words or phrases. In other words, it does not analyse language conditional on the context of the particular statement. However, more recent research by Li (2009) contrasts the measure of tone calculated using DICTION and a Naïve Bayesian machine learning approach. Li (2009) concludes that the machine learning algorithm and the dictionary approach capture the tone of the financial documents similarly.

Chapter 2: Range-based Analysis of Volatility Spillovers in European Financial Markets

2.1 Introduction

In this paper we study the dynamic linkages among European security markets based on the time series of daily high-low ranges of national equity market indices. The daily high-low range is defined as the log of the ratio of the intradaily high and low prices of the national market index. As is well documented, see Alizadeh et al. (2002), the daily range can provide a surprisingly accurate indirect measure of daily volatility (that is, daily return standard deviation). It is also readily available across markets with no publicly-available intraday price series. We build a dynamic model of daily range, and address a number of empirical questions based upon it. We also include the realized daily range of the US S&P500 index as an explanatory variable, but our focus is on explaining volatility dynamics and linkages in the European markets.

We use a dynamic linear model of expected daily ranges based on the conditional autoregressive range (CARR) model of Chou (2005) and Engle and Gallo (2006). We refine the CARR model to make it consistent with a discrete-interval model of daily return standard deviations in which the vector of daily return standard deviations depends linearly upon its lagged values and lagged realized ranges, and in which intraday prices follow standard multivariate Brownian motion. We estimate both our new version of the CARR model and an earlier version of Engle and Gallo (2006) on our dataset and compare their performance.

We estimate using data over the period January 11, 1991 to May 23, 2013 and find a number of interesting results. The linear dynamics in daily range appear similar whether estimated using the Feller or gamma distribution. The gamma distribution better fits the empirical distribution of tail events in daily range, but this distributional assumption sacrifices the theoretical link between daily range and daily standard deviation provided by the Feller distribution model. There are strong asymmetries in daily range dynamics: in all four markets, expected daily range is higher after a day with negative open-to-close return. There are some cross-market dynamics among the European markets, but the strongest cross-market dynamic influence comes from the US market: daily range in each of the European markets tends to be higher on a day after a high realized range in the US market.

We divide our sample into pre-crisis and crisis periods, using July 17th 2007 as the regime switch date based on the analysis of Cipollini and Gallo (2010). We find clear evidence for a regime shift. First, not surprisingly, both average and median daily ranges increase sharply in all four European markets. Second, there is a sharp increase in the contemporaneous correlations between the daily ranges of the markets. Third, the dynamic models of daily range have a notable and consistent change, in all four markets the influence of lagged US market daily range increases substantially during the crisis period.

In Section 2.2 we describe our econometric methodology. In Section 2.3 we introduce the data and provide some descriptive statistics. Section 2.4 presents the empirical analysis for the full sample period. In Section 2.5 we estimate allowing for a regime shift in July, 2007, reflecting the ongoing financial crisis. Section 2.6 presents some concluding remarks.

2.2 A Range-based Volatility Model: Theoretical Framework

2.2.1 Range as a Volatility Proxy

Our model uses two time indices: a discrete index t for days, and a continuous index τ for intraday time. Let p_τ, $0 \le \tau \le 1$ denote the n–vector of intraday log prices on n assets during day t (for notational simplicity the day t is left implicit for intraday time). Assume that the n–vector of realized daily ranges is the high minus low of day t intraday log prices:

$$hl_{it} = \max_{0\le\tau\le1} p_{i\tau} - \min_{0\le\tau\le1} p_{i\tau} \quad \text{for} \quad i = 1,\dots,n, \tag{2.1}$$

which is strictly positive as long as the price is not constant over the entire interval. Also important in our analysis is the n–vector of expected ranges

$$\mu_t = E\left[hl_t \middle| I_{t-1}\right], \tag{2.2}$$

where the expectation is conditional on all information at time t–1 (that is, the beginning of day t). Suppose that intraday log prices follow a standard Brownian motion during day t with standard deviations σ_t. In this case, Parkinson (1980) shows that scaled range is an unbiased proxy for return standard deviation, and in particular:

$$\mu_t = \left(\sqrt{\frac{8}{\pi}}\right)\sigma_t\,, \tag{2.3}$$

so that the expected range and standard deviation differ only by a scale factor.

Note that our theoretical model ignores the overnight (and weekend) closed periods in these markets. The high and low price observations only cover the period during which the market is open, so that the comparable volatility in Equation (2.3) is daily open-to-close return volatility rather than the more commonly used close-to-close return volatility. We will discuss this further in our empirical analysis.

2.2.2 A Linear Dynamic Model of Volatility and Expected Range

In this subsection we develop a modified variant of Chou's (2005) conditional autoregressive range (CARR) model. We begin with a foundational model of daily volatility (that is, return standard deviation), which produces a fully parametric specification of expected daily range. Let σ_t denotes the n–vector of standard deviations of returns for day t, and p_τ denotes the n–vector of log prices at intra-day time τ within day t. We assume that intraday prices follow standard multivariate Brownian motion with zero mean vector and time-constant correlation matrix C:

$$\left[p_{\tau+\Delta} - p_\tau\right] \sim MVN\left(0^n, \Delta^2\left[Diag(\sigma_t)\right]C\left[Diag(\sigma_t)\right]\right) \text{ for } 0 \le \tau < \tau + \Delta \le 1, \tag{2.4}$$

where $Diag(\sigma_t)$ denotes the diagonal matrix with the vector σ_t on its diagonal.

We impose a simple linear dynamic model on the n–vector of daily standard deviations, in particular:

$$\sigma_{i,t} = \omega_i^* + \sum_{j=1}^{n} \alpha_{i,j}^* hl_{j,t-1} + \sum_{j=1}^{n} \beta_{i,j} \sigma_{j,t-1} \qquad (2.5)$$

with all non-negative parameter elements. We assume that the parameter values are such that the time-series process for σ_t is covariance stationary. The vector of estimable parameters in Equation (2.5) (other than those set to zero by assumption) will be denoted Θ. Scaling ω_i^* and $\alpha_{i,j}^*$ by $\left(\sqrt{\frac{8}{\pi}}\right)$ and substituting Equation (2.3) into Equation (2.5) gives:

$$\mu_{i,t} = \omega_i + \sum_{j=1}^{n} \alpha_{i,j} hl_{j,t-1} + \sum_{j=1}^{n} \beta_{i,j} \mu_{j,t-1}, \qquad (2.6)$$

which is an equivalent expression of the dynamic system in terms of $\mu_{i,t}$ rather than $\sigma_{i,t}$. We assume that, conditional upon the fixed daily volatilities (2.5), the Brownian motion determining price processes within days is completely independent across days. Following Engle and Gallo (2006), the daily range innovation is the ratio of the realized range to its conditional expected value:

$$\varepsilon_{it} = \frac{hl_{it}}{\mu_{it}}, \; i = 1,...,n, \qquad (2.7)$$

and it follows immediately from the assumptions above that this is independently and identically distributed through time. We will derive its distribution in the next subsection.

The model, particularly in formulation (2.5), has close parallels with GARCH-family models. There are two distinctions between (2.5) and standard GARCH. First, the innovation for the dynamic model is the realized daily range rather than the squared close-to-close return, and second, the realized daily range drives standard deviation (and/or expected range) rather than variance.

2.2.3 Maximum Likelihood under Intradaily Brownian Motion

The model of the previous subsection has a known log likelihood function. As Alizadeh et al. (2002) note, extending Feller (1951), the distribution of the range under Brownian motion is given by:

$$\Pr(hl_{it} = y) = 8\sum_{k=1}^{\infty}(-1)^{k}\frac{k^2 e^y}{\sigma_{it}}\varphi\left(\frac{ke^y}{\sigma_{it}}\right), \tag{2.8}$$

where $\varphi(\cdot)$ is the standard normal density. Although the density function (2.8) involves an infinite sum, it is straightforward to compute numerically since the low-order additive terms dominate the sum (the multiplicative component $\varphi\left(\frac{ke^y}{\sigma_{it}}\right)$ goes to zero at an exponential rate in k); see Alizadeh et al. (2002). Since we assume intradaily constant-volatility Brownian motion, this provides the exact distribution of realized daily range, conditional upon knowing σ_{it}. Substituting Equation (2.7) into Equation (2.8) gives the likelihood function of the realized range innovations which are independently and identically distributed through time.

We assume that the initial value of σ_{it} for $t = 0$ is known. Given this and our other assumptions, the likelihood of the sample equals the product of (2.8) evaluated at observed hl_{it} for each t using the linear dynamic model (2.5) to define σ_{it} recursively. Recall that Θ denotes the vector of parameters in the linear dynamic model. Stating the log likelihood problem:

$$\hat{\Theta} = \arg\max_{\Theta}\sum_{t=1}^{T}\ln\left(\Pr(\varepsilon_{it}, \sigma_{it})\right),$$

where σ_{it} is given by Equation (2.5) and $\Pr(\varepsilon_{it}, \sigma_{it})$ denotes the function (2.8). Time subscript t runs from 1 to T. In large samples these maximum likelihood estimates are consistent and asymptotically normal, with the asymptotic covariance matrix consistently estimated by the inverse of the inner product of the derivative matrix of the log likelihood function with respect to Θ evaluated at $\hat{\Theta}$.

2.2.4 Estimation under an Alternative Distributional Assumption on Realized Range

One weakness of the specification described in the last two subsections is its reliance on constant-volatility intraday Brownian motion for log price; this is not supported by the evidence since daily equity index returns have positive excess kurtosis. Dropping this assumption invalidates Equation (2.8) as the distribution of daily range. In this subsection we describe an alternative estimation strategy developed by Engle and Gallo (2006). The Engle-Gallo specification does not require the assumptions of Brownian motion and intradaily constant volatility. They use the same linear dynamic model of expected daily range as above (2.6) but do not specify the inter-daily nor intra-daily process for log prices. They assume that the realized daily range has a gamma distribution:

$$\varepsilon_{i,t} \sim Gamma\left(\gamma_i, \frac{\gamma_i}{\mu_{i,t}}\right). \qquad (2.9)$$

Note that, in this application, the gamma distribution has only one free parameter rather than the usual two; this reflects the restriction from Equation (2.7) that $\varepsilon_{i,t}$ must have unit expectation since by definition $\mu_{i,t}$ is the expectation of $hl_{i,t}$.

The Engle-Gallo approach has two advantages over last two methods. One, already mentioned, it drops the assumptions of intraday constant volatility and Brownian motion for log prices. Two, it adds an additional parameter to capture the high kurtosis evident in realized daily range. In terms of disadvantages, it does not provide any specific link between daily range and the time-series properties of log price: the gamma distribution is assumed for daily range without specifying how this comes about through Equation (2.1) and the process for prices. Related to this, it gives a model of expected daily range only, not of daily standard deviation. Engle and Gallo (2006) note that another advantage of the gamma distribution in this context is that the nonlinear maximum likelihood optimization problem can be solved in two separate steps, but in our application we do not find this necessary.

We compare these two CARR specifications empirically below.

2.2.5 Spillover and Leverage Effects

We use as the base-case model the simplest specification:

$$\mu_{i,t} = \omega_i + \alpha_i hl_{i,t-1} + \beta_i \mu_{i,t-1} \qquad (2.10)$$

with all nonnegative coefficients and $\alpha_i + \beta_i < 1$. $\mu_{i,t}$ can be interpreted as the expectation of the range at time *t* for the asset ***i***. ω_i is the constant term of the equation for $\mu_{i,t}$; α_i is the autoregressive coefficient and β_i is the moving average coefficient. Following Engle and Gallo (2006), we also consider the so-called leverage effects,

$$\mu_{i,t} = \omega_i + \alpha_i^{up} Ind\left(r_{i,t-1} \geq 0\right) hl_{i,t-1} + \alpha_i^{down} Ind\left(r_{i,t-1} < 0\right) hl_{i,t-1} + \beta_i \mu_{i,t-1}, \qquad (2.11)$$

where $r_{i,t-1}$ is the close-to-close return on the asset on day *t*–1 and $Ind\left(r_{i,t-1} < 0\right)$ is a dummy variable which equals one if this return is negative and zero otherwise. α_i^{up} and α_i^{down} are parameters that capture the asymmetry. All four coefficients are restricted to be non-negative. We also consider a slightly different specification to capture the same type of leverage effect,

$$\mu_{i,t} = \omega_i + \alpha_i hl_{i,t-1} + \varphi_i r_{i,t-1} + \beta_i \mu_{i,t-1}. \qquad (2.12)$$

Note that, by definition of the range, $hl_{i,t-1} \geq \left|r_{i,t-1}\right|$ and so as long as $\left|\varphi_i\right| < \alpha_i$ and the other coefficients are non-negative this model belongs to the multiplicative error model class, see Engle (2002).

We also estimate the extended specification (2.6) including lagged cross-country realized ranges to test for spillover effects between markets. Note that in this case, as noted by Engle and Gallo (2006), full-information maximum likelihood requires that the system of equations (2.6) be estimated simultaneously, which also requires that the marginal distributions between the contemporaneous range innovations is specified. Instead of this, again following Engle and Gallo (2006), we restrict ourselves to limited information maximum likelihood, estimating each equation separately using the univariate likelihood objective function described above.

2.3 Simulation Evidence on the Range-based Volatility Estimators

To assess the properties of the range-based volatility estimators, we perform an extensive simulation analysis. We consider the range implied estimates of the standard deviation. Specifically, we use Parkinson (1980) range-based proxy for the return standard deviation, and in particular:

$$\sigma_i = \mu_i \left(\sqrt{\frac{\pi}{8}} \right), \qquad (2.13)$$

where σ_i and μ_i denote the daily standard deviations and the daily range of the log price processes for assets 1 and 2, respectively.

We consider two correlated log asset prices, which follow a bivariate random walk with homoskedastic and contemporaneously correlated innovations[1]. Subsequent log prices for asset i = 1, 2 are simulated using

$$\log P_{i,t+k/K} = \log P_{i,t+(k-1)/K} + \varepsilon_{i,t+k/K} \qquad i = 1,\ 2,\ \ k = 1,\ 2,...,K, \qquad (2.14)$$

where K is the number of prices per day. We assume that the shocks $\varepsilon_{i,t+k/K}$ are serially uncorrelated and normally distributed with mean zero and variance σ_i / K, where daily standard deviations σ_i of the log price processes are set equal to 0.0252 and 0.0149 for assets 1 and 2, respectively. σ_1 is calibrated as the average of the daily standard deviation of the DAX constituent assets over the period from January 2, 2003, to September 30, 2011. σ_2 is simply the sample average daily standard deviation of the DAX Index.

For each day, we calculate the high and low log prices for both assets i = 1, 2. The shocks $\varepsilon_{1,t+k/K}$ and $\varepsilon_{2,t+k/K}$ are contemporaneously correlated with

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[1] The random walk process (discrete time version of Brownian motion) for the log-prices follows from the assumption that prices follow a geometric Brownian motion. Strictly speaking, this would imply that the random walk process contains a drift, but we abstain from this fact here. This drift is probably negligible.

correlation coefficient ρ_{12}, which we set equal to 0.5. The simulation experiment uses $K \in \{25,\ 100,\ 500,\ 1{,}000\}$ observations per day, where price observations are equidistant and occur synchronously for the two assets. We simulate the prices for 10,000 days in all the experiments presented below. Table 2.1 shows that the range-implied estimates of the standard deviation are downward bias. This result is consistent with the facts that the range of a discretely sampled process is strictly less than the range of a true underlying process. The range-implied estimates of the standard deviation are close to the theoretical values of standard deviation when K gets larger.

Table 2.1. Monte Carlo experiment for the range-implied standard deviation

	Asset 1	Asset 2
Theoretical value of st. dev	0.0252	0.0149
Range-implied estimates of st. dev		
K = 25	0.0217	0.0130
K = 100	0.0234	0.0138
K = 500	0.0243	0.0145
K = 1,000	0.0246	0.0145

Notes: The Table shows the results of a simulation experiment where 10,000 days of K log prices are simulated from a normal distribution with mean zero and variance σ_i/K, where daily standard deviations σ_i of the log price processes are set equal to 0.0252 and 0.0149 for assets 1 and 2, respectively. All experiments use 10,000 Monte Carlo Replications. The shocks $\varepsilon_{1,t+k/K}$ and $\varepsilon_{2,t+k/K}$ are contemporaneously correlated with correlation coefficient ρ_{12}, which we set equal to 0.5. For the each day, we calculate high and low log prices for both assets i = 1, 2; these prices are then used to calculate the range-based estimates of standard deviation.

For each of the experiments we also calculate the simulated estimation standard deviation (Table 2.2).

Table 2.2. Simulated estimation standard deviation

	Asset 1	Asset 2
K = 25	0.0094	0.0058
K = 100	0.0122	0.0072
K = 500	0.0137	0.0082
K = 1,000	0.0141	0.0083

Notes: The Table shows the results of a simulation experiment where 10,000 days of K log prices are simulated from a normal distribution with mean zero and variance σ_i/K, where daily standard deviations σ_i of the log price processes are set equal to 0.0252 and 0.0149 for assets 1 and 2, respectively. All experiments use 10,000 Monte Carlo Replications. The shocks $\varepsilon_{1,t+k/K}$ and $\varepsilon_{2,t+k/K}$ are contemporaneously correlated with correlation coefficient ρ_{12}, which we set equal to 0.5. For each day, we calculate the high and low log prices for both assets i = 1, 2.

2.4 Data and Descriptive Statistics

Our data set contains four European stock indices, which are the CAC 40 index (France), DAX 30 index (Germany), AEX index (the Netherlands), and IBEX 35 index (Spain), and as an explanatory variable the S&P500 index. All of these series are downloaded from the Datastream database. Each series has 5,388 daily observations over the sample period from January 11, 1991 to May 23, 2013. When price data for a particular trading day in one or more of the five countries are not available (for example, due to a national holiday in that country), we delete that date entirely from our sample. In total 455 days were deleted from the initial data set (8% of the days) to eliminate these missing observations in one or more of the markets and create a balanced panel.

Descriptive statistics are shown in Table 2.3. The table shows that the daily range distributions are positively skewed and leptokurtic relative to the normal distribution. Autocorrelations of realized range decay slowly, which is consistent with the pattern observed for other daily volatility measures such as squared daily return.

Table 2.4 reports the cross-autocorrelation matrices of the vector of the daily range series. The cross-autocorrelations indicate a near-symmetry of lead/lag relationship between four European markets. So, for example, the correlation between the contemporaneous range in Germany and lagged range in Spain (0.532) is nearly identical to the correlation between the contemporaneous range in Spain and lagged range in Germany (0.553). Also note that the contemporaneous correlations increase during the crisis period compared to the pre-crisis period. The only exception is the correlation coefficient between the contemporaneous range in Spain and the contemporaneous range in the Netherlands (0.721) and the correlation coefficient between the contemporaneous range in Spain and the contemporaneous range in the US (0.582), which are the same during the pre-crisis and the crisis period. When we take a look at the pairs of the autocorrelations containing Spain, we observe the decrease in the autocorrelations during the crisis period compared to the pre-crisis period. This finding suggests that Spain tends to trigger very little or no contagion among the core countries during the crisis period, where contagion is commonly defined as a significant increase in cross-market interdependencies after a large shock hits one country or a group of countries. Our results are also consistent with Kabaska and Gatwoski (2012) study which analyses contagion among several European sovereigns using CDS data and come to the same conclusion.

Table 2.3. Descriptive statistics of the daily range

	France	Germany	Netherlands	Spain	USA
Mean	0.026	0.025	0.023	0.026	0.021
Median	0.022	0.019	0.018	0.022	0.017
Maximum	0.148	0.178	0.186	0.213	0.174
Minimum	0.005	0.000	0.000	0.003	0.003
Standard deviation	0.016	0.020	0.017	0.018	0.016
Skewness	2.228	2.217	2.541	2.137	3.186
Kurtosis (excess)	7.784	7.443	10.022	8.550	17.658
25-%ile	0.016	0.011	0.012	0.014	0.012
75-%ile	0.031	0.031	0.028	0.033	0.026
ACF(1)	0.610	0.738	0.691	0.628	0.620
ACF(5)	0.529	0.687	0.625	0.541	0.582
ACF(20)	0.411	0.577	0.504	0.412	0.457

Notes: The table reports the descriptive statistics for the daily high-low price range of stock indices, including CAC 40 (France), DAX 30 (Germany), AEX (the Netherland), IBEX 35 (Spain), and S&P500 (USA) over the sample period from January 11, 1991 to May 23, 2013. The sample size is 5,387 observations.

Table 2.4. Cross-autocorrelation matrices for five national stock market indices daily range

		Panel A			
Pre-crisis period					
$\mathbf{Y_0}$	$hl_{FRA,t}$	$hl_{GER,t}$	$hl_{NETH,t}$	$hl_{SPA,t}$	$hl_{USA,t}$
$hl_{FRA,t}$	1.000				
$hl_{GER,t}$	0.764	1.000			
$hl_{NETH,t}$	0.796	0.826	1.000		
$hl_{SPA,t}$	0.746	0.692	0.721	1.000	
$hl_{USA,t}$	0.576	0.649	0.623	0.582	1.000
Crisis period					
$\mathbf{Y_0}$	$hl_{FRA,t}$	$hl_{GER,t}$	$hl_{NETH,t}$	$hl_{SPA,t}$	$hl_{USA,t}$
$hl_{FRA,t}$	1.000				
$hl_{GER,t}$	0.914	1.000			
$hl_{NETH,t}$	0.923	0.881	1.000		
$hl_{SPA,t}$	0.821	0.729	0.721	1.000	
$hl_{USA,t}$	0.759	0.766	0.780	0.581	1.000
		Panel B			
Pre-crisis period					
$\mathbf{Y_1}$	$hl_{FRA,t-1}$	$hl_{GER,t-1}$	$hl_{NETH,t-1}$	$hl_{SPA,t-1}$	$hl_{USA,t-1}$
$hl_{FRA,t}$	0.582	0.581	0.577	0.512	0.467
$hl_{GER,t}$	0.574	0.755	0.661	0.532	0.534
$hl_{NETH,t}$	0.573	0.679	0.698	0.535	0.520
$hl_{SPA,t}$	0.527	0.553	0.540	0.604	0.482
$hl_{USA,t}$	0.473	0.531	0.498	0.479	0.513
Crisis period					
$\mathbf{Y_1}$	$hl_{FRA,t-1}$	$hl_{GER,t-1}$	$hl_{NETH,t-1}$	$hl_{SPA,t-1}$	$hl_{USA,t-1}$
$hl_{FRA,t}$	0.615	0.605	0.615	0.501	0.617
$hl_{GER,t}$	0.628	0.672	0.631	0.484	0.640
$hl_{NETH,t}$	0.608	0.608	0.646	0.457	0.650
$hl_{SPA,t}$	0.520	0.482	0.482	0.561	0.455
$hl_{USA,t}$	0.600	0.621	0.639	0.444	0.683

Notes: Autocorrelation matrices of the vector of daily ranges of five national stock market indices, $X \equiv [hl_{FRA,t}, hl_{GER,t}, hl_{NETH,t}, hl_{SPA,,t}, hl_{USA,t}]$. The k-th order autocorrelation matrix is defined by $\mathrm{Y(k)} \equiv D^{-1/2}E[(X_{t-k} - \mu)(X_t - \mu)']D^{-1/2}$, where $D \equiv Diag(\sigma_1^2,\ldots,\sigma_5^2)$. Hence, the ($i$, j) element of Y(k) corresponds to the correlation between $hl_{i,t-k}$ and $hl_{j,t}$. Following Cipollini and Galo (2010), we choose July 17, 2007 as the regime break point. Hence, we assume that the pre-crisis period extends from January 11, 1991 to July 17, 2007, and the crisis period is from July 18, 2007 to May 23, 2013

2.4.1 Comparison to Close-to-close and Open-to-close Standard Deviations

Table 2.5 shows the sample variances of close-to-close, open-to-close, and close-to-open returns for each of the markets. Ignoring the negligible differences in sample mean, in the absence of return autocorrelation the close-to-close return variance will equal the sum of close-to-open and open-to-close variance, and this is approximately the case. It is interesting to note that the close-to-open variance variance is higher for the European market indices than for the US index. This is not a surprising result; US market moves during the European evening can have a big impact on European market opening values the next (European) morning. The effect is asymmetrical; the US market opening prices are on average fairly close to previous-day closing prices, indicating that they are not as influenced by US-closed-time activity in Asian and European markets.

Table 2.5. Sample variances of close-to-close, open to close, and close-to-open returns

	France	Germany	Netherlands	Spain	USA
Variance (close-to-close)	0.00022	0.00023	0.00020	0.00022	0.00014
Variance (open-to-close)	0.00008	0.00005	0.00008	0.00007	0.00001
Variance (close-to-open)	0.00014	0.00016	0.00013	0.00016	0.00013
Variance ratio: open-to-close/close-to-close	0.6607	0.7063	0.6433	0.7236	0.9284
St. dev. (open-to-close)	0.0119	0.0126	0.0114	0.0126	0.0116
Range-implied open-to-close st.dev.	0.0162	0.0155	0.0143	0.0164	0.0134

Notes: The sample period is from January 11, 1991 to May 23, 2013. In the absence of return autocorrelation the close-to-close return variance will equal the sum of close-to-open and open-to-close variance. We use the mean daily range to compute implied standard deviation; range-implied standard deviation under the Feller/normal congruent distributions: $\sigma_t = \mu_\tau \sqrt{\frac{\pi}{8}}$.

If prices follow zero-mean, fixed-volatility Brownian motion then $\sqrt{\frac{\pi}{8}}$ times the mean daily range is equal to daily return standard deviation. We use the mean daily range statistics from Table 2.3 to compute implied standard deviations in this way, and compare them to the sample standard deviations of the open-to-close returns. In all cases, the range-based standard deviation exceeds the sample-return-based standard deviation.

2.5 Estimation and Testing Given a Single Regime

2.5.1 Estimation of the Univariate Models of Dynamic Range

We begin with the estimation of the base-case model (2.10). Note that there are two variants of the base case model depending upon whether we use the Feller distribution or the gamma distribution for the realized range innovations; using the gamma distribution adds an extra estimated parameter. Table 2.6 shows the model with a Feller distribution in Panel A and with a gamma distribution model in Panel B. The shared parameter estimates are quite similar in the two models; the main difference comes from the extra parameter of the gamma distribution model. We now make a more detailed evaluation of these two models by comparing their one-step-ahead risk forecasts.

Table 2.6. Maximum likelihood estimation of a univariate dynamic model of daily range

Panel A: Estimation using a Feller distribution				
	France	Germany	Netherlands	Spain
ω	0.00014 (42.239)	0.00005 (46.682)	0.00009 (36.024)	0.00016 (137.344)
β	0.822 (2,051.313)	0.808 (4,383.096)	0.813 (2,145.931)	0.771 (4,377.313)
α	0.105 (400.556)	0.118 (935.667)	0.112 (456.852)	0.134 (1,039.930)

Panel B: Estimation using a Gamma distribution				
	France	Germany	Netherlands	Spain
ω	0.00046 (7.094)	0.00018 (5.829)	0.00033 (6.725)	0.00051 (8.816)
β	0.807 (105.895)	0.806 (110.406)	0.785 (96.362)	0.763 (88.609)
α	0.1756 (26.640)	0.1871 (26.802)	0.201 (27.006)	0.217 (27.575)
γ	6.889 (52.044)	6.275 (56.658)	6.406 (54.539)	5.989 (73.360)

Notes: The Table shows the maximum likelihood estimates of univariate dynamic models of daily range. See equation (2.10) for the definitions of the coefficients. The model in Panel A uses: $hl_{it} = \mu_{i,t}\varepsilon_{i,t},\ \varepsilon_{i,t}$ follows a Feller distribution. The model in Panel B uses: $hl_{it} = \mu_{i,t}\varepsilon_{i,t},\ \varepsilon_{i,t} \sim Gamma\left(\gamma_i, \frac{\gamma_i}{\mu_{i,t}}\right)$. The numbers in the parentheses are *t*-statistics. Sample period is from January 11, 1991 to May 23, 2013.

2.5.2 Analyzing the Distributional Characteristics of Daily Range

Recall that realized daily range equals expected daily range conditional upon yesterday's information times a unit mean i.i.d. innovation:

$$hl_{it} = \mu_{it}\varepsilon_{it}, \qquad (2.15)$$

where ε_{it} follows a Feller distribution under our initial specification, or a gamma distribution with parameter γ under the Engle and Gallo (2006) specification. We use Equation (2.15) to examine the one-day-ahead value-at-risk hit rates of our two dynamic models from the last subsection. For each time period, we find the upper limit c_{it} such that the probability (under the given prediction model) that the realized range equals or exceeds it equals α (for $\alpha = .01,\ .05$ and $.10$)

$$c_{it} \text{ s.t. } \Pr\left(hl_{it} \geq c_{it} \middle| \mu_{it}\right) = \alpha.$$

In common parlance, c_{it} for $\alpha = .01,\ .05$ and $.10$ is the value-at-risk for the specified trading strategy at confidence level 99%; 95%; and 90%. Since we are using realized daily range, this is the value-at-risk for the daily loss on the worst potential intraday trade, not the value-at-risk of daily buy-and-hold return.

If the forecasting model is correctly specified, then the dummy variable which equals one if $hl_{it} \geq c_{it}$ and zero otherwise has an i.i.d. binomial distribution with an expected value of α: This is called the hit rate for the value-at-risk forecast. Table 2.7 shows the results. Across all countries, both models have too-high hit rates, particularly for $\alpha = .01$. In most cases (with exceptions only for the 90% value-at-risk using the Gamma distribution) we can reject with 95% confidence that the value-at-risk is correctly given by the model. The performance of the Feller-distribution-based model is notably worse than that of the gamma-distribution-based model in terms of the excessive proportion of hits, but both models are clearly rejected in most cases. Note, as shown above, the shared parameters of the two models are quite similar in their estimated values. The difference between the performance of the two models in Table 2.7 comes from the slightly better ability of the gamma distribution to capture the fairly thick tails of the distribution of realized range.

Table 2.7. Hit rates for VaR events at 90%, 95%, and 99% confidence for two forecasting models of daily range

Model	For 1%	For 5%	For 10%
	France		
Gamma	2.230 (9.075)	6.403 (4.725)	10.709 (1.735)
Feller	3.211 (16.311)	8.593 (12.101)	13.103 (7.592)
	Germany		
Gamma	1.967 (7.134)	6.329 (4.476)	10.542 (1.326)
Feller	3.805 (20.693)	9.744 (15.978)	14.514 (11.045)
	Netherlands		
Gamma	1.707 (5.216)	6.125 (3.789)	11.154 (2.824)
Feller	3.415 (17.816)	9.725 (15.914)	14.681 (11.453)
	Spain		
Gamma	1.890 (6.566)	5.698 (2.351)	10.319 (0.781)
Feller	3.712 (20.007)	9.577 (15.415)	15.052 (12.361)
	USA		
Gamma	1.745 (5.496)	5.234 (0.788)	10.171 (0.418)
Feller	4.306 (24.389)	10.783 (19.477)	16.314 (15.449)

Note: The Table examines the one-day-ahead value-at-risk hit rates of Feller-distribution-based model and Gamma-distribution-based model. For each time period, we find the upper limit c_{it} such that the probability that realized range equals or exceeds it equals α (for α=.01,.05, and .10), c_{it} s.t. $\Pr(hl_{it} \geq c_{it} | \mu_{it}) = \alpha$. If the forecasting model is correctly specified, then the dummy variable which equals one if $hl_{it} \geq c_{it}$ and zero otherwise has an i.i.d. binomial distribution with an expected value of α. Sample period is from January 11, 1991 to May 23, 2013.

Figures 2.1 through 2.10 show the same finding graphically. They show the sample densities of realized range innovations (2.7) and compare them to the theoretical density; in the case of the gamma distribution this differs across countries, dependent upon the estimated $\hat{\gamma}$, whereas for the Feller distribution it is the same for all countries. The better fit of the gamma distribution to the upper tail of realized range seems evident from the graphs.

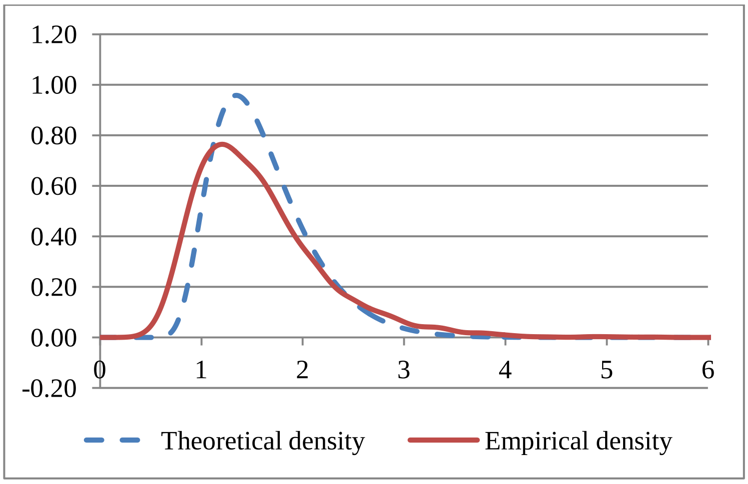

Figure 2.1. Empirical and theoretical densities of range innovations using the Feller distribution model: France

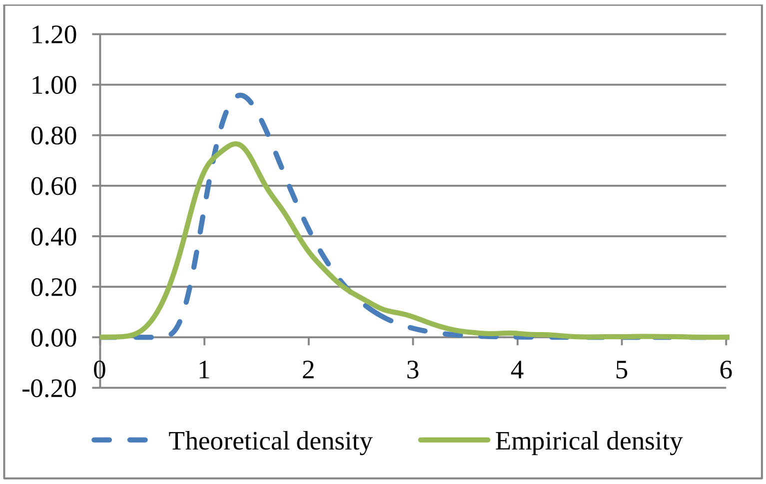

Figure 2.2. Empirical and theoretical densities of range innovations using the Feller distribution model: Germany

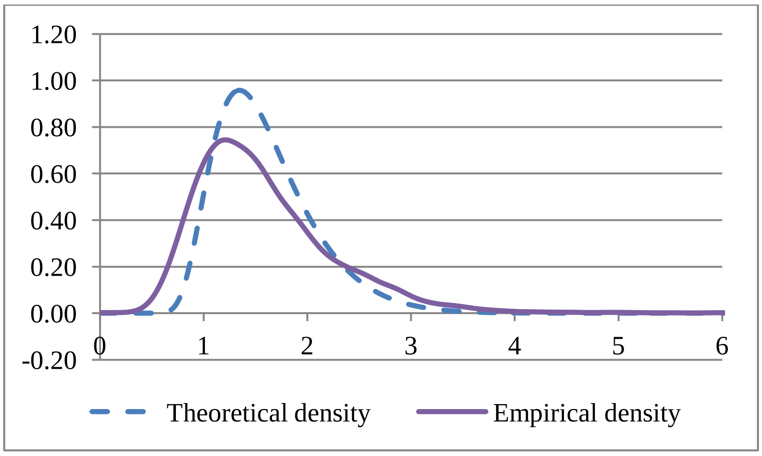

Figure 2.3. Empirical and theoretical densities of range innovations using the Feller distribution model: the Netherlands

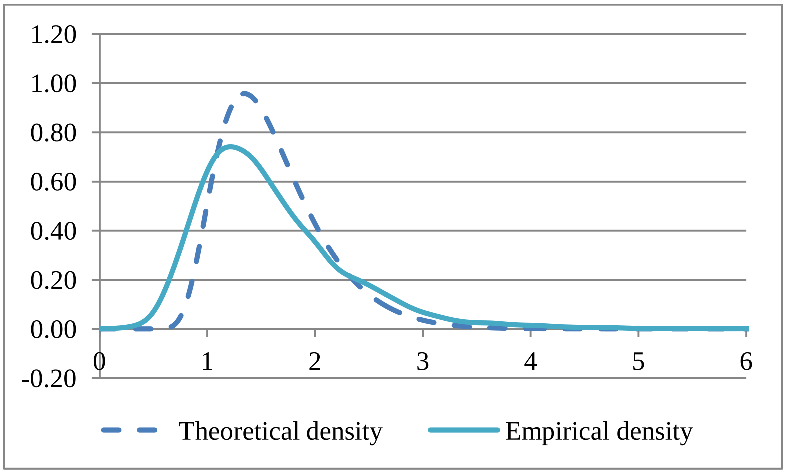

Figure 2.4. Empirical and theoretical densities of range innovations using the Feller distribution model: Spain

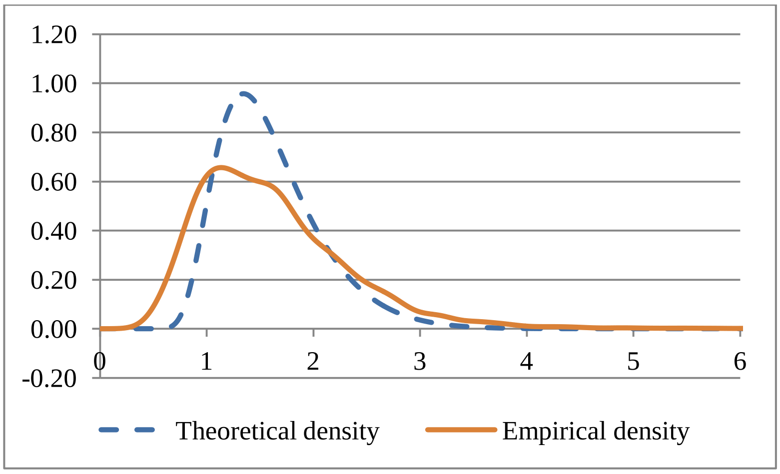

Figure 2.5. Empirical and theoretical densities of range innovations using the Feller distribution model: USA

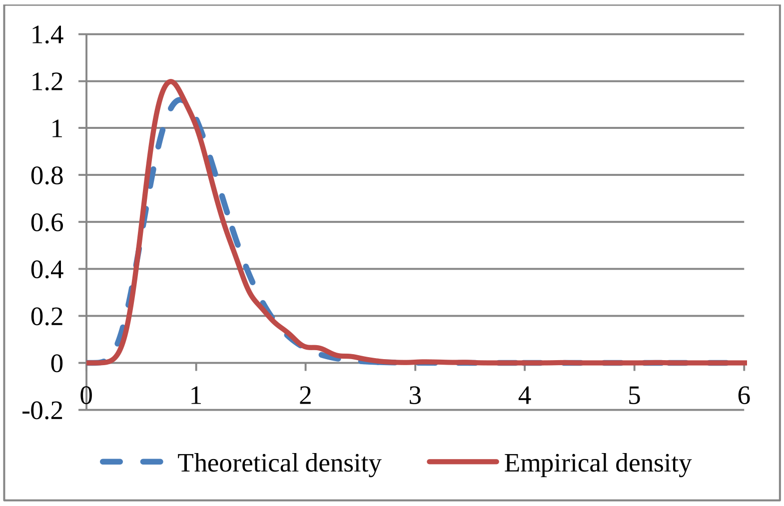

Figure 2.6. Empirical and theoretical distributions of range innovations using the Gamma distribution model: France

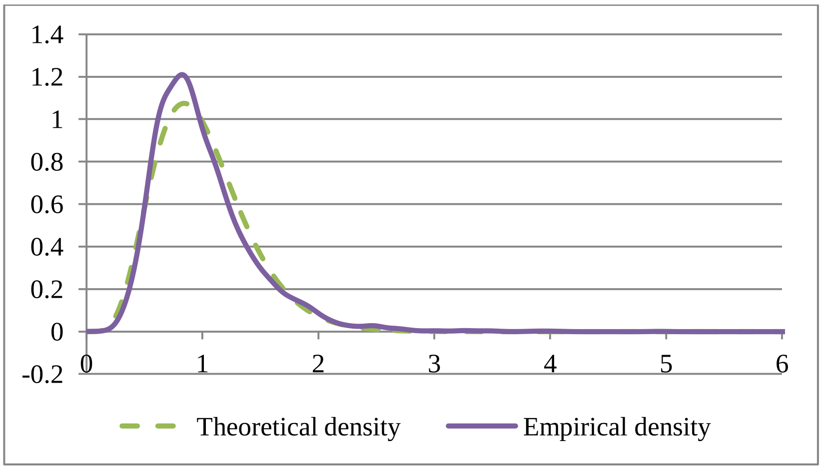

Figure 2.7. Empirical and theoretical distributions of range innovations using the Gamma distribution model: Germany

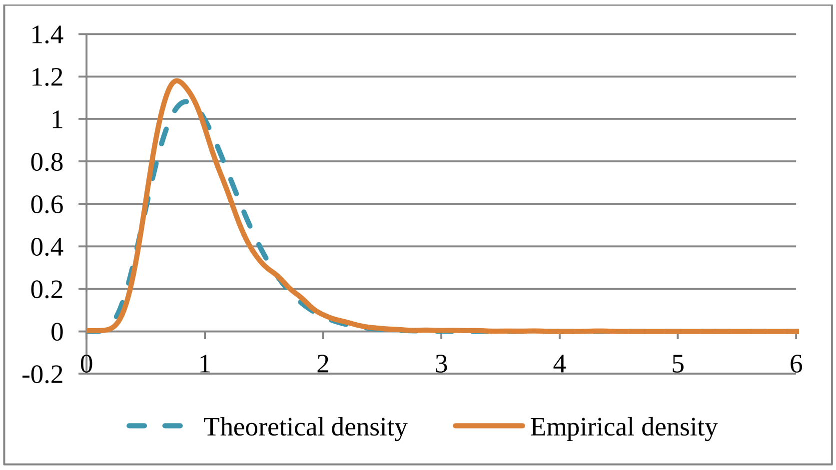

Figure 2.8. Empirical and theoretical distributions of range innovations using the Gamma distribution model: the Netherlands

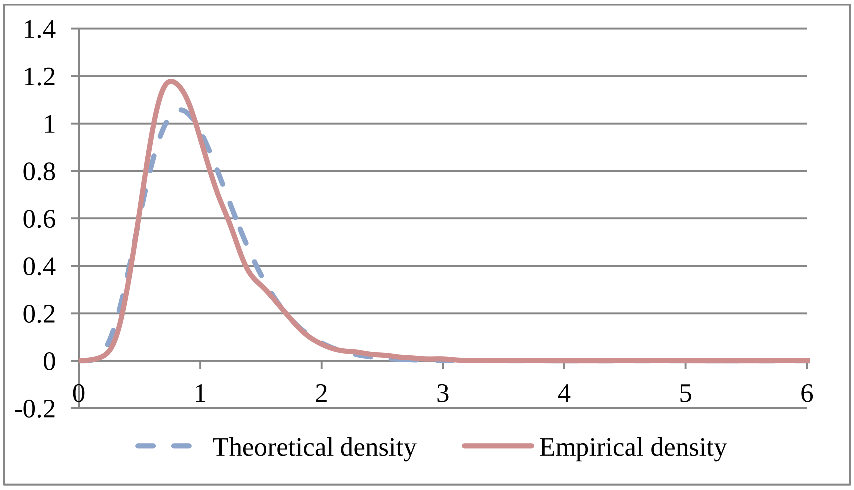

Figure 2.9. Empirical and theoretical distributions of range innovations using the Gamma distribution model: Spain

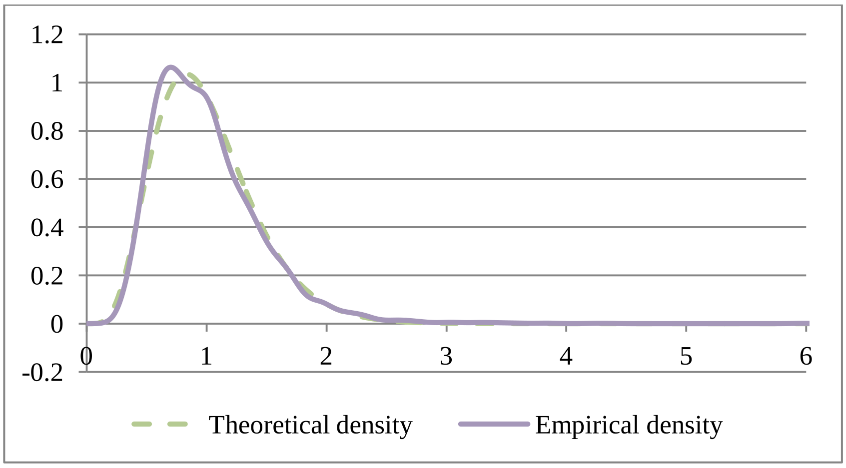

Figure 2.10. Empirical and theoretical distributions of range innovations using the Gamma distribution model: USA

Table 2.8. Maximum likelihood estimation of a univariate dynamic model of daily range with additional return-based explanatory variables

	France		Germany		Netherlands		Spain	
ω	0.00050 (8.656)	0.00053 (9.623)	0.00022 (7.511)	0.00025 (8.391)	0.0038 (8.661)	0.00039 (8.848)	0.00054 (10.340)	0.00054 (10.235)
β	0.826 (127.893)	0.834 (130.764)	0.821 (109.935)	0.818 (112.296)	0.807 (101.446)	0.808 (103.026)	0.788 (96.264)	0.787 (97.160)
α	0.120 (20.159)	0.146 (26.076)	0.142 (17.245)	0.173 (24.579)	0.142 (17.163)	0.175 (24.114)	0.157 (18.543)	0.193 (25.736)
φ*(close-to-close return)*		-0.086 (-17.058)		-0.044 (-10.293)		-0.067 (-14.037)		-0.071 (-13.110)
α^{down}	0.067 (13.454)		0.055 (10.258)		0.065 (12.888)		0.069 (12.402)	
γ	7.057 (52.524)	7.147 (52.564)	6.366 (57.075)	6.354 (57.247)	6.539 (54.834)	6.376 (53.647)	6.115 (73.743)	6.118 (75.150)

Notes: The Table shows the maximum likelihood estimates of a univariate dynamic model of daily range with additional return-based explanatory variable. See equations (2.11) and (2.12) for the definitions of the coefficients. The model uses: $hl_{it} = \mu_{i,t}\varepsilon_{i,t},\ \ \varepsilon_{i,t} \sim Gamma\left(\gamma_i, \frac{\gamma_i}{\mu_{i,t}}\right)$. The numbers in the parentheses are *t*-statistics. Sample period is from January 11, 1991 to May 23, 2013.

2.5.3 Leverage Effects and Volatility Spillovers Across Markets

In this section we estimate using the gamma distribution for the realized range innovations. Table 2.8 shows estimates for the two models, Equations (2.11) and (2.12), with leverage effects. In the first specification, the impact of yesterday's realized range on today's expected range is higher if yesterday's market return is negative. In the second specification, today's expected range is negatively related to yesterday's market return. These two specifications are quite similar in practice, since realized range tends to be strongly collinear with the absolute value of realized return. Using either leverage specification, we find significant evidence for substantial leverage effects in the dynamics of daily range.

We next show estimates of the models (Table 2.9) including cross-market lagged range as an explanatory variable, to test for volatility spillovers. The influence of lagged cross-market range tends to be much smaller than the influence of lagged own-market range. We find positive statistically significant range-based volatility spillover effects coming from Spain. This result is consistent with the paper by Alter and Beyer (2013) that shows that the core countries are highly sensitive to shocks from periphery countries such as Spain, Portugal, and Italy. We also find statistically significantly negative coefficient for the realized range on yesterday's French market which implies that the French equity market is the net receiver of potential spillovers. This result is also consistent with the finding of Alter and Beyer (2013) that finds a negative total net spillover effect. The lagged US market range has the most reliable influence, both in terms of uniform statistical significance across the European countries, and in terms of the magnitude of the estimated coefficients. Note that, due to time zone differences, the realized range on yesterday's US market includes price moves during trading time after the close of yesterday's European markets, but before the current day's market open.

Table 2.9. Single-equation maximum likelihood estimation of multivariate models of daily range

	France	Germany	Netherlands	Spain
ω	0.00050	0.00017	0.00029	0.00029
	(6.496)	(2.999)	(4.139)	(3.730)
β	0.784	0.802	0.755	0.744
	(88.414)	(101.556)	(77.275)	(78.005)
α	0.155	0.186	0.179	0.206
	(20.864)	(23.358)	(20.303)	(22.696)
$FRA_{t\text{-}1}$		-0.010	-0.009	0.021
		(-2.953)	(-2.120)	(5.567)
$GER_{t\text{-}1}$	0.002		0.020	0.008
	(0.443)		(4.475)	(1.485)
$NETH_{t\text{-}1}$	0.004	0.003		-0.002
	(0.577)	(0.684)		(-0.227)
$SPA_{t\text{-}1}$	0.017	0.006	0.012	
	(4.229)	(1.982)	(3.387)	
$USA_{t\text{-}1}$	0.024	0.008	0.030	0.016
	(4.676)	(2.159)	(6.148)	(2.720)
γ	6.952	6.288	6.485	6.029
	(51.677)	(56.500)	(53.498)	(58.686)

Notes: The Table shows the maximum likelihood estimates of multivariate models of daily range, based on equation (2.6). The model uses: $hl_{it} = \mu_{i,t}\varepsilon_{i,t},\ \ \varepsilon_{i,t} \sim Gamma\left(\gamma_i, \frac{\gamma_i}{\mu_{i,t}}\right)$. The numbers in the parentheses are *t*-stats. Sample period is from January 11, 1991 to May 23, 2013.

2.6 Testing for a Regime Shift During the Financial Crisis

The latter part of our sample is characterized by unusual market turbulence associated with the global financial crisis. We re-estimate with an assumed regime break differentiating the pre-crisis and crisis periods. Following Cipollini and Gallo (2010), we choose July 17, 2007 as the regime break point. This date corresponds to the announcement by Bear Stearns of the collapse of two hedge funds, and was followed by suspension of payments by BNP Paribas and increased support facilities by the ECB and Fed in early August 2007. We also applied the Chow stability test to the chosen sub-periods. The results rejected the hypothesis of no break for all European markets at hand. Hence, we assume that the crisis period extends from July 18, 2007 to the end of our sample on May 23, 2013.

Table 2.10 gives the descriptive statistics in the pre-crisis and crisis periods. Not surprisingly, both the mean and median of daily range increases sharply in all four markets. Table 2.4 shows both contemporaneous and lagged auto-correlations and cross-correlations. There is a notable increase in contemporaneous correlations between the markets. Autocorrelations do not show a pattern: some increase and some decrease. First-order cross-correlations show a pattern similar to contemporaneous correlations, that is, increasing in most cases.

Table 2.10. Pre-crisis and crisis period descriptive statistics of the daily ranges

	France	Germany	Netherlands	Spain	USA
Pre-crisis period (January 11, 1991 to July 17, 2007)					
Mean	0.0240	0.0226	0.0209	0.0229	0.0194
Median	0.0203	0.0166	0.0162	0.0188	0.0162
Maximum	0.1404	0.1735	0.1860	0.1823	0.1353
Minimum	0.0047	0.0004	0.0009	0.0026	0.0028
Standard deviation	0.0142	0.0189	0.0160	0.0154	0.0124
Crisis period (July 18, 2007 to May 23, 2013)					
Mean	0.0309	0.0306	0.0278	0.0352	0.0211
Median	0.0263	0.0252	0.0230	0.0304	0.0209
Maximum	0.1478	0.1778	0.1489	0.2130	0.1740
Minimum	0.0052	0.0037	0.0000	0.0083	0.0045
Standard deviation	0.0186	0.0205	0.0184	0.0196	0.0211

Notes: The table reports the descriptive statistics for the daily high-low price range of stock indices, including CAC 40 (France), DAX 30 (Germany), AEX (the Netherland), IBEX 35 (Spain), and S&P500 (USA). Following Cipollini and Galo (2010), we choose July 17, 2007 as the regime break point. Hence, we assume that the pre-crisis period extends from January 11, 1991 to July 17, 2007, and the crisis period is from July 18, 2007 to May 23, 2013.

Table 2.11 shows the model with cross-country linkages (2.6) estimated for the full sample with the inclusion of a multiplicative dummy variable DC_t for each cross-country coefficient. The dummy variable is one in the crisis period and zero in the pre-crisis period; the associated coefficients capture the change in the coefficient in the crisis period. There is no sign of an increase in the cross-market dynamic linkages across the European markets, in fact, several dummy coefficients indicate a significant decrease. Particularly notable is the increased influence of yesterday's realized US range on today's expected European range – this is significantly positive for all four European countries. So the influence of the lagged US market increased during the crisis period, but the cross-market influences among these European countries did not. Table 2.12 shows the full model estimated separately on the crisis period and pre-crisis period. The results mirror those in Table 2.11. The only notable change between the pre-crisis and crisis period is that the influence of the lagged US market range increased in all markets.

2.7 Conclusion

This chapter examines the daily risk dynamics and inter-market linkages of four European stock markets using daily range data. Daily range can provide an accurate indirect measure of daily volatility and is readily available across markets with no publicly-available intraday price series. We compare the conditional autoregressive range model of Engle and Gallo (2006) in which the realized range has a gamma distribution to a new formulation in which intraday returns are normally distributed and realized range has a Feller distribution. The two models give similar estimates for the autoregressive range dynamics, but the gamma-distribution-based model better captures the leptokurtotic feature observed in daily range data.

In addition to strong autoregressive dynamics, the expected range varies inversely with the previous day's return. There are also some spillover effects, so that the previous day's realized range in other European market positively influences the next day's expected range. These spillover effects are not uniform across the markets; the strongest spillover comes from the previous day's realized range of the US market index. We find statistically significantly negative coefficient for the realized range on yesterday’s French market which implies that the French equity market is the net receiver of potential spillovers. This result is also consistent with the finding of Alter and Beyer (2013) who also find a negative total net spillover effect. We also compare the pre-crisis (January 11, 1991 to July 17, 2007) and European financial crisis (July 18, 2007 to May 23, 2013) sub-periods of our sample. In all four markets, average daily range increased sharply during the crisis period, and the contemporaneous correlations between the markets increased in most cases. Spillover effects between European markets did not seem to change, but the influence of yesterday’s US market range on realized range in European markets increased.

Table 2.11. Extended model estimation (single-equation ML) using a Gamma distribution

	France	Germany	Netherlands	Spain
ω	0.00054	0.00019	0.00033	0.00028
	(6.095)	(2.892)	(4.029)	(2.945)
β	0.777	0.798	0.749	0.723
	(82.842)	(96.873)	(73.559)	(68.934)
α	0.163	0.191	0.186	0.197
	(20.031)	(22.835)	(20.119)	(20.083)
$FRA_{t\text{-}1}$		-0.007	-0.008	0.035
		(-2.068)	(-1.747)	(8.238)
$GER_{t\text{-}1}$	0.004		0.021	0.009
	(0.811)		(4.558)	(1.531)
$NETH_{t\text{-}1}$	0.004	0.006		0.007
	(0.522)	(1.190)		(0.842)
$SPA_{t\text{-}1}$	0.022	0.004	0.016	
	(3.527)	(1.085)	(3.188)	
$USA_{t\text{-}1}$	0.012	0.002	0.019	0.020
	(1.946)	(0.520)	(3.515)	(2.965)
$DC_{t\text{-}1}$	0.0003	0.0003	0.0004	0.0010
	(1.828)	(2.032)	(1.879)	(4.214)
$FRA_{t\text{-}1}\ DC_{t\text{-}1}$	-0.082	-0.010	-0.007	-0.056
	(-2.842)	(-0.393)	(-0.255)	(-1.199)
$GER_{t\text{-}1}\ DC_{t\text{-}1}$	0.016	-0.032	0.0002	-0.004
	(0.891)	(-1.863)	(0.013)	(-0.1331)
$NETH_{t\text{-}1}\ DC_{t\text{-}1}$	-0.004	-0.019	-0.056	-0.039
	(-0.154)	(-0.865)	(-2.348)	(-1.090)
$SPA_{t\text{-}1}\ DC_{t\text{-}1}$	0.004	0.003	-0.011	0.033
	(0.405)	(0.295)	(-1.139)	(1.836)
$USA_{t\text{-}1}\ DC_{t\text{-}1}$	0.063	0.053	0.068	0.034
	(4.460)	(4.156)	(4.937)	(1.730)
γ	76.984	6.318	6.517	6.084
	(51.363)	(56.240)	(53.137)	(57.309)

Notes: The Table shows the maximum likelihood estimates of extended models of daily range. The model is the same as in Table 2.9 with the addition of multiplicative dummies for cross-country coefficients during the crisis period. The numbers in the parentheses are *t*-statistics. Sample period is from January 11, 1991 to May 23, 2013.

Table 2.12. Extended model estimation (single-equation ML) using a Gamma distribution over the pre-crisis period and over the crisis period

	France		Germany		Netherlands		Spain	
	Pre-crisis	Crisis	Pre-crisis	Crisis	Pre-crisis	Crisis	Pre-crisis	Crisis
ω	0.00040	0.00174	0.00016	0.00100	0.00029	0.00098	0.00027	0.00193
	(5.138)	(5.555)	(2.571)	(4.085)	(3.580)	(4.252)	(2.688)	(5.680)
β	0.812	0.669	0.818	0.719	0.767	0.700	0.725	0.692
	(89.226)	(26.515)	(93.984)	(33.624)	(68.654)	(31.374)	(59.278)	(34.561)
α	0.144	0.097	0.172	0.218	0.176	0.152	0.196	0.246
	(19.339)	(2.530)	(19.818)	(7.898)	(18.354)	(5.293)	(17.795)	(11.145)
FRA_{t-1}			-0.006	-0.030	-0.007	-0.023	0.0344	-0.019
			(-1.914)	(-0.912)	(-1.599)	(-0.786)	(7.720)	(-0.428)
GER_{t-1}	0.002	0.039			0.019	0.032	0.009	0.012
	(0.542)	(1.583)			(4.128)	(1.587)	(1.422)	(0.461)
$NETH_{t-1}$	0.003	0.008	0.005	-0.015			0.007	-0.042
	(0.405)	(0.236)	(1.150)	(-0.549)			(0.821)	(-1.277)
SPA_{t-1}	0.017	0.033	0.004	0.007	0.014	0.003		
	(3.101)	(2.613)	(1.092)	(0.637)	(2.932)	(0.322)		
USA_{t-1}	0.010	0.109	0.001	0.076	0.017	0.104	0.020	0.060
	(1.896)	(5.842)	(0.343)	(4.816)	(3.287)	(7.083)	(2.840)	(3.330)
γ	7.128	6.649	6.313	6.329	6.393	6.876	5.827	6.869
	(44.335)	(26.231)	(49.713)	(26.377)	(46.067)	(26.346)	(49.944)	(25.874)

Notes: The models are the same as in Table 2.7 but estimated separately on the pre-crisis and crisis periods. The numbers in the parentheses are *t*-statistics. Following Cipollini and Galo (2010), we choose July 17, 2007 as the regime break point. Hence, we assume that the pre-crisis period extends from January 11, 1991 to July 17, 2007, and the crisis period is from July 18, 2007 to May 23, 2011.

Chapter 3: Measuring Equity Risk Exposures with Range-based Correlations

3.1 Introduction

Our objective in this paper is to use information extracted from the daily opening, closing, high, and low prices of the stocks to improve the estimation of the current betas and the predictions of the future betas. We create a new time-varying beta measure called "range-based beta", which is based on the daily range-based volatility and covariance estimators of Rogers and Zhou (2008) for estimating market beta. Within this context, the range-based beta is the ratio of the range-based covariance of stock and market to the range-based market variance. In light of the success of the range-based volatility estimator, it is natural to inquire whether the realized range beta is more efficient than the return-based beta**.** Rogers and Zhou (2008) construct an unbiased correlation estimator which is a quadratic function of the high, low, and closing log-price of the two assets, and which has the smallest Mean Squared Error (MSE) in the class of quadratic estimators. In addition, we improve the specification of betas by combining the parametric and non-parametric approaches to modelling time variation in betas. Since the main strengths of each approach are the most important weaknesses of the other, we show that a combination of the two methods leads to more accurate betas than those obtained from each of the two methods separately. MSE is used as a measure of accuracy for the beta estimation. We estimate both our new range-based beta measure and betas extracted using traditional methodologies and compare their performance. Specifically, we compare our range-based betas with betas extracted from the conditional CAPM with time-varying betas. This technique estimates beta based on traditional (co)variance estimates from historical stock returns and takes this estimate as a forecast for the future. We also consider the commonly used historical rolling window beta method. In contrast to the historical return-based methodology that is subject to the critical assumption that betas are stable over time, the information in range-betas allows us to construct ex ante beta predictors assuming only the beta is stable during each day. These range-based betas reflect current day's market information, and, hence, avoid the weakness of historical betas, which are not as responsive to changing market conditions.

We analyse the constituents of the DAX index for the period 2003-2011. We find that the range-based beta measure yields estimates of firm-level betas competitive with historical betas. The use of intraday high and low prices for beta measurement is complicated by infrequent trading. Trading does not occur continuously, that is, in practice we observe transactions at irregularly spaced points in time (Engle, 2000). For the range-based estimators, non-trading introduces a bias as the observed intraday high and low prices are likely to be below and above their 'true' values. Therefore, we expect the range-based beta to be closer to the 'true' beta for highly liquid assets. Hence, we sort stocks into three portfolios according to their turnover measure. We find that the range-based beta approach yields betas competitive with historical betas for the portfolios sorted according to their turnover measure.

The range-based beta is appealing for the ease of its estimation. The construction of the range-based beta requires only the current's day high, low, closing, and opening prices. In addition, this paper is first to develop the range-based covariance and correlation measures that can be applied for equities.

We proceed with the following steps. First, we propose a new way to model range-based correlations, which are based on the range-based covariance and variance estimators of Rogers and Zhou (2008). Second, we estimate the range-based covariance and correlation measures and compare them with the close-to-close return-based measures. Third, we compare the range-based betas with the betas generated by the rolling window model and by the conditional CAPM with time-varying coefficients. Fifth, we perform cross-sectional analysis. Concluding remarks and directions for future research are presented in the final section.

3.2 A Single Factor Model

In this section we present the underlying stock market model – a linear factor model – and its asset pricing implications and discuss the importance as well as ways to estimate factor betas. Our economy contains *N* traded assets, $i = 1,..., N$. Suppose that there is a single market factor that enters linearly in the pricing equation such as in the Sharpe-Lintner version of CAPM model. Under this model, the specification for the return of asset *i* is at time *t*:

$$r_{it} = \alpha_{it} + \beta_i r_{Mt} + \varepsilon_{it,} \quad t = 1,...T, \tag{3.1}$$

where $\alpha_{it} = (1 - \beta_i)r_0$ and r_0 is a risk-free rate, r_{Mt} denotes the common factor market return. ε_{it} is the "non-systematic" risk component. The standard APT structure assumes constant betas, idiosyncrasies uncorrelated with the factor(s) and idiosyncrasies uncorrelated with each other:

$$E(r_{Mt}, \varepsilon_{it}) = 0, \qquad \forall i, \tag{3.2}$$

$$E(\varepsilon_{jt}, \varepsilon_{it}) = 0, \qquad \forall i \neq j. \tag{3.3}$$

The beta coefficient β_i can be represented through the Security Characteristic Line (SCL). For the ease of exposition, it will be assumed that markets are efficient and the expected value of the returns in excess of the compensation for the risk is zero for all portfolios. It is also assumed that the effective risk-free rate does not change significantly and hence will be assumed to be zero. The resulting equation of the SCL is

$$r_{it} = \beta_i r_{Mt} + \varepsilon_{it}. \tag{3.4}$$

Now, the SCL represents the relationship between the return of a given asset *i* at time *t* with the return of the market r_{Mt} and a sensitivity measure of beta β_i. Beta is a sensitivity measure that describes the relationship of an asset's return in reference to the return of a financial market or index. Beta is defined as

$$\beta_i = \frac{Cov(r_{it}, r_{Mt})}{Var(r_{Mt})}. \tag{3.5}$$

Specifically, beta measures the statistical variance or systemic risk of an asset that cannot be mitigated through diversification.

3.2.1 Range-based Volatility and Correlation

In this section we show how one can use information extracted from the daily opening, closing, high and low prices of the stocks to obtain range-based volatilities and correlations, and then use these predictors in the computations of beta.

Formally, we consider two assets, where the log of the asset prices follows a bivariate zero drift Brownian motion, and we allow for the possibility that the asset returns are correlated

$$dP = \sigma_{Pt} dW \,, \tag{3.6}$$

$$dM = \sigma_{Mt} dZ \,, \tag{3.7}$$

$$E\left(\begin{bmatrix} dW \\ dZ \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{3.8}$$

$$E\left(\begin{bmatrix} (dW)^2 & dZdW \\ dWdZ & (dZ)^2 \end{bmatrix}\right) = \begin{bmatrix} d\tau & \rho_{PMt} d\tau \\ \rho_{PMt} d\tau & d\tau \end{bmatrix} \quad \sigma_{PMt} = \sigma_{Pt} \cdot \sigma_{Mt} \cdot \rho_{PMt} \,, \tag{3.9}$$

where W and Z are zero drift Brownian Motions.[1] P and M denote log-prices of assets "*P*" and "*M*", respectively. Hence we can interpret dP and dM as the continuously compounded returns. Equations (3.6)-(3.7) describe the evolution of log-price processes within a time interval, $0 \le \tau \le T_0$. We think of this interval as one trading day, but it could be defined over any interval. Our model also uses a discrete index t for days. The parameters σ_{Pt}, σ_{Mt}, and ρ_{PMt} stay constant during the trading day t, but may vary from day to day.

For simplicity we further assume that P and M are standard Brownian motions, that is $\sigma_{Pt} = \sigma_{Mt} = 1$. In this case, $\rho_{PMt} = \sigma_{PMt}$ during the day t. We next

[1] This assumption, used by various authors, is quite innocent if the data is being sampled intra daily, as the growth rate is negligible in comparison with fluctuations.

apply **Theorem 1** of Rogers and Zhou (2008) where the correlation over a fixed time interval [0,1] ρ_{PMt} is defined as follows:

$$\rho_{PMt} = \frac{1}{2} S_{Pt} S_{Mt} + \frac{1}{2(1-2b)} \left(H_{Pt} + L_{Pt} - S_{Pt}\right)\left(H_{Mt} + L_{Mt} - S_{Mt}\right), \qquad (3.10)$$

where the constant b is equal to $2\log 2 - 1 \cong 0.386294$. $H_{Pt} \equiv \max_{0 \le t \le 1} P_\tau$ and $H_{Mt} \equiv \max_{0 \le \tau \le 1} M_\tau$ denote the high log-prices of assets *P* and *M*, $L_{Pt} \equiv \min_{0 \le t \le 1} P_\tau$ and $L_M \equiv \min_{0 \le \tau \le 1} M(\tau)$ denote the low log-prices of assets *P* and *M*, $S_P = P(1)$ and $S_{Mt} = M(1)$ denote the close log-prices of assets *P* and *M*. Rogers and Zhou (2008) construct an unbiased range-based correlation estimator which is a quadratic function of the high, low, and closing (log-)price of the two assets, and which has smallest MSE. Rogers and Zhou (2008) construct various moments for correlation, subject to the constraint that the estimator has no bias if $\rho = -1,\ 0,\ 1$. This produces a new estimator whose variance is half that of the obvious estimator based solely on closing prices. They also present simulation evidences that this advantage appears to be preserved for other values of ρ and is partly robust to departures from Gaussian returns. The form of the estimator is, moreover, insensitive to errors produced by discrete sampling of the underlying Brownian motions, a problem encountered with some other range-based estimators. Also note that if we are trying to produce an estimate of the covariance matrix of more than two Brownian motions, estimating each entry by means of Equation (3.10), then the matrix will be of rank 2 and nonnegative definite. Another problem identified in the earlier literature with estimators based on high and low values occurs when we observe the Brownian motions discretely, at *N* equally spaced times, say we observe $H^{(N)} \equiv \sup\{X(i/N): i = 0,...,N\}$ and $L^{(N)} \equiv \inf\{X(i/N): i = 0,...,N\}$, and these substantially underestimate the supremum and overestimate the infimum. A correction is known to deal with this (see Broadie et al., 1997), but we see that as we only ever need to calculate $H + L$, the discretization errors cancel out on average because of the observation that $H - H^{(N)}$ and $L^{(N)} - L$ have the same distribution, by symmetry.

Rogers and Zhou (2008) Theorem 1 and the proof of the Theorem 1 are described in Appendix A.

We relax the assumption that the time starts at day 0, which implies that opening prices are not equal to 0. We also relax the standardization that *P* and *M* are standard Brownian motions, that is $\sigma_{Pt} = \sigma_{Mt} = 1$ during the day, the covariance estimator σ_{PMt} from (3.10) is then given by:

$$\sigma_{PMt} = \frac{1}{2}(S_{Pt} - O_{Pt})(S_{Mt} - O_{Pt}) + \frac{1}{2(1-2b)}(H_{Pt} + L_{Pt} - S_{Pt} - O_{Pt})(H_{Mt} + L_{Mt} - S_{Mt} - O_{Mt}), \tag{3.11}$$

where O_{Pt} and O_{Mt} denote opening log-prices for assets *P* and *M*, respectively. All other variables are defined as before.

Based on the covariance estimator in (3.11), the variance estimator for the asset *P* is simply

$$hl_{Pt}^{RZ} = \frac{1}{2}(S_{Pt} - O_{Pt})^2 + \frac{1}{2(1-2b)}(H_{Pt} + L_{Pt} - S_{Pt} - O_{Pt})^2. \tag{3.12}$$

Note that this estimator is a linear combination of Garman and Klass (1980) volatility estimator, which utilizes the open, close, high, and low prices. The Garman-Klass estimator hl_{Pt}^{GK} is defined as

$$hl_{Pt}^{GK} = 0.511(H_{Pt} - L_{Pt})^2 - 0.019(S_{Pt}(H_{Pt} - L_{Pt}) - 2H_{Pt}L_{Pt}) - 0.383S_{Pt}^2. \tag{3.13}$$

A close-to-close volatility estimator has by definition an efficiency gain (ratio of estimated variance) equal to 1. Garman-Klass volatility estimator is theoretically 7.4 times more efficient than simple close-to-close volatility estimator (see Appendix B for the deviation).

Thus far, we have said little about the theoretical properties of the range-based volatility and correlation estimators introduced by Rogers and Zhou (2008). One obvious point is that our variance estimator is unbiased under the same conditions that deliver unbiasedness of the Garman-Klass variance estimator (see Appendix B for the unbiasedness properties of the Garman-Klass estimator), because the Rogers and Zhou (2008) and the Garman-Klass variance estimators are linear combinations. Namely, for the Wiener process defined by Equation (3.6)-(3.9), the Garman-Klass variance estimator is unbiased only if the drift is equal to zero. In general, $E[hl_{PMt}] \neq \sigma^2$ if $\mu \neq 0$. This is a shortcoming of the Garman-Klass variance estimator. Conversion to correlation, however, will introduce bias due to the nonlinearity of the transformation. A similarly related point is that the estimated variance-covariance matrix $\hat{\Sigma}$, in general, is not guaranteed to be positive definite. However, as Brandt and Diebold (2006) point out, positive definiteness is rarely violated in practice. However, we are not interested in the theoretical properties of the range-based volatility and correlation estimates under abstract conditions surely violated in practice, but rather on their performance in realistic situations involving small samples, discrete sampling, and market microstructure noise. As we argued previously, we have reasons to suspect the good performance of the range-based approach, because of both its high efficiency due to the use of the information in the intraday sample path and its robustness to microstructure noise.

Finally, from (3.9) we can express the correlation ρ_{PMt} and plugging the value for σ_{PMt} (3.11) and hl_{Pt}^{RZ} (3.12), the range-based correlation is defined as

$$\hat{\rho}_{PMt} = \frac{\frac{1}{2}(S_{Pt} - O_{Pt})(S_{Mt} - O_{Mt}) + \frac{1}{2(1-2b)}(H_{Pt} + L_{Pt} - S_{Pt} - O_{Pt})(H_{Mt} + L_{Mt} - S_{Mt} - O_{Mt})}{\sqrt{\frac{1}{2}(S_{Pt} - O_{Pt})^2 + \frac{1}{2(1-2b)}(H_{Pt} + L_{Pt} - S_{Pt} - O_{Pt})^2}\sqrt{\frac{1}{2}(S_{Mt} - O_{Mt})^2 + \frac{1}{2(1-2b)}(H_{Mt} + L_{Mt} - S_{Mt} - O_{Mt})^2}}, \tag{3.14}$$

where $b = 2\log 2 - 1 \cong 0.386294$ and the rest of the variables is in the usual notation.

3.3 Empirical Results

3.3.1 Data Description

We consider 21 individual stocks in the DAX index (constituents in October 2011) obtained from Datastream, where the data consists of high, low, opening and closing transaction prices sampled at the daily frequency. For all the stocks the sample period runs from January 2, 2003, to September 30, 2011. Table 3.1 reports some sample statistics on the distribution of the 21 ranges of individual stocks and the DAX index based on daily frequency, in addition to the close-to-open squared return (Table 3.2). The range data exhibit significant departure from the normal distribution for most cases. Interestingly, this departure is smaller compared with return data. The most volatile stocks in the sample are BEIERSDORF and VOLKSWAGEN VZ, whereas the least volatile are E.ON N and ADIDAS N.

Table 3.1. Summary statistics of the range-based volatility measure

Name	Mean	Skewness	Kurtosis	Std. Dev.	Max
Components for DAX					
ADIDAS N	0.00029	5.924	52.948	0.00055	0.0078
BAYER N	0.00044	16.201	350.263	0.00174	0.0484
BEIERSDORF	0.00040	46.514	2,183.465	0.00608	0.2858
BMW	0.00042	10.411	159.845	0.00105	0.0217
COMMERZBANK	0.00078	9.483	139.557	0.00207	0.0443
DAIMLER N	0.00047	21.180	643.881	0.00148	0.0506
DEUTSCHE BANK N	0.00051	7.021	73.009	0.00125	0.0192
E.ON N	0.00030	7.157	72.151	0.00064	0.0085
FRESENIUS MED CARE	0.00026	26.284	941.423	0.00081	0.0309
FRESENIUS	0.00043	4.385	30.007	0.00070	0.0079
HEIDELBERGCEMENT	0.00070	6.802	76.858	0.00148	0.0268
HENKEL VZ	0.00028	22.071	671.881	0.00085	0.0294
LINDE	0.00030	19.817	592.187	0.00082	0.0276
MAN	0.00055	8.335	108.563	0.00126	0.0247
MERCK	0.00036	13.029	295.675	0.00078	0.0219
MUNICHRE	0.00036	11.570	231.751	0.00100	0.0263
RWE	0.00028	14.838	321.166	0.00078	0.0212
K+S N	0.00057	7.570	91.902	0.00131	0.0244
SIEMENS N	0.00038	38.298	1,660.2	0.00212	0.0931
THYSSENKRUPP	0.00049	5.1367	40.553	0.00091	0.0108
VOLKSWAGEN VZ	0.00070	19.085	523.684	0.00250	0.0803
DAX Index	0.000018	10.264	171.632	0.000436	0.01018

Notes: The Table reports the summary statistics for the range data for the sample January 2003 to September 2011, including altogether 2,228 observations. We report the sample mean, skewness, kurtosis, standard deviation, minimum, and maximum for the range-based volatility. The range-based volatility estimator is defined in Equation (3.12).

Table 3.2. Summary statistics of the return-based (close-to-open) volatility measure

Name	Mean	Skewness	Kurtosis	Std. Dev.	Max
Components for DAX					
ADIDAS N	0.0011	27.798	1,100.853	0.034	1.363
BAYER N	0.0004	1.169	33.900	0.022	0.337
BEIERSDORF	0.0006	27.159	1068.206	0.028	1.092
BMW	0.0003	0.084	7.656	0.021	0.138
COMMERZBANK	-0.0006	-0.480	12.796	0.032	0.206
DAIMLER N	0.0001	0.221	10.973	0.023	0.194
DEUTSCHE BANK N	-0.0002	0.215	13.168	0.027	0.212
E.ON N	0.0005	0.604	46.286	0.022	0.312
FRESENIUS MED CARE	0.0012	27.612	1090.267	0.027	1.083
FRESENIUS	0.0014	22.483	828.939	0.030	1.110
HEIDELBERGCEMENT	-0.0001	-0.031	14.256	0.028	0.188
HENKEL VZ	0.0009	25.341	971.319	0.029	1.104
LINDE	0.0005	0.298	8.859	0.019	0.155
MAN	0.0007	0.245	44.681	0.028	0.421
MERCK	0.0005	-0.390	9.133	0.019	0.101
MUNICHRE	0.0000	0.004	10.369	0.020	0.135
RWE	0.0002	0.152	12.012	0.018	0.155
K+S N	0.0013	-0.145	7.887	0.026	0.150
SIEMENS N	0.0002	-0.327	16.076	0.022	0.216
THYSSENKRUPP	0.0004	-0.174	82.440	0.029	0.489
VOLKSWAGEN VZ	0.0007	-0.546	12.504	0.026	0.180
DAX Index	0.0003	0.0004	9.045	0.015	0.108

Notes: The Table reports the summary statistics for the log close-to-open returns for the sample January 2003 to September 2011, including altogether 2,228 observations. We report the sample mean, skewness, kurtosis, standard deviation, minimum, and maximum for the log close-to-close returns.

In Figures 3.1 and 3.2, we first provide a time-series plot of the daily realized market variance calculated using the return-based (close-to-open) and the range-based approach (3.12), respectively. One can see very clearly that both models exhibit similar patterns. The average daily realized market variance is 0.002 for both models.

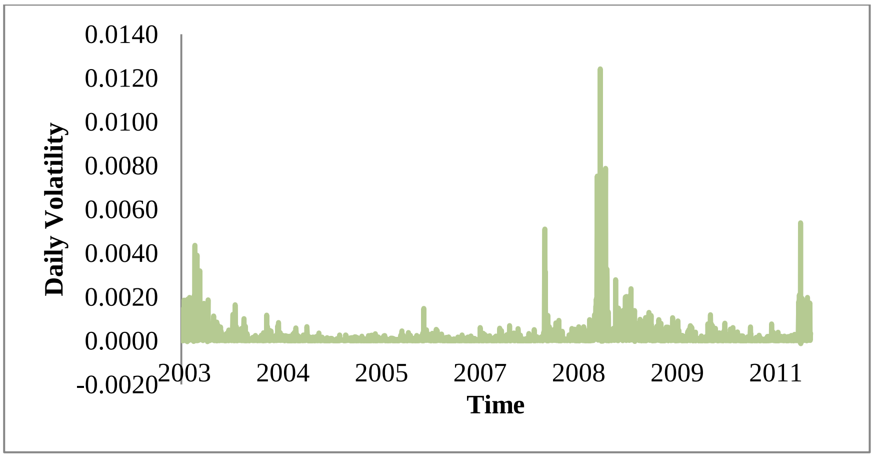

Figure 3.1. Market Index volatility calculated using the return-based (close-to-open) approach

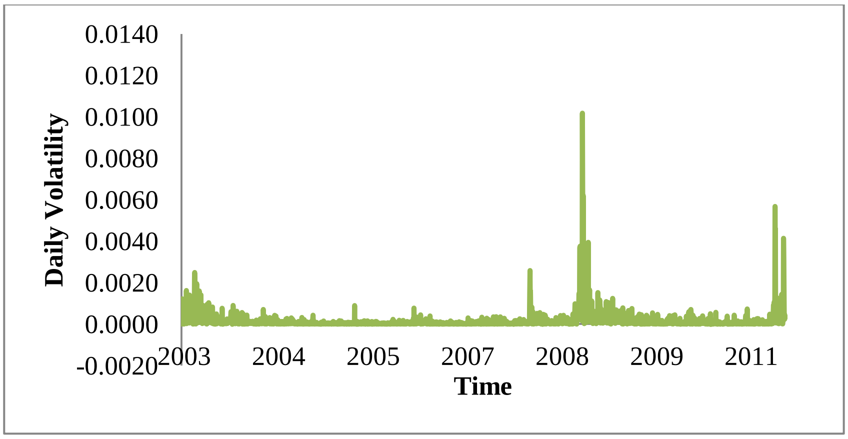

Figure 3.2. Market Index volatility calculated using range-based approach
Notes: The range based volatility is estimated using the Rogers and Zhou (2008) variant of the volatility estimator (3.12).

3.3.2 Unconditional Correlation Estimates

We next employ Rogers and Zhou's (2008) unconditional estimators of the correlations of the two stocks that use the daily opening, closing, high and low prices of each. We compute the range-based daily correlation estimates for each pair of stock *P* with the market index *M*, proxied by the DAX index, ρ_{PM}. We then find the sample average of these correlations over time

$$\rho_{iPM} = \frac{1}{T}\sum_{t=1}^{T}\rho_{iPMt} , \tag{3.15}$$

where T is time and i denotes individual constituents in the DAX index. Table 3.3 reports the daily average correlation estimates for each pair of stock P with the market index. Table 3.3 reports both the range-based and traditional close-to-close return-based correlation ρ_{PM}^{return}. Table 3.3 also presents the range-based and return-based covariance and variance estimates which are used for the calculation of the correlation coefficient estimates.

The range-based correlation estimates are downwards biased because the range of the discretely sampled process is strictly less than the range of the underlying process. A similar point is mentioned by Brandt and Diebold (2006). The magnitude of the bias decreases as the frequency increases. The correlation coefficient between range-based and return-based correlations is 0.9323 which suggests that our range-based correlation measure is quite close to the traditional return-based correlation measure. The range-based covariance estimates are also downwards biased compared with the close-to-close return-based estimates. The high correlation coefficient between range-based and return-based estimates suggests that both models are good at capturing variability of the asset prices. In addition, range-based and return-based volatility estimates are similar for most of the DAX components.

We also average correlations across stock markets to compute a synthetic equally weighted index of their average correlation

$$\rho_{t} = \frac{1}{n}\sum_{P_i=1}^{n}\rho_{iPMt} , \tag{3.16}$$

here, n is the number of individual stocks in the DAX index and i denotes individual constituents in the DAX index. In Figure 3.3, we plot the realized daily average correlation amongst the individual stocks with the market index. The realized correlation appears highly persistent. Figure 3.3 also shows that the index correlations tend to spike up after joint negative events, which is consistent with the results reported by Kearney and Poti (2005). Contrary to the evidence of Cappiello et al. (2003) and others, this phenomenon is not well captured by a linear specification. The range-based correlation is highly appealing due to the ease of estimation and can be treated as an alternative to the Dynamic Conditional Correlation model of Engle (2002).

Table 3.3. Range-based and return-based correlations

Name	ρ_{PM}	ρ_{PM}^{return}	ρ_{PM} / ρ_{PM}^{return}	σ_{PM}	σ_{PM}^{return}	hl_{Pt}^{RZ}	Vol_{Pt}^{ret}
Components for DAX							
ADIDAS N	0.348	0.523	0.665	0.0001	0.0001	0.0003	0.0003
BAYER N	0.424	0.571	0.743	0.0001	0.0001	0.0004	0.0004
BEIERSDORF	0.233	0.313	0.744	0.0001	0.0001	0.0004	0.0002
BMW	0.448	0.584	0.767	0.0001	0.0001	0.0004	0.0004
COMMERZBANK	0.427	0.466	0.917	0.0001	0.0002	0.0008	0.0008
DAIMLER N	0.524	0.650	0.806	0.0002	0.0002	0.0005	0.0004
DEUTSCHE BANK N	0.535	0.599	0.893	0.0002	0.0002	0.0005	0.0005
E.ON N	0.392	0.582	0.675	0.0001	0.0001	0.0003	0.0003
FRESENIUS MED CARE	0.204	0.252	0.809	0.00004	0.0000	0.0003	0.0002
FRESENIUS	0.163	0.244	0.666	0.0001	0.0001	0.0004	0.0003
HEIDELBERGCEMEN T	0.257	0.388	0.663	0.0001	0.0001	0.0007	0.0006
HENKEL VZ	0.304	0.434	0.702	0.0001	0.0001	0.0003	0.0002
LINDE	0.353	0.533	0.663	0.0001	0.0001	0.0003	0.0003
MAN	0.404	0.594	0.681	0.0001	0.0002	0.0006	0.0005
MERCK	0.202	0.306	0.661	0.0001	0.0001	0.0004	0.0003
MUNICHRE	0.478	0.565	0.846	0.0001	0.0001	0.0004	0.0003
RWE	0.413	0.559	0.738	0.0001	0.0001	0.0003	0.0002
K+S N	0.309	0.393	0.786	0.0001	0.0001	0.0006	0.0005
SIEMENS N	0.554	0.716	0.773	0.0002	0.0002	0.0004	0.0003
THYSSENKRUPP	0.451	0.589	0.766	0.0001	0.0002	0.0005	0.0004
VOLKSWAGEN VZ	0.349	0.367	0.952	0.0001	0.0001	0.0007	0.0006
DAX Index						0.0002	0.0002
C-S correlation between coefficient estimates		0.932			0.7153		0.9347

Notes: The Table reports the range-based correlations (ρ_{PM}) estimated using (3.14), the return-based (close-to-open) correlations (ρ_{PM}^{return}), the ratio of the range-based to the return-based correlation$(\rho_{PM} / \rho_{PM}^{return})$, the range-based covariance (σ_{PM}), defined by (3.11), the return-based covariance (σ_{PM}^{return}), the range-based variance (hl_{Pt}^{RZ}) defined in (3.12), and the return-based variance (Vol_{Pt}^{ret}). The last raw of the table also presents the cross-sectional correlations between corresponding range-based and return-based measures. The sample covers January 2003 to September 2011, including altogether 2,228 observations.

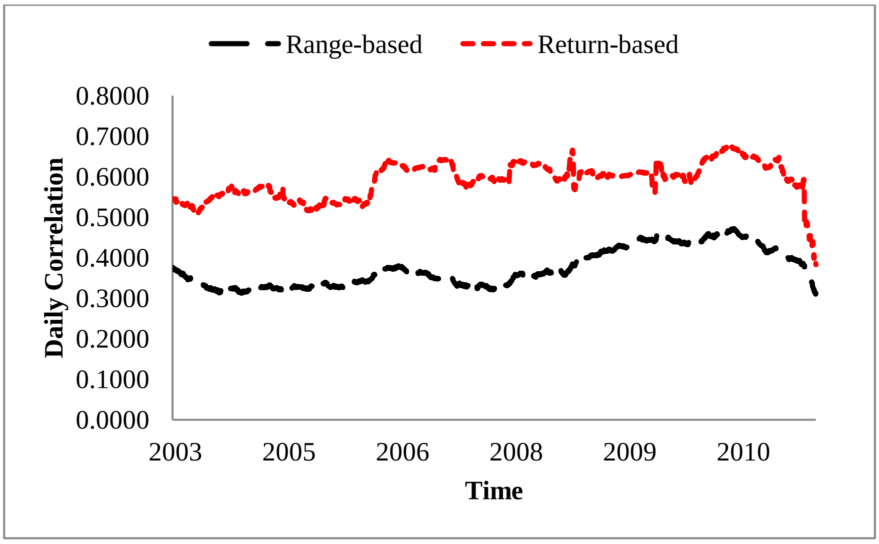

Figure 3.3. Realized daily average range-based and return-based correlation amongst the individual stocks with the market index, one-year rolling average

3.3.3 Range-implied vs. Traditional Betas

In this section we analyse the relation between stock market betas, measured using our proposed range-based methodology and using the traditional rolling window betas and the conditional CAPM with time-varying betas.

For the range-based betas, we use the estimated range-implied volatilities and correlations analysed in the previous section. Specifically, we estimate the range-based market beta $\beta_{PM,t}$ for stock *P* as:

$$\beta_{PMt} = \frac{\frac{1}{2}(S_{Pt} - O_{Pt})(S_{Mt} - O_{Mt}) + \frac{1}{2(1-2b)}(H_{Pt} + L_{Pt} - S_{Pt} - O_{Pt})(H_{Mt} + L_{Mt} - S_{Mt} - O_{Mt})}{\frac{1}{2}(S_{Mt} - O_{Mt})^2 + \frac{1}{2(1-2b)}(H_{Mt} + L_{Mt} - S_{Mt} - O_{Mt})^2}. \quad (3.17)$$

We then find the average range-based betas over time:

$$\beta_{iPM} = \frac{1}{T}\sum_{t=1}^{T}\beta_{iPMt}, \quad (3.18)$$

where T is time and the rest of the variables are defined in Section 3.2.1.

Historical rolling window betas are calculated using the approach presented by Baker et al. (2011). Specifically, for our analysis we compute historical betas using daily stock and index returns with 1.5 year rolling windows length. At the end of each day within our sample period, we compute the stock–to-market covariances as well as the market variance and use them in Equation (3.5) to produce market beta predictions for each stock. To be consistent with the estimation procedure of historical rolling window betas, the range-based beta is also estimated using 1.5 year rolling window. The main difference between the two measures is that the range-based betas vary each day in the estimation period of 1.5 years, whereas the historical rolling window method is constant over the same estimation period.

Jagannathan and Wang (1996) and Lettau and Ludvigson (2001) show that the conditional CAPM with a time-varying beta outperforms the unconditional CAPM with a constant beta. Therefore, we also estimate the conditional CAPM with time-varying betas. The model is:

$$r_{it} = \alpha_{it} + \beta_{it} r_{Mt} + \varepsilon_{it,} \quad \varepsilon_{it} \sim N\left(0, \sigma_{\varepsilon}^2\right), \tag{3.19}$$

$$\beta_{it+1} = \beta_{it} + \upsilon_{it}, \qquad \upsilon_{it} \sim N\left(0, \sigma_{\upsilon}^2\right), \tag{3.20}$$

where β_{it} refers to the conditional beta of stock *i* defined in Equation (3.5), r_{Mt} denotes the common factor market return, and the innovations $\{\varepsilon_{it}, \upsilon_{it}\}$ are mutually independent. This CAPM allows for time-varying β_{it} that evolves as a random walk over time.

Figures 3.4-3.6 present the estimated betas using the range-based beta model, the historical rolling window method, and the conditional CAPM with time-varying betas approach for Adidas, BMW, and Volkswagen, respectively. It can be seen that the estimated betas in the range-based beta model are close to the beta estimates from the historical rolling window method. Also note that the range-based beta estimates are lower than the beta estimates from the rolling window method on average. This is consistent with the results reported in Table 3.3. Specifically, discretely sampled range-based beta estimator is strictly less than the range-based beta of the true underlying process. The conditional CAPM with time-varying betas yields highly volatile beta estimates for Adidas and BMW.

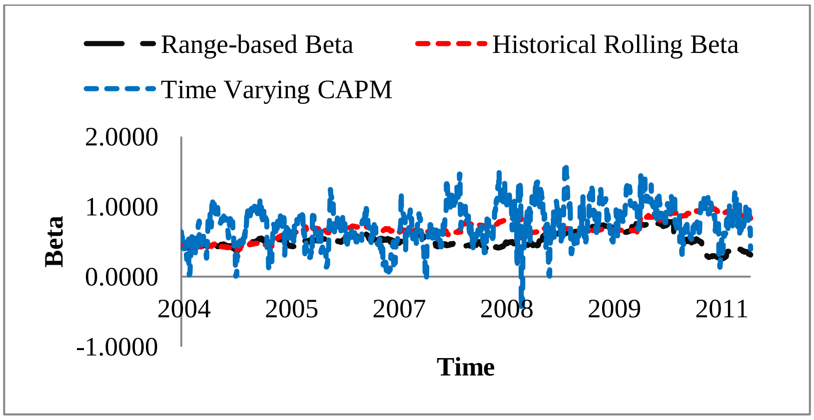

Figure 3.4. Average betas of ADIDAS N

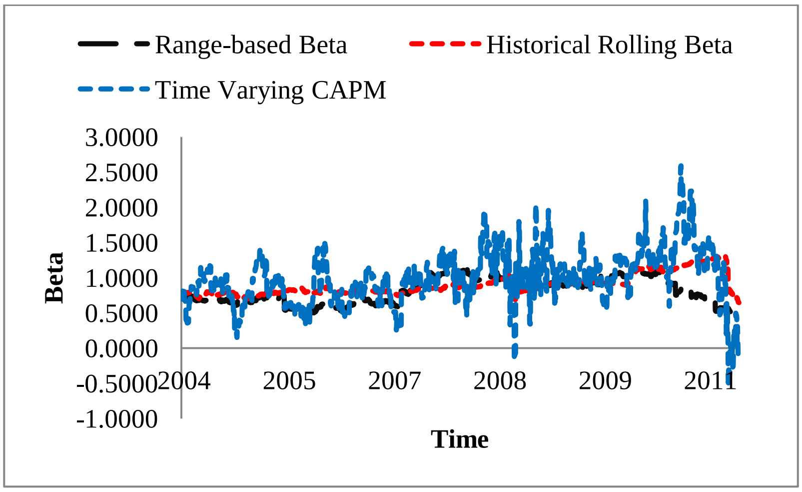

Figure 3.5. Average betas of BMW

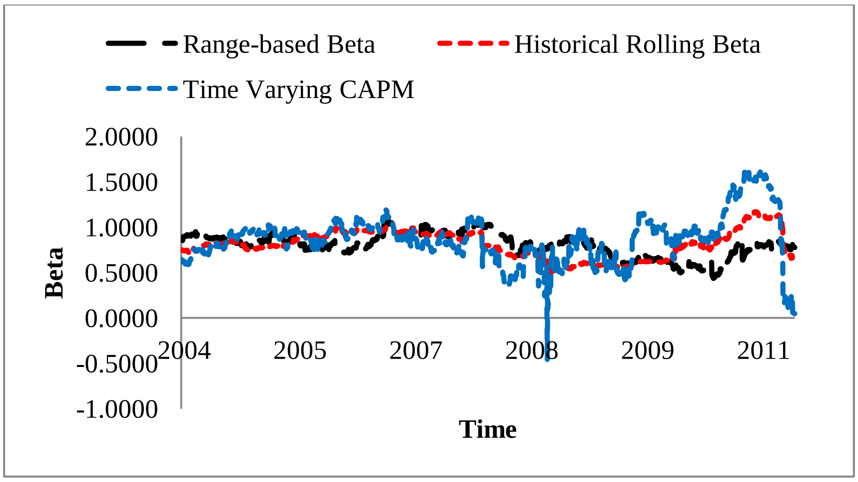

Figure 3.6. Average betas of VOLKSWAGEN VZ

Table 3.4 reports the average beta estimates for all stocks using the range-based approach and the rolling window method. Results for the beta estimates using the conditional CAPM with time-varying betas are the same as when using the rolling window method. The range-based beta estimates are close to the beta estimates using the conditional CAPM with time-varying betas. A high correlation coefficient of 0.9339 confirms that both the range-based beta measure and the return-based beta measure reflect the same market information. The ratios of the average range-based beta to the average return-based close-to-close beta estimates are close to one on average.

3.4.4 Portfolio Sorting

We follow Fama and French (1993) and form portfolios based on pre-ranking betas, where betas are computed using the range-based betas, the historical rolling window betas, and the conditional CAPM with time-varying betas. Specifically, at the end of each day within our sample period, we sort the constituent assets of the DAX Index based on their estimated betas. Next we group stocks into three portfolios according to their expected beta, either the range-based, the historical, or the conditional CAPM with time-varying betas. The low beta portfolio thereby contains seven stocks with the lowest expected market betas, the medium beta portfolio contains the stocks with the medium expected market betas, and the high beta portfolio contains seven stocks with the highest expected market betas. Next day for each portfolio using post formation betas we compute the realized portfolio return over the next day and the expected betas. We repeat this procedure each day.

Table 3.4. Average beta estimates

Market	Range-based beta $\beta_{PM,t}$	Close-to-open return-based beta Covar($r_P r_M$)/var(r_M)	Range-based/return-based
ADIDAS N	0.501	0.649	0.772
BAYER N	0.778	0.807	0.963
BEIERSDORF	0.419	0.334	1.255
BMW	0.780	0.840	0.929
COMMERZBANK	1.013	0.982	1.031
DAIMLER N	0.964	0.960	1.003
DEUTSCHE BANK N	1.019	1.003	1.016
E.ON N	0.568	0.718	0.791
FRESENIUS MED CARE	0.305	0.276	1.103
FRESENIUS	0.429	0.332	1.294
HEIDELBERGCEMENT	0.775	0.728	1.065
HENKEL VZ	0.501	0.490	1.023
LINDE	0.687	0.660	1.040
MAN	0.822	1.020	0.806
MERCK	0.392	0.383	1.022
MUNICHRE	0.779	0.728	1.070
RWE	0.641	0.652	0.983
K+S N	0.801	0.683	1.172
SIEMENS N	0.963	0.940	1.025
THYSSENKRUPP	0.894	0.927	0.965
VOLKSWAGEN VZ	0.793	0.691	1.147
Correlation between range-based and return-based	**0.934**		
Spearman's rank-order correlation	**0.917**		

Notes: The Table reports the range-based betas, the return-based betas, and the ratio between the range-based beta and the return-based beta. The Table also presents the cross-sectional correlations and the Spearman's rank-order correlation between the range-based and the return-based measures. The sample covers January 2003 to September 2011, including altogether 2,228 observations.

In Table 3.5 we provide the summary of the mean expected betas and the realized returns for the beta-sorted portfolios. Table 3.5 also reports the Sharpe ratios. The general conclusion is that all estimation methods produce betas that move around a very similar mean value. We find that for both the range-based betas and the rolling window betas the returns increase when beta increases. In addition, for the conditional CAPM with time-varying coefficients we have one decreasing return fragment across beta sorted portfolios. Daily return differences between the extreme portfolios are 0.06%, 0.06%, and 0.08% with beta differences of 0.61, 0.58, and 0.81 for the range-based beta approach, rolling window based measure, and betas extracted using conditional CAPM with time-varying coefficients, respectively. Daily return difference between extreme portfolios for the range-based beta approach translates into 15% per annum. This result indicates that stocks in the highest range-based beta portfolio generate about 15% more annual return compared to stocks in the lowest range-based portfolio. In terms of Sharpe ratio, we find that the range-based beta method yields the highest Sharpe ratio for the medium beta portfolio. For the lowest beta portfolio the rolling window method is superior. For the highest beta portfolio the conditional CAPM approach generates the highest Sharpe ratio. This result suggests that the range-based beta approach can be chosen when investors' portfolio is composed of stocks with an average beta of 0.75.

In sum, all of these results confirm the existence of a positive and significant relation between the range-based beta and one-day-ahead returns on the DAX stocks.

Table 3.5. The mean expected betas and the realized returns for the beta-sorted portfolios

Portfolio	Low	Medium	High
Range-beta			
Expected beta	0.4211	0.7235	1.0322
Realized return	0.0001	0.0005	0.0007
Sharpe ratio	0.0042	0.0258	0.0328
Rolling Window			
Expected beta	0.4891	0.7974	1.0724
Realized return	0.0002	0.0004	0.0008
Sharpe ratio	0.0103	0.0181	0.0350
Conditional CAPM with Time-varying Betas			
Expected beta	0.4287	0.7869	1.2430
Realized return	0.0002	0.0001	0.0010
Sharpe ratio	0.0076	0.0064	0.0469

Notes: The Table reports the mean expected betas, the realized returns, and the Sharpe ratios for the beta-sorted portfolios. The sample covers January 2003 to September 2011, including altogether 2,228 observations.

3.3.5 Mean Squared Error

Accuracy measures on the predictability of beta involve the beta predictions computed using the simplified SCL from CAPM, referenced in Equation (3.4),

$$r_{it} = \hat{\beta}_i r_{Mt} + \varepsilon_{it}, \tag{3.21}$$

where r_{Mt} is the observed market return at time *t*, $\hat{\beta}_i$ is defined as the 1-day forward corresponding beta prediction at time *t*, ε_{it} is a random error term and r_{it} represents the predicted return on asset *i* at time *t*. The Mean Square Error (MSE) measures the test of return accuracy from the predicted beta that is conditioned on the observed market return. MSE is defined as

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(r_{it} - \tilde{r}_{it})^2, \tag{3.22}$$

where n is the number of predictions contained, r_{it} is the predicted out of sample realized return on asset t at time t and $\tilde{r}_{it}$ is the observed return on asset i at the corresponding time t.

To test whether improvements in MSE are statistically significant using the analysed models, we use Diebold and Mariano's (1995) test. Denote the difference in squared errors of the two forecasts, r_{it}^1 and r_{it}^2 as $d_t = \left(r_{it}^1 - \tilde{r}_{it}\right)^2 - \left(r_{it}^2 - \tilde{r}_{it}\right)^2$, which is also known as 'loss differentials'. If the range-based beta approach is a better forecasting tool, one would expect that, on average, the loss differentials d_t would be positive. Consequently, one would expect negative values if the alternative method is superior. Following this intuition, the Diebold and Mariano (1995) test considers the null hypothesis $H_0 : E(d_t) = 0$; positive values of the statistic suggest that the forecasts from the range-based beta model have lower mean-squared errors, while negative values favour the alternative benchmark.

In Table 3.6 we report both the MSE and the Mean Absolute Error (MAE) of the portfolio composed of all 21 constituent DAX components. We find that the range-based beta yields quite competitive results. The MSE using the range-based beta is 2.9%, whereas the MSE from the betas extracted from the historical rolling window approach and the conditional CAPM with time-varying coefficients are 2.88% and 2.81%, respectively.

Table 3.6. MSE and MAE of DAX constituents

	MSE	MAE
Range-beta	0.000290	0.0004
Rolling Window	0.000288	0.0004
Conditional CAPM with Time-varying Betas	0.000281	0.0004

Notes: The Table reports the MSE and the MAE of the portfolio composed of all 21 constituent DAX components. MAE = $r_{it} - \beta_{it-1} r_{Mt}$ and MSE = $(r_{it} - \beta_{it-1} r_{Mt})^2$. The sample covers January 2003 to September 2011, including altogether 2,228 observations.

In Tables 3.7-3.9 we provide the summary statistics of the mean expected betas and the realized returns for the beta-sorted portfolios. The Diebold and Mariano (1995) test statistic (denoted *DM*) is asymptotically normal and standard critical values are used. We also re-estimate the models with an assumed regime break differentiating the pre-crisis and crisis periods. Following Cipollini and Gallo (2009), we choose July 17, 2007 as the regime break point. This date corresponds to the announcement by Bear Stearns of the collapse of two hedge funds, and was followed by the suspension of payments by BNP Paribas and increased support facilities by the ECB and Fed in early August 2007. Hence, we assume that the crisis period extends from July 18, 2007 to the end of our sample on September 30, 2011. For the low beta portfolio (Table 3.7) we find that the range-based beta approach yields the lowest mean squared tracking error (0.00023), which is also significantly lower than the MSE from the conditional CAPM (0.00028) over the full sample period. For the lowest beta portfolio the mean squared error from the range-based beta approach is insignificantly different from the rolling window approach (0.00024). During the pre-crisis period for the low beta portfolio the MSE from the range-base method (0.00016) is insignificantly different from the rolling window approach (0.00016). The range-based beta model generates significantly higher MSE than the conditional CAPM with time-varying coefficients (0.00015). In addition, during the post-crisis period the MSE from the range-based beta model (0.00028) is significantly lower than the conditional CAPM with time-varying coefficients (0.00038) and insignificantly lower than the rolling window approach (0.00030). For the medium beta portfolio (Table 3.8) the range-based beta approach generates insignificantly different MSE (0.00039) than the other two alternative models over the full sample period. The same pattern holds for the two sub-samples, with the exception of the pre-crisis period where the rolling window approach (0.00011) is superior to the range-based beta method (0.00012). For the highest beta portfolio (Table 3.9) the range-beta method (0.00036) is insignificantly different from the rolling window method (0.00034) and the range-based beta method is significantly higher than the conditional CAPM with time-varying coefficients (0.00030) over the full sample. Over the pre-crisis sub-sample the range-based beta approach (0.00009) is superior to the conditional CAPM with time-varying coefficients (0.0001). Over the post-crisis period the range-based beta model (0.00056) generates significantly higher MSE than other two models.

Table 3.7. Low beta portfolio

	Beta	MSE	Beta	MSE	Beta	MSE
	Full sample		Pre-crisis		Post-crisis	
Range-beta						
Mean	0.421	0.00023	0.407	0.00016	0.432	0.00028
St. Dev	0.168	0.00073	0.159	0.00040	0.174	0.00090
Rolling Window						
Mean	0.489	0.00024	0.475	0.00016	0.499	0.00030
St. Dev	0.159	0.00178	0.172	0.000390	0.147	0.00231
DM stat, (*p*-value)		(0.340)		(0.989)		(0.336)
Conditional CAPM with Time-varying Betas						
Mean	0.439	0.00028	0.462	0.00015	0.423	0.00038
St. Dev	0.206	0.00191	0.182	0.00035	0.220	0.00249
DM stat, (*p*-value)		(0.002)		(0.027)		(0.001)

Notes: The Table reports the sample statistics for the low beta portfolios. The sample covers January 2003 to September 2011, including altogether 2,228 observations. DM indicates test statistic from the Diebold and Mariano (1995) test. The null hypothesis posits equal forecasting accuracy between the two models. The statistic is asymptotically normal. The statistic is computed as $DM = d / \hat{\sigma}(d)$ where d is the sample average of d_t and $\hat{\sigma}(d)$ is a heteroscedastic and autocorrelation(HAC)-consistent estimate of the standard deviation of d.

Table 3.8. Medium beta portfolio

	Beta	MSE	Beta	MSE	Beta	MSE
	Full sample		Pre-crisis		Post-crisis	
Range-beta						
Mean	0.723	0.00029	0.684	0.00012	0.751	0.00040
St. Dev	0.116	0.00192	0.126	0.00033	0.108	0.00250
Rolling Window						
Mean	0.797	0.00028	0.785	0.00011	0.805	0.00041
St. Dev	0.136	0.00118	0.128	0.00028	0.141	0.00152
DM stat, (*p*-value)		(0.982)		(0.004)		(0.736)
Conditional CAPM with Time-varying Betas						
Mean	0.790	0.00026	0.791	0.00012	0.790	0.00036
St. Dev	0.209	0.00108	0.142	0.00036	0.247	0.00138
DM stat, (*p*-value)		(0.230)		(0.899)		(0.232)

Notes: The Table reports the sample statistics for the low beta portfolios. The sample covers January 2003 to September 2011, including altogether 2,228 observations. DM indicates test statistic from the Diebold and Mariano (1995) test. The null hypothesis posits equal forecasting accuracy between the two models. The statistic is asymptotically normal. The statistic is computed as $DM = d/\hat{\sigma}(d)$ where d is the sample average of d_t and $\hat{\sigma}(d)$ is a heteroscedastic and autocorrelation(HAC)-consistent estimate of the standard deviation of d.

Table 3.9. High beta portfolio

	Beta	MSE	Beta	MSE	Beta	MSE
	Full sample		Pre-crisis		Post-crisis	
Range-beta						
Mean	1.031	0.00036	0.942	0.00009	1.096	0.00056
St. Dev	0.156	0.00217	0.104	0.00023	0.155	0.00283
Rolling Window						
Mean	1.072	0.00034	0.988	0.00010	1.133	0.00051
St. Dev	0.174	0.00192	0.119	0.00030	0.182	0.00250
DM stat, (*p*-value)		(0.140)		(0.073)		(0.082)
Conditional CAPM with Time-varying Betas						
Mean	1.229	0.00030	1.103	0.00010	1.322	0.00044
St. Dev	0.367	0.00140	0.224	0.00026	0.420	0.00184
DM stat, (*p*-value)		(0.004)		(0.009)		(0.002)

Notes: The Table reports the sample statistics for the low beta portfolios. The sample covers January 2003 to September 2011, including altogether 2,228 observations. DM indicates test statistic from the Diebold and Mariano (1995) test. The null hypothesis posits equal forecasting accuracy between the two models. The statistic is asymptotically normal. The statistic is computed as $DM = d / \hat{\sigma}(d)$ where d is the sample average of d_t and $\hat{\sigma}(d)$ is a heteroscedastic and autocorrelation(HAC)-consistent estimate of the standard deviation of d.

Figures 3.7-3.9 plot the average betas for low beta, medium beta, and high beta portfolios. Figures 3.7-3.9 present significant time-series variation as well as stationary and mean-reverting behaviour of the beta extracted from the range-based approach, the historical rolling window method, or from the conditional CAPM with time-varying coefficients. The figures also show that the betas extracted from either of three methods generally have positive time trend during the financial market and economic downturn, with a negative drift and stabilizing behaviour during ordinary periods. Also note that the betas extracted from the conditional CAPM with time-varying coefficients are varying with greater amplitude and frequency, whereas are smoothed with the two other methods. Figures 3.10-3.12 plot average MSE for the beta sorted portfolios. Note the high peak of MSE during the beginning of 2008 which reflects recent financial crisis. For the low beta portfolio the rolling window beta method was worse at predicting the market. In contrast, the range-based beta is the closest at reflecting the variability of low beta stocks of DAX index. For medium beta portfolio, both the rolling window approach and the conditional CAPM with time-varying coefficients are not able to capture the market variability as compared to the range-based specification. Finally, for high beta portfolio the range-based method predicts the market with high MSE of 0.0003 over the full sample period.

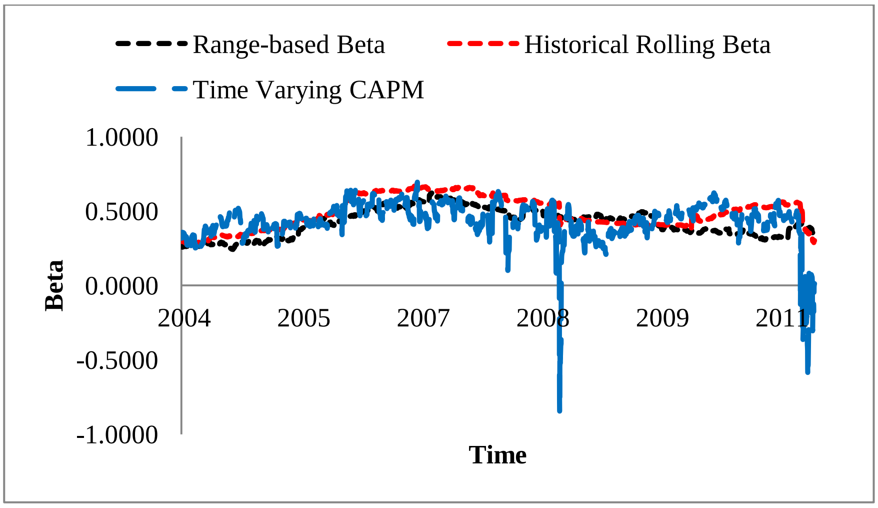

Figure 3.7. Average betas of the low beta portfolios

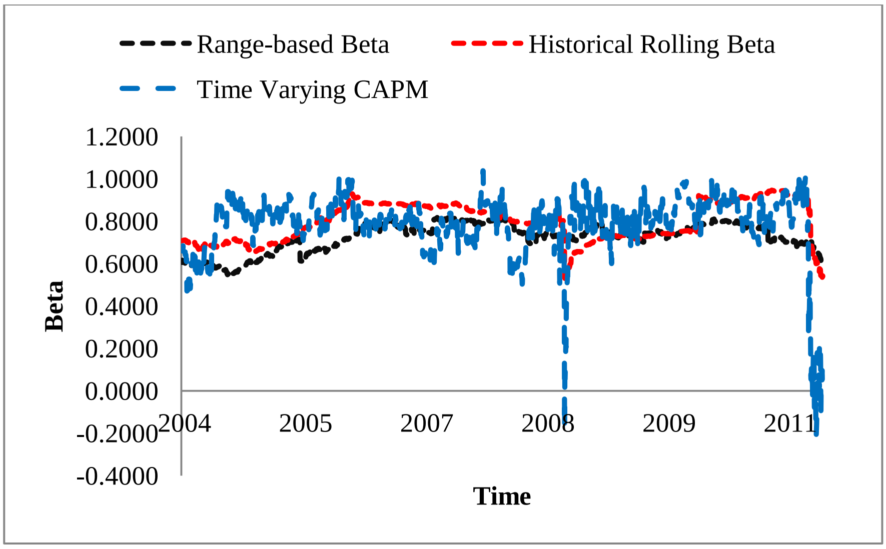

Figure 3.8. Average betas of the medium beta portfolios

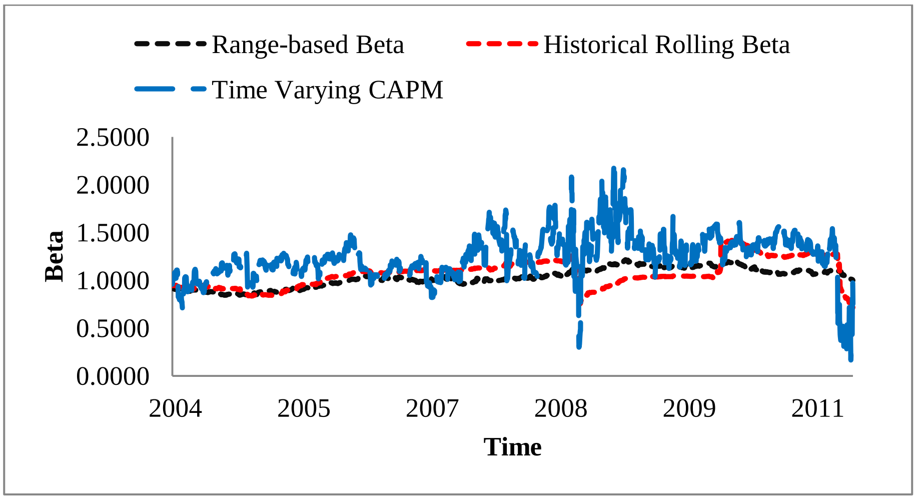

Figure 3.9. Average betas of the high beta portfolio

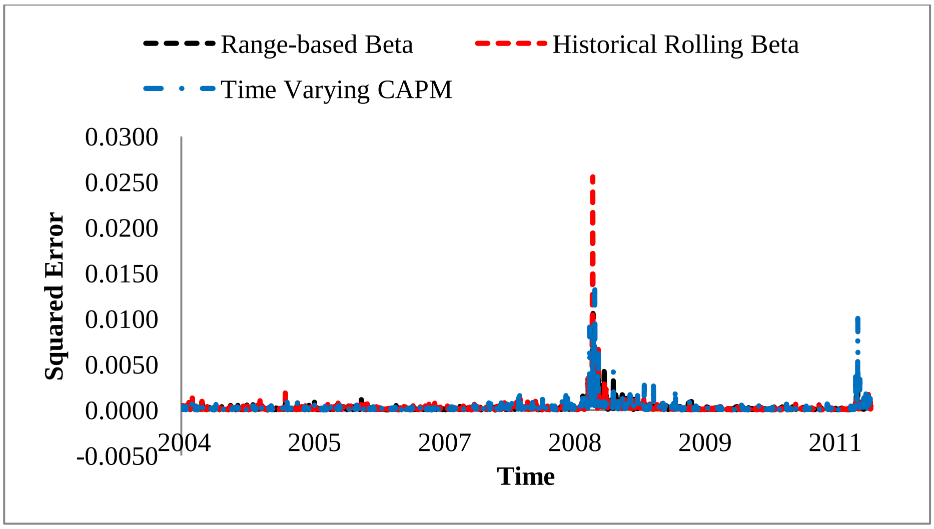

Figure 3.10. Average MSE of low beta portfolio

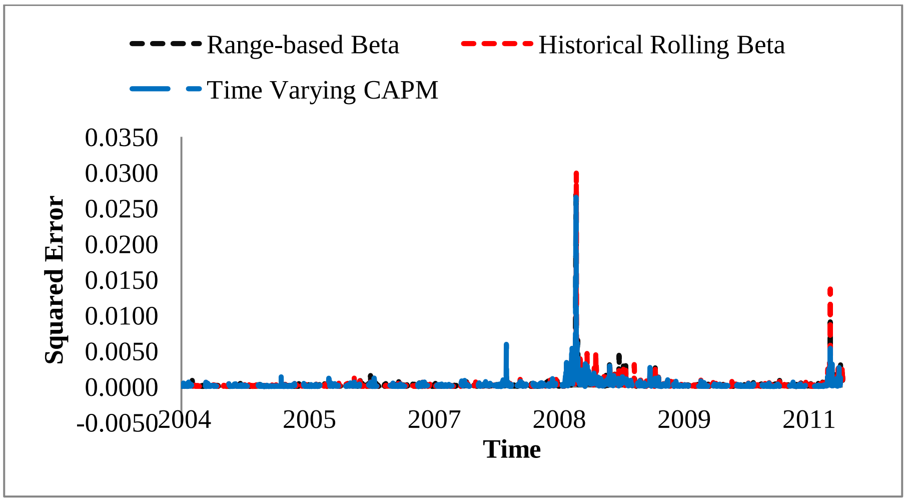

Figure 3.11. Average MSE of medium beta portfolio

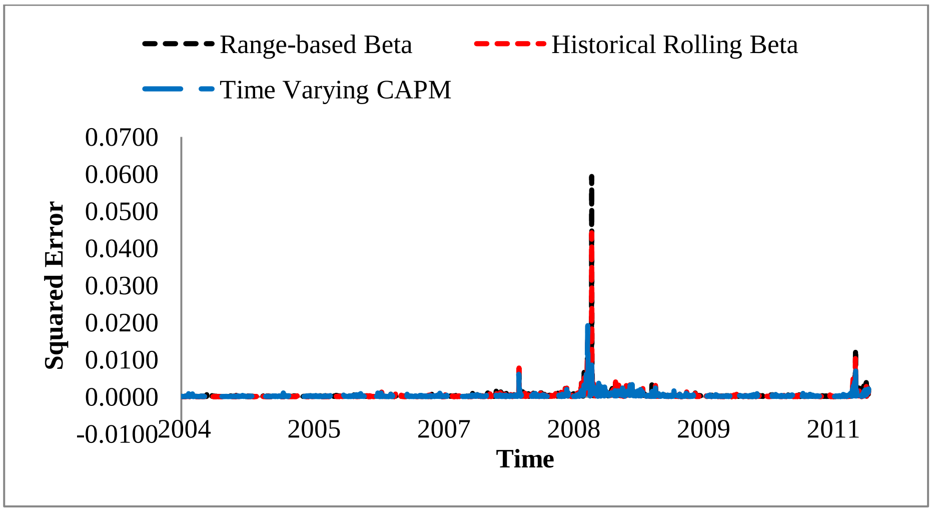

Figure 3.12. Average MSE of high beta portfolio

Garman and Klass (1980) and Marsh and Rosenfeld (1986) find that the range-based volatility estimates are downward biased due to non-trading. In practice transaction prices are observed in discrete time and occur infrequently and irregularly during a trading day. The 'true' high (low) of the underlying continuous time price process is unlikely to be recorded as a transaction and the observable high (low) price underestimates (overestimates) the unrecorded 'true' high (low). Therefore, we expect the range-based beta to be closer to the 'true' beta for highly liquid assets. Hence, we create portfolios sorted on turnover by volume measure. As a result, we create low turnover by volume, medium turnover by volume, and high turnover by volume portfolios. Results are reported in Table 3.10. First note, that the MSE increases as turnover increases suggesting that high liquidity assets are harder to predict. What is more, the range-based approach yields not statistically different MSE, which is not statistically different from the MSE generated by other two models for all three portfolios over the full sample period. The same pattern holds over the sub-periods.

Table 3.10. Turnover sorted portfolios

	Beta	MSE	Beta	MSE	Beta	MSE
	Full sample		Pre-crisis		Post-crisis	
Low Turnover						
Range-beta	0.546	0.00023	0.507	0.00013	0.575	0.00030
Rolling Window	0.580	0.00022	0.545	0.00013	0.606	0.00029
DM stat, (*p*-value)		(0.143)		(0.0000)		(0.020)
Conditional CAPM with Time-varying Betas	0.609	0.00023	0.581	0.00013	0.629	0.00029
DM stat, (*p*-value)		(0.708)		(0.130)		(0.811)
Turnover by Volume	3.596		5.110		2.494	
Medium Turnover						
Range-beta	0.748	0.00026	0.729	0.00011	0.747	0.00037
Rolling Window	0.813	0.00026	0.799	0.00011	0.823	0.00036
DM stat, (*p*-value)		(0.010)		(0.006)		(0.023)
Conditional CAPM with Time-varying Betas	0.839	0.00025	0.831	0.00011	0.845	0.00036
DM stat, (*p*-value)		(0.243)		(0.780)		(0.234)
Turnover by Volume	19.016		27.103		13.135	
High Turnover						
Range-beta	0.883	0.00039	0.798	0.00014	0.944	0.00057
Rolling Window	0.966	0.00039	0.906	0.00013	1.010	0.00058
DM stat, (*p*-value)		(0.164)		(0.000)		(0.095)
Conditional CAPM with Time-varying Betas	1.012	0.00038	0.944	0.00014	1.062	0.00056
DM stat, (*p*-value)		(0.854)		(0.694)		(0.867)
Turnover by Volume	131.910		99.406		155.549	

Notes: The Table reports the sample statistics for the turnover sorted portfolios. The sample covers January 2003 to September 2011, including altogether 2,228 observations. DM indicates test statistic from the Diebold and Mariano (1995) test. The null hypothesis posits equal forecasting accuracy between the two models. The statistic is asymptotically normal. The statistic is computed as $DM = d/\hat{\sigma}(d)$ where d is the sample average of d_t and $\hat{\sigma}(d)$ is a heteroscedastic and autocorrelation(HAC)-consistent estimate of the standard deviation of d.

3.3.6 Bias Corrected Range-based Correlations

Finally, to assess the properties of the range-based correlation estimators, we perform an extensive simulation analysis. We consider two correlated log asset prices, which follow a bivariate random walk with homoskedastic and contemporaneously correlated innovations[2]. For each day and without loss of generality, the initial log prices for both assets are set equal to 0. The Rogers and Zhou (2008) formula defines the range-based estimator over a fixed time interval [0,1], which also means that opening log asset prices are equal to zero. Subsequent log prices for asset i = 1, 2 are simulated using

$$\log P_{i,t+k/K} = \log P_{i,t+(k-1)/K} + \varepsilon_{i,t+k/K} \qquad i = 1,\ 2,\ \ k = 1,\ 2,...,K, \qquad (3.23)$$

where K is the number of prices per day.

The simulation experiment uses $K \in \{500,\ 1{,}000\}$ observations per day, where the price observations are equidistant and occur synchronously for the two assets. We first consider the case where the shocks $\varepsilon_{i,t+k/K}$ are serially uncorrelated and normally distributed with mean zero and variance 1/K. For each day, we calculate the open, close, high, and low log prices for both assets i = 1, 2. The number of simulated trading days N (or Monte Carlo replications) equals 1,000 and 10,000. The shocks $\varepsilon_{1,t+k/K}$ and $\varepsilon_{2,t+k/K}$ are contemporaneously correlated with correlation coefficient ρ_{12}, which we set equal to 0.25, 0.5, and 0.75, resulting in the range-based beta measure between assets of 0.25, 0.5, and 0.75, respectively. Setting opening prices ($O_{1,t}$ and $O_{2,t}$) equal to 0, Equation (3.17) for the range-based beta $\beta_{12,t}$ of the first stock is:

$$\hat{\beta}_{12,t} = \frac{\frac{1}{2}S_{1,t}S_{2,t} + \frac{1}{2(1-2b)}\left(H_{1,t} + L_{1,t} - S_{1,t}\right)\left(H_{2,t} + L_{2,t} - S_{2,t}\right)}{\frac{1}{2}S_{2,t}^2 + \frac{1}{2(1-2b)}\left(H_{2,t} + L_{2,t} - S_{2,t}\right)^2}, \qquad (3.24)$$

[2] The random walk process (discrete time version of Brownian motion) for the log-prices follows from the assumption that prices follow a geometric Brownian motion. Strictly speaking, this would imply that the random walk process contains a drift, but we abstain from this fact here. This drift is probably negligible.

where the constant b is equal to $2\log 2 - 1 \cong 0.386294$ and the notation is as before. The results of the simulation are presented in Table 3.11. The-range based beta estimates are slightly downward biased which is consistent with the fact that the range of the discretely sampled process is strictly less than the range of the underlying diffusion.

Table 3.11. Monte Carlo experiment for the range-based beta

	Parameter values		
Theoretical values of beta	0.25	0.50	0.75
Range-based beta estimates			
K = 500, N = 1,000	0.32	0.47	0.75
K = 500, N = 10,000	0.21	0.50	0.70
K = 1,000, N = 10,000	0.22	0.45	0.68

Notes: The Table shows the results of a simulation experiment which uses $K \in \{500,\ 1{,}000\}$, and K is a number of (log) prices per day. Log prices for asset i = 1, 2 are simulated using log $P_{i,t+k/K}$ = log $P_{i,t+(k-1)/K}$ + $\varepsilon_{1,t+k/K}$, i = 1, 2, k = 1, 2,...,K. The shocks $\varepsilon_{1,t+k/K}$ are serially uncorrelated and normally distributed with mean zero and variance 1/K. The shocks $\varepsilon_{1,t+k/K}$ and $\varepsilon_{2,t+k/K}$ are contemporaneously correlated with correlation coefficient ρ_{12}, which we set equal to 0.25, 0.5, and 0.75. N denotes the number of Monte Carlo replications. For each day, we calculate the open, close, high, and low log prices for both assets i = 1, 2; these prices are then used to calculate the range-based betas.

Finally, for each $\rho_{12} = 0,\ 0.1,...,1$, we generate 10,000 Monte Carlo replications of correlated Brownian motions with K set equal to 1,000. For each day, we again store the opening, close, high, and low log prices for both assets i = 1, 2. The range-based correlation ρ_{12} is estimated using Rogers and Zhou (2008) formula which defines the correlation measure over a fixed time interval [0,1]:

$$\rho_{12} = \frac{\frac{1}{2}(S_{1t})(S_{2t}) + \frac{1}{2(1-2b)}(H_{1t} + L_{1t} - S_{1t})(H_{2t} + L_{2t} - S_{2t})}{\sqrt{\frac{1}{2}(S_{1t})^2 + \frac{1}{2(1-2b)}(H_{1t} + L_{1t} - S_{1t})^2}\sqrt{\frac{1}{2}(S_{2t})^2 + \frac{1}{2(1-2b)}(H_{2t} + L_{2t} - S_{2t})^2}}, \tag{3.25}$$

where $b = 2\log 2 - 1 \cong 0.386294$ and the opening prices (O_{1t} and O_{2t}) are set equal to 0. The results are reported in Table 3.12. The most striking result is the fact that the estimator ρ_{12} is biased, even for moderately small values of the true correlation. Observe that the bias is always in the direction of underestimating the magnitude of the correlation (Figure 3.13).

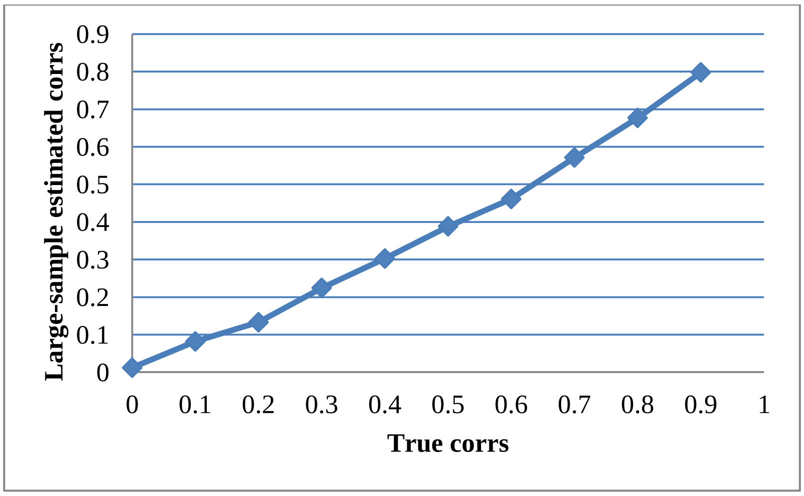

Figure 3.13. Bias correction

Table 3.12. Monte Carlo experiment for the range-based correlations

Theoretical correlation values	Range-based correlation estimates	St. dev
0.0	0.0119	0.7045
0.1	0.0817	0.7105
0.2	0.1328	0.6988
0.3	0.2243	0.6892
0.4	0.3024	0.6666
0.5	0.3881	0.6423
0.6	0.4609	0.6128
0.7	0.5712	0.5580
0.8	0.6767	0.4950
0.9	0.7978	0.3911
1.0	0.3696	0.1396

Notes: The Table shows the results of a simulation experiment where 10,000 days of 1,000 log prices are simulated from a normal distribution with mean zero and variance σ_i/K, where daily standard deviations σ_i of the log price processes are set equal to 0.0252 and 0.0149 for assets 1 and 2, respectively. All experiments use 10,000 Monte Carlo Replications. The shocks $\varepsilon_{1,t+k/K}$ and $\varepsilon_{2,t+k/K}$ are contemporaneously correlated with correlation coefficient ρ_{12}, which we set equal to 0.5. For each day, we calculate the open, close, high, and low log prices for both assets i = 1, 2; these prices are then used to calculate the range-based estimates of correlation.

We next improve the range-based correlation estimates computed in Table 3.3 using results in Figure 3.13. For instance, the range-based correlation estimator of 0.4 is approximately equal the bias corrected range-based correlation estimator of 0.5. Table 3.13 presents the results. Note that after the bias correction we observe improvement in the range-based correlation estimates. The range-based correlation estimates are now close to the return-based correlations. The ratio of the bias corrected range-based correlations to the return-based correlations is one, on average. The same bias correction technique can be applied to the range-based beta measures which will further improve the performance of the return accuracy from the predicted beta that is conditioned on the observed market return.

Table 3.13. Bias corrected range-based correlations

Components for DAX	ρ_{PM}	ρ_{PM}^{return}	ρ_{PM} / ρ_{PM}^{return}	ρ_{PM}^{Adj}	ρ_{PM}^{Adj} / ρ_{PM}^{return}
ADIDA+S N	0.348	0.523	0.665	0.454	0.869
BAYER N	0.424	0.571	0.743	0.549	0.962
BEIERSDORF	0.233	0.313	0.744	0.310	0.990
BMW	0.448	0.584	0.767	0.566	0.969
COMMERZBANK	0.427	0.466	0.917	0.553	1.187
DAIMLER N	0.524	0.650	0.806	0.662	1.019
DEUTSCHE BANK N	0.535	0.599	0.893	0.676	1.129
E.ON N	0.392	0.582	0.675	0.508	0.872
FRESENIUS MED CARE	0.204	0.252	0.809	0.290	1.151
FRESENIUS	0.163	0.244	0.666	0.232	0.950
HEIDELBERGCEMENT	0.257	0.388	0.663	0.342	0.881
HENKEL VZ	0.304	0.434	0.702	0.404	0.932
LINDE	0.353	0.533	0.663	0.461	0.865
MAN	0.404	0.594	0.681	0.523	0.881
MERCK	0.202	0.306	0.661	0.287	0.939
MUNICHRE	0.478	0.565	0.846	0.604	1.069
RWE	0.413	0.559	0.738	0.535	0.957
K+S N	0.309	0.393	0.786	0.403	1.026
SIEMENS N	0.554	0.716	0.773	0.700	0.978
THYSSENKRUPP	0.451	0.589	0.766	0.584	0.992
VOLKSWAGEN VZ	0.349	0.367	0.952	0.452	1.232
C-S correlation between coefficient estimates		0.932			0.933

Notes: The Table reports the range-based correlations (ρ_{PM}), the return-based (close-to-open) correlations (ρ_{PM}^{return}), the ratio of the range-based to the return-based correlation $(\rho_{PM} / \rho_{PM}^{return})$, the range-based correlation corrected for the bias (ρ_{PM}^{Adj}), and the ratio of the bias corrected range-based to the return-based correlation $(\rho_{PM}^{Adj} / \rho_{PM}^{return})$. The last raw of the table also presents the cross-sectional correlations between corresponding range-based and return-based measures. The sample covers January 2003 to September 2011, including altogether 2,228 observations.

3.3.7 Robustness Checks

We also perform robustness checks by using a 60 day rolling window for estimating the historical and the range-based beta estimates (Table 3.14). Results are generally consistent with the use of 1.5 year rolling window approach. In particular, Table 3.14 shows that all the range-based betas, the rolling window betas, and the conditional CAPM with time-varying coefficients produce betas that move around a very similar mean value. In terms of the Sharpe ratio, we find that the conditional CAPM with time-varying betas yields the lowest Sharpe ratio.

In Tables 3.15-3.17 we provide the summary statistics of the mean expected betas and the realized returns for the beta-sorted portfolios. The Diebold and Mariano (1995) test statistic (denoted *DM*) is asymptotically normal and the standard critical values are used. We also re-estimate the models with an assumed regime break differentiating the pre-crisis and crisis periods. For the low beta portfolio the mean squared error from the range-based beta approach is insignificantly different from the rolling window approach and the conditional CAPM over the full sample period and the two sub-samples. For the medium beta portfolio the MSE from the range-based beta approach is significantly lower than other two methods over the full sample period and over the post-crisis sub-period. The MSE from the range-based beta method is insignificantly different from the rolling window approach and the conditional CAPM over the pre-crisis sub-sample for the medium beta portfolio. Finally, for the highest beta portfolio, the conditional CAPM with time varying coefficient is superior than the other two methods over the full sample period and two sub-samples.

Table 3.14. The mean expected betas and the realized returns for the beta-sorted portfolios, 60 day rolling window

Portfolio	Low	Medium	High
Range-beta			
Expected beta	0.233	0.708	1.199
Realized return	0.00006	0.0001	0.0006
Sharpe ratio	0.006	0.008	0.039
Rolling Window			
Expected beta	0.416	0.788	1.144
Realized return	0.0003	-0.00001	0.0005
Sharpe ratio	0.028	0.001	0.0322
Conditional CAPM with Time-varying Betas			
Expected beta	0.415	0.771	1.197
Realized return	0.0001	-0.0002	0.0009
Sharpe ratio	0.018	0.018	0.056

Notes: The Table reports the mean expected betas, the realized returns, and the Sharpe ratios for the beta-sorted portfolios. The sample covers January 2003 to September 2011, including altogether 2,228 observations. The range-based beta and the historical beta estimates are constructed using 60-day rolling window.

Table 3.15. Low beta portfolio

	Beta	MSE	Beta	MSE	Beta	MSE
	Full sample		Pre-crisis		Post-crisis	
Range-beta						
Mean	0.233	0.00026	0.213	0.00018	0.253	0.00033
St. Dev	0.327	0.00082	0.291	0.00043	0.460	0.00108
Rolling Window						
Mean	0.416	0.00028	0.402	0.00018	0.431	0.00039
St. Dev	0.210	0.00184	0.219	0.00040	0.198	0.00259
DM stat, (*p*-value)		(0.183)		(0.156)		(0.104)
Conditional CAPM with Time-varying Betas						
Mean	0.415	0.00028	0.408	0.00018	0.423	0.00038
St. Dev	0.210	0.00178	0.200	0.00042	0.220	0.00249
DM stat, (*p*-value)		(0.164)		(0.324)		(0.103)

Notes: The Table reports the sample statistics for the low beta portfolios. The range-based beta and the historical beta estimates are constructed using 60-day rolling window. The sample covers January 2003 to September 2011, including altogether 2,228 observations. DM indicates test statistic from the Diebold and Mariano (1995) test. The null hypothesis posits equal forecasting accuracy between the two models. The statistic is asymptotically normal. The statistic is computed as $DM = d / \hat{\sigma}(d)$ where d is the sample average of d_t and $\hat{\sigma}(d)$ is a heteroscedastic and autocorrelation(HAC)-consistent estimate of the standard deviation of *d*.

Table 3.16. Medium beta portfolio

	Beta	MSE	Beta	MSE	Beta	MSE
	Full sample		Pre-crisis		Post-crisis	
Range-beta						
Mean	0.708	0.00024	0.702	0.00016	0.715	0.00032
St. Dev	0.204	0.00085	0.200	0.00041	0.207	0.00113
Rolling Window						
Mean	0.788	0.00025	0.762	0.00015	0.816	0.00036
St. Dev	0.185	0.00103	0.134	0.00042	0.222	0.00139
DM stat, (*p*-value)		(0.072)		(0.165)		(0.014)
Conditional CAPM with Time-varying Betas						
Mean	0.771	0.00026	0.753	0.00015	0.790	0.00037
St. Dev	0.209	0.00102	0.161	0.00041	0.247	0.00138
DM stat, (*p*-value)		(0.015)		(0.590)		(0.005)

Notes: The Table reports the sample statistics for the low beta portfolios. The range-based beta and the historical beta estimates are constructed using 60-day rolling window. The sample covers January 2003 to September 2011, including altogether 2,228 observations. DM indicates test statistic from the Diebold and Mariano (1995) test. The null hypothesis posits equal forecasting accuracy between the two models. The statistic is asymptotically normal. The statistic is computed as $DM = d / \hat{\sigma}(d)$ where d is the sample average of d_t and $\hat{\sigma}(d)$ is a heteroscedastic and autocorrelation(HAC)-consistent estimate of the standard deviation of *d*.

Table 3.17. High beta portfolio

	Beta	MSE	Beta	MSE	Beta	MSE
	Full sample		Pre-crisis		Post-crisis	
Range-beta						
Mean	1.199	0.00039	1.149	0.00015	1.250	0.00064
St. Dev	0.343	0.00284	0.306	0.00039	0.372	0.00401
Rolling Window						
Mean	1.144	0.00030	1.040	0.00015	1.251	0.00047
St. Dev	0.290	0.00125	0.194	0.00036	0.331	0.00173
DM stat, (*p*-value)		(0.0001)		(0.059)		(0.0003)
Conditional CAPM with Time-varying Betas						
Mean	1.197	0.00029	1.076	0.00014	1.322	0.00044
St. Dev	0.363	0.00133	0.242	0.00037	0.420	0.00184
DM stat, (*p*-value)		(0.000)		(0.018)		(0.0001)

Notes: The Table reports the sample statistics for the low beta portfolios. The range-based beta and the historical beta estimates are constructed using 60-day rolling window. The sample covers January 2003 to September 2011, including altogether 2,228 observations. DM indicates test statistic from the Diebold and Mariano (1995) test. The null hypothesis posits equal forecasting accuracy between the two models. The statistic is asymptotically normal. The statistic is computed as $DM = d / \hat{\sigma}(d)$ where d is the sample average of d_t and $\hat{\sigma}(d)$ is a heteroscedastic and autocorrelation(HAC)-consistent estimate of the standard deviation of *d*.

In Table 3.18 we create portfolios based on the turnover measure. We find that the MSE from the range-based approach is insignificantly different from the rolling window approach and the conditional CAPM with time-varying coefficient over the full sample period. This result again suggests that the range-based approach could be chosen as an alternative when analysing highly liquid assets.

Table 3.18. Turnover sorted portfolios

	Beta	MSE	Beta	MSE	Beta	MSE
	Full sample		Pre-crisis		Post-crisis	
Low Turnover						
Range-beta	0.530	0.00024	0.519	0.00018	0.540	0.00031
Rolling Window	0.577	0.00023	0.535	0.00017	0.619	0.00029
DM stat, (*p*-value)		(0.000)		(0.000)		(0.000)
Conditional CAPM with Time-varying Betas	0.578	0.00023	0.530	0.00017	0.628	0.00029
DM stat, (*p*-value)		(0.000)		(0.000)		(0.000)
Turnover by Volume	4.021		5.508		2.495	
Medium Turnover						
Range-beta	0.742	0.00026	0.746	0.00015	0.738	0.00038
Rolling Window	0.810	0.00024	0.780	0.00014	0.841	0.00035
DM stat, (*p*-value)		(0.000)		(0.000)		(0.001)
Conditional CAPM with Time-varying Betas	0.817	0.00025	0.792	0.00014	0.843	0.00036
DM stat, (*p*-value)		(0.008)		(0.000)		(0.044)
Turnover by Volume	21.278		29.205		13.138	
High Turnover						
Range-beta	0.868	0.00038	0.799	0.00017	0.939	0.00060
Rolling Window	0.962	0.00036	0.889	0.00016	1.038	0.00056
DM stat, (*p*-value)		(0.060)		(0.000)		(0.133)
Conditional CAPM with Time-varying Betas	0.989	0.00035	0.916	0.00016	1.063	0.00054
DM stat, (*p*-value)		(0.115)		(0.0004)		(0.156)
Turnover by Volume	132.329		109.727		155.540	

Notes: The Table reports the sample statistics for the turnover sorted portfolios. The range-based beta and the historical beta estimates are constructed using 60-day rolling window. The sample covers January 2003 to September 2011, including altogether 2,228 observations. DM indicates test statistic from the Diebold and Mariano (1995) test. The null hypothesis posits equal forecasting accuracy between the two models. The statistic is asymptotically normal. The statistic is computed as $DM = d / \hat{\sigma}(d)$ where d is the sample average of d_t and $\hat{\sigma}(d)$ is a heteroscedastic and autocorrelation(HAC)-consistent estimate of the standard deviation of d.

3.4 Conclusions

Many applications of modern finance theory require precise beta estimates for individual stocks. However, as noted by Campbell et al. (2001), “firm-specific betas are difficult to estimate and may well be unstable over time”. In this paper we therefore create a new range-based beta measure which uses the information on the daily opening, closing, high, and low prices. The range-based beta is appealing for the ease of its estimation. The construction of the range-based beta requires only the current day high, low, closing, and opening prices. We also combine our new estimation methodology with non-parametric approach for modelling the changes in beta. We estimate both our new range-based beta measure and betas extracted using traditional methodologies and compare their performance.

We analyse constituents of the DAX index in the period 2003-2011. We demonstrate that our approach yields competitive estimates of firm-level betas compared with traditional methods. Moreover, we document strong cross-sectional variation in betas of firms that are grouped together in portfolios sorted on betas. Consequently, aggregating stocks into portfolios conceals important information contained in individual stock betas and reduces the cross-sectional variation in betas.

Since our framework is flexible, it can be readily extended to include multiple risk factors, where factor betas can be estimated based on the range-based variance and covariance.

Chapter 4: Relative Performance of Options-implied and Range-based Volatility Estimates in Euro Area Countries

4.1 Introduction

In this study we assess the information content of the implied volatility by considering implied volatility indices that are constructed based on the concept of model-free implied variance proposed by Demeterfi et al. (1999). In particular, using the range-based volatility estimator as a proxy for the realized variance we study the linkages between the range-based volatility and the implied volatility measure for the sample of five equity indices over the period January 3, 2000 to November 26, 2012. We assess the two-way relationships between the range-based volatility and the implied volatility, both within the index and accounting for spillovers between indices. Moreover, we study the evolution of spillovers between the range-based volatility and the implied volatility over time, identifying the net receivers and transmitters of shock and quantifying their magnitude using impulse response analysis. Finally, we consider average variance risk premium estimate defined as the simple average of the differences between the realized return variance and the implied variance.

We find that implied volatility does contain information in forecasting realized range-based volatility. The historical range-based volatility, on the other hand, has less explanatory power than the implied volatility in predicting realized range-based volatility.

The chapter is organized as follows: Section 4.2 overviews the main definitions and calculation methodology for the range-based volatility measure. In Section 4.3, we discuss the construction of volatility indexes. Section 4.4 presents the data used in this study and overviews the statistical properties of the implied and the realized range-based volatility estimators. In addition, in Section 4.4, the relationship between the implied and range-based volatility measure is examined using monthly non-overlapping samples. Finally, we conduct variance risk premium analysis. Concluding remarks are presented in Section 4.5.

4.2 Identification and Estimation

Consider the general continuous-time stochastic volatility model for the logarithmic stock price process p_t,

$$dp_t = \mu_t dt + \sigma_t dW_t , \qquad (4.1)$$

where W_t is a standard Brownian process. We assume that μ_t is general and may depend, for instance, on σ_t and p_t. The process σ_t is also general. Note that the functional forms of $\mu_t(\cdot)$ and $\sigma_t(\cdot)$ are completely flexible as long as they avoid arbitrage. The point-in-time volatility σ_t^2 entering the stochastic volatility model above is latent and its consistent estimation through filtering is complicated by a host of market microstructure noise. Alternatively, the model-free realized volatility measures afford a simple way of quantifying the integrated volatility over non-trivial time intervals. It is common in the literature to compute the realized volatility (RV) by summing the squared high-frequency returns over the $[t, t+\Delta]$ time-interval:

$$RV_{t,t+\Delta} = \sum_{i=\Delta}^{n} r_{i\Delta,\Delta}^2 . \qquad (4.2)$$

It follows then by the theory of quadratic variation (see, e.g., Andersen et al. (2003a), for a recent survey of the realized volatility literature),

$$\lim_{n\to\infty} RV_{t,t+\Delta} \overset{a.s.}{\to} \int_t^{t+\Delta} \sigma_s^2 ds , \qquad (4.3)$$

where *a.s.* denotes almost sure convergence and n is number of periods over the interval Δ. In other words, when n is large relative to Δ, the measurement error in the realized volatility should be small, that is:

$$RV_{t,t+\Delta} \approx \int_t^{t+\Delta} \sigma_s^2 ds . \qquad (4.4)$$

In our context, the quadratic variation equals the integrated volatility denoted by IV_t:

$$RV_{t,t+\Delta} \xrightarrow{p} IV_{t,t+\Delta} , \qquad (4.5)$$

where p denotes convergence in probability.

Using option prices, it is also possible to construct a model-free measure of the risk-neutral expectation of the integrated volatility. In particular, the time t volatility measure computed as a weighted average, or integral, of a continuum of Δ-maturity options is

$$IV_{t,t+\Delta}^{*} = 2\int_0^{\infty} \frac{C(t+\Delta, K) - C(t, K)}{K^2} dK , \qquad (4.6)$$

where $C(t, K)$ denotes the price of a European call option maturing at time t with strike price K. As shown by Demeterfi et al. (1999), this model-free implied volatility then equals the true risk-neutral expectation of the integrated volatility,

$$IV_{t,t+\Delta}^{*} = E^{*}\left(RV_{t,t+\Delta} \middle| G_t\right), \qquad (4.7)$$

where $E^{*}(\cdot)$ refers to the expectation under the risk-neutral measure Q and G_t denotes an information set. Although the original derivation of this important result in Demeterfi et al. (1999) assumes that the underlying price path is continuous, this same result has been extended by Jiang and Tian (2005) to the case of jump diffusions. Moreover, Jiang and Tian (2005) also demonstrates that the integral in the formula for $IV_{t,t+\Delta}^{*}$ may be accurately approximated from a finite number of options in empirically realistic situations.

The choice of volatility proxy, however, is less obvious, as financial markets are not frictionless and microstructure bias sneaks into the realized volatility, when n is too large. To illustrate, with noisy prices RV is biased and inconsistent, see, e.g., Zhou (1996), Bandi and Russell (2004, 2005), Aït-Sahalia et al. (2005), and Hansen and Lunde (2006). In empirical work, the benefits of more frequent sampling is traded off against the damage caused by cumulating noise, and by using various criteria for picking the optimal sampling frequency (e.g., at the 5-, 10-, or 30-minute frequency), whereby data are discarded.

The limitations of the realized volatility motivate our choice of using the range-based volatility proxy. Using a terminology similar to the above, we define the intraday range at sampling times t_{i-1} and t_i. For the ith interval of length Δ on the period t, for $i = 1,2,..,I$ with $I = 1/\Delta$ assumed to be integer, we observe the high log price $\sup_{(i-1)\Delta<j<i\Delta} p_{t-1+j}$ and the low log price $\inf_{(i-1)\Delta<j<i\Delta} p_{t-1+j}$. Under the assumption of a fully observed continuous time log-price path, Parkinson (1980) proposes the scaled high-low range estimator for the variance:

$$hl_t = \frac{1}{4\log 2}\left(\sup_{(i-1)\Delta<j<i\Delta} p_{t-1+j} - \inf_{(i-1)\Delta<j<i\Delta} p_{t-1+j}\right). \qquad (4.8)$$

Parkinson (1980)'s estimator is expected to be a more accurate measure of realized volatility than the sum of daily stock returns, because intraday prices theoretically contain more volatility information than daily closing prices.

4.3 Construction of Volatility Indexes

Over recent years, the derivatives exchanges have started constructing implied volatility indices. In 1993, the CBOE introduced an implied volatility index, named VIX. In 1994, the German Futures and Options Exchange launched an implied volatility index (VDAX) based on DAX index options. In 1997, the French Exchange market MONEP created a volatility index (VCAC) that reflects the synthetical at-the-money implied volatilities of the CAC-40 index options. In 2005, the DJ EURO STOXX Volatility Index (VSTOXX) with 30-day maturity was introduced. The VSTOXX Index covers Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal, and Spain. France, Germany, the Netherlands, and the DJ EURO STOXX volatility indexes are constructed following the new CBOE methodology for the VIX index. Therefore, we next overview the main definitions and the calculation methodology for the new VIX.

4.3.1 Old Volatility Indexes

The old VIX index is based on the Black-Scholes implied volatility of S&P 100 options. To construct the old VIX, two puts and two calls for strikes immediately above and below the current index are chosen. Near maturities (greater than eight days) and second nearby maturities are chosen to achieve a complete set of eight options. By inverting the Black-Scholes pricing formula using current market prices, an implied volatility is found for each of the eight options. These volatilities are then averaged, first the puts and the calls, then the high and low strikes. Finally, an interpolation between maturities is done to compute a 30 calendar day (22 trading day) implied volatility.

Because the Black-Scholes model assumes the index follows a geometric Brownian motion with constant volatility, when in fact it does not, the old VIX will only approximate the true risk-neutral implied volatility over the coming month. In reality the price process is likely more complicated than geometric Brownian motion. Limiting it to a very specific form and deducing an implied volatility from market prices may lead to substantial error in the estimation.

4.3.2 New Methodology

Demeterfi et al. (1999) develop a model-free risk-neutral implied volatility over a future time period. Suppose call options with a continuum of strike prises (*K*) for a given maturity (*T*) are traded on an underlying asset. Following Demeterfi et al. (1999) and Britten-Jones and Neuberger (2000), we consider the asset price and forward option price, denoted by *S* and $C\left(T, K^{rT}\right)$, respectively. Britten-Jones and Neuberger (2000) model-free implied volatility is defined as follows:

$$E_0^Q\left[\int_0^T\left(\frac{dS_t}{S_t}\right)^2\right] = 2\int_0^\infty \frac{C\left(T, Ke^{rT} - \max\left(S_0 - K, 0\right)\right)}{K^2}dK, \qquad (4.9)$$

where the integrated return variance between the current date 0 and a future date *T* is fully specified by the set of prices of call options expiring on date *T*. The expectation (E_0^Q) is taken under the risk-neutral measure. r is the risk-free interest to expiry. The price (S_t) and volatility processes are not assumed to follow a specific model, but only required to satisfy the following assumptions: (1) Markovian, (2) continuity, and (3) no-arbitrage. S_0 is the stock price at time t=0.

The new volatility index is a typical application of model-free implied volatility which was launched by the CBOE in September 2003 and is calculated, based upon the following formula, using S&P 500 index options:

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{rT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2, \tag{4.10}$$

where the superscript F denotes the forward index level derived from index prices, and K_0 is the first strike below F in the definition of VIX. K_0 is the strike price used to determine $Q(K_i)$ which is call or put option price. $Q(K_i)$ is the call price with strike K_i if $K_i \geq K_0$, otherwise it is the put price; and ΔK_i is the interval between the strike prices, defined as $\frac{K_{i+1} - K_{i-1}}{2}$.[1] The last term in Equation (4.10) is intended to adjust for the fact that there is no exact at-the-money option. ΔK_i is the interval between strikes on either side of K_i. And T is the time to maturity which is now based in minutes instead of days as in the old VIX. One of the advantages of this approach is that all available out-of-the money call and put options are utilized instead of just the eight used in the old VIX.

Carr and Wu (2006) show that the new VIX squared approximates the conditional expectation of the annualized return variance under the risk-neutral measure over the next 30 calendar days:

$$VIX_t^2 \cong E_t^Q\left[\sigma_t^2\right], \tag{4.11}$$

[1] ΔK_i for the lowest strike is defined as the difference between the lowest strike and the next higher strike. Similarly, ΔK_i for the highest strike is the difference between the highest strike and the next lower strike.

where σ_t^2 denotes the annualized return variance from time t to 30 calendar days later and the other notation stays the same as before. Hence, VIX_t^2 approximates the 30-day variance swap rate. Variance swap contracts are actively traded over the counter on major equity indexes. At maturity, the long side of the variance swap contract receives a realized variance and pays a fixed variance rate, which is the variance swap rate. At the time of entry, the contract has zero value. Hence, by no-arbitrage, the variance swap rate equals the expected value of the realized variance under the risk-neutral measure.

4.4 Empirical Results

4.4.1 Data Description

We consider European implied volatility indices for France, Germany, the Netherlands, and the DJ EURO STOXX 50 Index. In addition, we analyse the U.S. (VIX) volatility index. The construction algorithm of all implied volatility indices is based on the concept of model-free implied variance proposed by Demeterfi et al. (1999). The indices represent the 30-day variance swap rate once they are squared (see Carr and Wu, 2006). VCAC, VDAX-New, VAEX, and VSTOXX are constructed from the market prices of options on the CAC 40 (France), the DAX (Germany), the AEX (Netherlands), and the DJ EURO STOXX 50 index, respectively. The new VIX is based on the S&P 500 implied volatility. The data for all the implied volatility indices are obtained from DATASTREAM.

The volatility index is the (annualized) implied volatility of a non-traded (synthetic) option contract with one month to maturity. This measure is believed to be less affected by the problems that pollute standard implied volatility measures extracted from the corresponding index contracts. Examples of the market microstructure noise are the potential nonsynchronous measurement of option and index levels, early exercise and dividends, bid-ask spreads as well as the wild card option (see Christensen and Prabhala, 1998 for discussion).

Early empirical research in option pricing typically involves severely overlapped daily samples. As options expire on a fixed calendar date, implied volatilities calculated from the same option over two consecutive business days are likely to be highly correlated because the time horizons differ by just one day or at most several days (over the weekend or holidays). As demonstrated by Christensen, Hansen and Prabhala (2001), such overlapped samples may lead to the so-called overlapping data errors problem and render the *t*-statistics and other diagnostic statistics in the linear regression invalid. Therefore, following Christensen and Prabhala (1998), we use monthly non-overlapping observations to control the correlation structure of the regression errors by taking the closing value of each month. We also multiply the implied volatility measures by a constant factor equal to $\left(\frac{252}{365}\right)^{1/2}$ to account for the difference between trading days and calendar days in a year (Schwert, 2002).

We compute the range-based volatility over the remaining life (one month) of the option as

$$\sigma_t = \sqrt{\frac{252}{n_t}\sum_{j=1}^{n_t} R_j}\,, \tag{4.12}$$

where n_t is the number of trading days in month t. As defined in Chapter 2, the discrete version of the Parkinson scaled range (hl_{it}) is

$$hl_{it} = \frac{1}{4\ln 2}\left(\max_{0\le\tau\le1} \ln p_{i\tau} - \min_{0\le\tau\le1} p_{i\tau}\right)^2 \text{ for } i = 1,\ldots,n, \tag{4.13}$$

where τ is a continuous index τ for intraday time and t is a discrete index for days; p_τ, $0 \le \tau \le 1$ denotes the n–vector of intraday log prices on n assets during day t.

In sum, after the data transformations we have σ_t^{IMP}, which is the annualized (assuming 252 trading days per year) implied standard deviation for a synthetic, at-the-money, option contract with one month to maturity, as given by the option implied volatility indexes, and σ_t, which is the annualized range-based volatility measure of the equity indexes over the remaining life (one month) of the option. Both series contain 155 non-overlapping observations.

4.4.2 Statistical Properties of the Volatility Indexes

In this section we characterize some of the statistical properties of the volatility indexes. Figure 4.1 illustrates the time evolution of the associated realized one-month range-based volatility series for the DJ EURO STOXX 50 along with the implied volatility measure over the period January 2000 to November 2012. As explained in the previous sections, the implied volatility may be viewed as an indicator of future monthly volatility while the realized range-based volatility provides a measure of the actual realized volatility over that month. A couple of points are evident from the graph. First, there is good coherence between the implied volatility and the ensuring market volatility. However, since the realized range-based volatility is recorded daily but represents monthly (future) volatility, there is a great deal of induced serial correlation in this series. Hence, this feature must be interpreted with some care. Second, it is evident that the implied volatility series almost uniformly exceeds the subsequent realized range-based volatility. This is consistent with earlier work establishing the presence of a substantial negative variance risk premium in the implied volatility measures. In other words, investors are on average willing to pay a sizeable premium to acquire a positive exposure to future equity-index volatility. In addition, the VSTOXX seems to oscillate in long swings between a quite volatile regime with high index values and a more stable regime with low index values. High volatility characterizes the periods ranging from 1999 to 2003 and from mid 2007 onwards. In contrast, low volatility seems dominant from 2004 to mid 2007. This is consistent with Whaley's (2000) claim that one may interpret the volatility index as the investor's fear gauge. There are a series of financial crises in the periods featuring a high volatility index, e.g., the internet burst in 2000, the 9/11 terrorist attack in 2001, the corporate scandals in 2002, the quantitative long/short equity hedge funds meltdown in the first week of August 2007, and the subsequent credit crunch and global financial crisis.

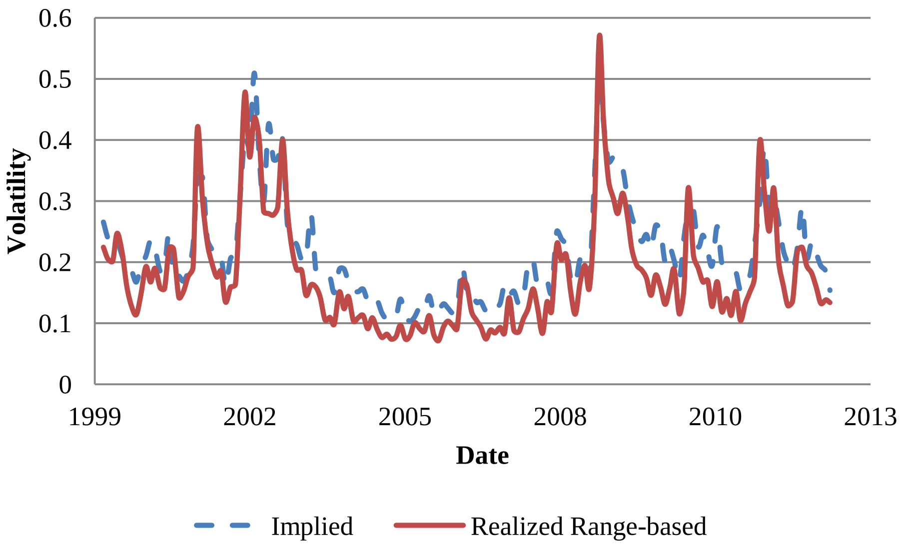

Figure 4.1. Implied and realized range-based volatility for the DJ EURO STOXX 50 index
Sample size ranges from January 3, 2000 to November 26, 2012, monthly frequency. Implied and realized range-based volatilities are annualized and given in decimal form.

Figure 4.2 illustrates the time evolution of the associated change in realized one-month range-based volatility series for the DJ EURO STOXX 50 along with the change in implied volatility measure. First note that the variability of the market represented by the first difference in the realized range-based volatility is close to the variability of the market represented by the first difference in implied volatility measure. Also note that the change in the realized range based volatility measure has higher amplitude of the variability and greater extremes during periods of financial turbulence.

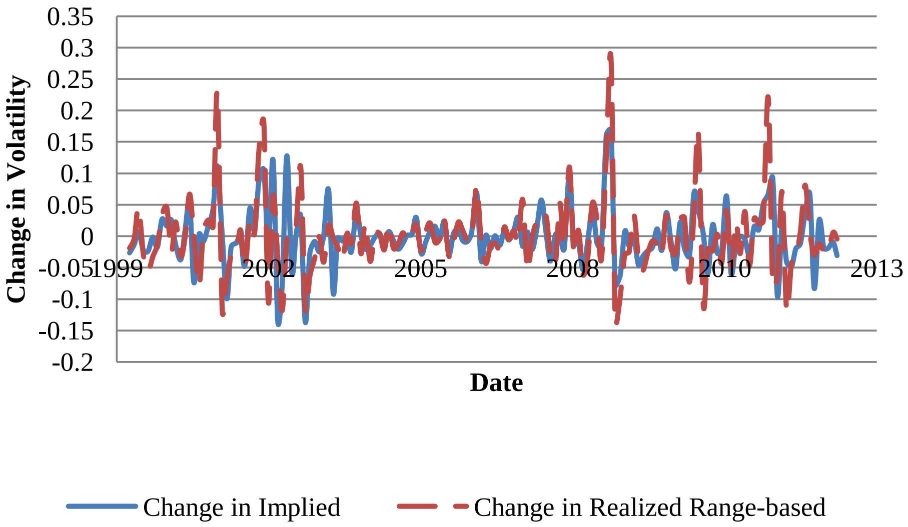

Figure 4.2. Changes in implied and realized range-based volatility for the DJ EURO STOXX 50 index

Sample size ranges from January 3, 2000 to November 26, 2012, monthly frequency.

Tables 4.1 and 4.2 document the results of our preliminary descriptive statistics. In particular, it reports the sample mean, standard deviation, minimum, maximum, skewness, and kurtosis for the time series of the implied volatility and the range-based volatility as well as the *p*-value of the Jarque-Bera test for normality. Both implied volatility and range-based volatility measures display heavy tails and positive skewness. As typically the case with volatility measures, a simple logarithmic transformation (not reported here) would almost lead to normality (see, e.g., Andersen et al., 2003a). Comparing the volatility index with the corresponding range based volatility measure, we find that on average, the volatility index is approximately 18 percentage points higher than the corresponding range based volatility. Note also that the realized range-based volatility measures, as expected, are more volatile than the implied measures. They have higher standard deviation, skewness, and kurtosis statistics. Such discrepancies are, of course, typical when comparing series that represent expectations of future realizations versus the actual ex-post realizations.

Table 4.1 also evaluates the persistence of the volatility indexes through a battery of testing procedures. It reports the *p*-values of the Augmented Dickey-Fuller (ADF), Dickey-Fuller GLS (DF-GLS), and Phillips-Perron (PP) tests for unit root. We select the number of lags in the ADF and DF-GLS tests using the

Bayesian information criterion, whereas we run the PP test using the quadratic spectral kernel with Andrews (1991) bandwidth choice. We reject the null hypothesis of a unit root for the volatility indexes of European markets with the ADF, ADF-GLS and PP tests in the period of January 3, 2000 to November 26, 2012.

Table 4.1. Descriptive statistics for the volatility indexes

Sample statistics	France	Germany (GDAXNEW)	Netherlands	the DJ EURO STOXX 50	USA
Mean	0.203	0.211	0.203	0.215	0.154
Minimum	0.098	0.102	0.048	0.103	0.067
Maximum	0.499	0.537	0.510	0.510	0.392
Standard Deviation	0.074	0.082	0.087	0.081	0.068
Skewness	1.303	1.484	1.407	1.227	0.981
Kurtosis	4.874	5.183	4.698	4.501	3.434
Jarque-Bera	0.000	0.000	0.000	0.000	0.000
ADF	0.042	0.004	0.088	0.059	0.057
DF-GLS	0.042	0.045	0.045	0.044	0.033
PP	0.004	0.004	0.010	0.005	0.016

Notes: The sample period runs from January 3, 2000 to November 26, 2012, including altogether 3,193 time-series observations. We report the sample mean, minimum, maximum, standard deviation, skewness, and kurtosis for the volatility indexes, as well as the p-values of the Jarque-Bera test for normality and of the Augmented Dickey-Fuller (ADF), Dickey-Fuller GLS (DF-GLS), and Phillips-Perron (PP) tests for unit root. The implied volatility indices are based on the approach of model-free implied variance proposed by Demeterfi et al. (1999). VCAC, VDAX-New, VAEX, and VSTOXX are constructed from the market prices of options on the CAC 40 (France), the DAX (Germany), the AEX (Netherlands), and the DJ EURO STOXX 50 index, respectively. New VIX is based on S&P 500 implied volatility.

Table 4.2. Descriptive statistics for the range-based volatility

Sample statistics	France	Germany	Netherlands	the DJ EURO STOXX 50	USA
Mean	0.168	0.187	0.159	0.178	0.145
Minimum	0.068	0.070	0.062	0.071	0.063
Maximum	0.511	0.597	0.514	0.568	0.635
Standard deviation	0.081	0.101	0.088	0.090	0.081
Skewness	1.403	1.550	1.657	1.555	2.552
Kurtosis	5.305	5.430	5.751	5.743	12.901
Jarque-Bera	0.000	0.000	0.000	0.000	0.000
ADF	0.0003	0.0007	0.0006	0.0003	0.0003
DF-GLS	0.0506	0.0606	0.0551	0.0566	0.0516
PP	0.0003	0.0009	0.0006	0.0003	0.0003

Notes: The Table reports the sample statistics for the range-based volatility measure. The sample period runs from January 3, 2000 to November 26, 2012, including altogether 3,193 time-series observations. We report the sample mean, minimum, maximum, standard deviation, skewness, and kurtosis for the range-based volatility estimators, as well as the p-values of the Jarque-Bera test for normality and of the Augmented Dickey-Fuller (ADF), Dickey-Fuller GLS (DF-GLS), and Phillips-Perron (PP) tests for unit root. The range-based volatility measure is constructed using Parkinson version of the range-based volatility, as defined in Equation (4.13).

Table 4.3 reports the correlations matrix of monthly 30-day volatility for the model-free implied volatility and the range based volatility estimates. Each volatility index is positively correlated with its corresponding range based volatility estimate. The highest correlation is between the range-based volatility measure and the implied volatility measure for the Netherlands. In addition, the volatility indexes are highly correlated. The highest correlation coefficient is between VSTOXX and VCAC implied volatility measures (0.991). The range based volatility estimates are also highly correlated. The highest correlation coefficient is again between the range based volatility estimates on the DJ EURO STOXX 50 index and the CAC 40 (0.916).

Table 4.4 shows the cross-correlations. First note that the realized range-based volatility measures have the lowest serial correlation at monthly frequency where the measurement overlap ceases to have an effect. The cross-correlations confirm the presence of the leverage effect. Each volatility index is positively correlated with its corresponding subsequent range based volatility estimate. Also, the end of the month implied volatility measure is a better predictor of the next month realized range-based volatility. We can also see that the first-order autocorrelation is the highest between the range-based volatility measure and the implied volatility measure for Germany.

Table 4.3. Volatility correlations

Panel A					
Dependent Variable	France (range)	Germany (range)	Netherlands (range)	the DJ EURO STOXX 50 (range)	USA (range)
France (range)	1.000				
Germany (range)	0.949	1.000			
Netherlands (range)	0.959	0.962	1.000		
the DJ EURO STOXX 50 (range)	0.987	0.970	0.972	1.000	
USA (range)	0.904	0.837	0.860	0.880	1.000
France (vol. Index)	0.920	0.908	0.921	0.932	0.800
Germany (vol. Index)	0.896	0.937	0.928	0.922	0.787
Netherlands (vol. Index)	0.880	0.899	0.938	0.901	0.800
the DJ EURO STOXX 50 (vol. Index)	0.916	0.906	0.914	0.930	0.800
USA (vol. Index)	0.841	0.839	0.798	0.835	0.817

Panel B					
Dependent Variable	France (vol. Index)	Germany (vol. Index)	Netherlands (vol. Index)	the DJ EURO STOXX 50 (vol. Index)	USA (vol. Index)
France (vol. Index)	1.000				
Germany (vol. Index)	0.971	1.000			
Netherlands (vol. Index)	0.953	0.965	1.000		
the DJ EURO STOXX 50 (vol. Index)	0.991	0.978	0.951	1.000	
USA (vol. Index)	0.841	0.828	0.804	0.845	1.000

Notes: The Table reports the correlations of the range-based volatility proxies and implied volatilities. The sample period runs from January 3, 2000 to November 26, 2012, monthly frequency. The implied volatility indices are based on the approach of model-free implied variance proposed by Demeterfi et al. (1999). VCAC, VDAX-New, VAEX, and VSTOXX are constructed from the market prices of options on the CAC 40 (France), the DAX (Germany), the AEX (Netherlands), and the DJ EURO STOXX 50 index, respectively. New VIX is based on S&P 500 implied volatility.

Table 4.4. Volatility cross-correlations

Panel A					
Dependent Variable	France (range)	Germany (range)	Netherlands (range)	the DJ EURO STOXX 50 (range)	USA (range)
France (range) at *t*-1	0.650	0.639	0.641	0.655	0.589
Germany (range) at *t*-1	0.634	0.692	0.657	0.658	0.560
Netherlands (range) at *t*-1	0.642	0.658	0.687	0.656	0.571
the DJ EURO STOXX 50 (range) at *t*-1	0.639	0.648	0.648	0.658	0.575
USA (range) at *t*-1	0.592	0.575	0.579	0.602	0.633
France (vol. Index) at *t*-1	0.727	0.760	0.746	0.743	0.669
Germany (vol. Index) at *t*-1	0.703	0.777	0.750	0.729	0.637
Netherlands (vol. Index) at *t*-1	0.700	0.759	0.762	0.725	0.654
the DJ EURO STOXX 50 (vol. Index) at *t*-1	0.733	0.771	0.754	0.752	0.670
USA (vol. Index) at *t*-1	0.674	0.720	0.670	0.681	0.674

Panel B					
Dependent Variable	France (vol. Index)	Germany (vol. Index)	Netherlands (vol. Index)	the DJ EURO STOXX 50 (vol. Index)	USA (vol. Index)
France (vol. Index) at *t*-1	0.980	0.955	0.945	0.970	0.829
Germany (vol. Index) at *t*-1	0.954	0.987	0.962	0.963	0.826
Netherlands (vol. Index) at *t*-1	0.946	0.962	0.988	0.948	0.815
the DJ EURO STOXX 50 (vol. Index) at *t*-1	0.975	0.967	0.951	0.984	0.845
USA (vol. Index) at *t*-1	0.841	0.838	0.826	0.855	0.985

Notes: The Table reports the cross-correlations of the range-based volatility proxies and implied volatilities. The sample period runs from January 3, 2000 to November 26, 2012, monthly frequency. The implied volatility indices are based on the approach of model-free implied variance proposed by Demeterfi et al. (1999). VCAC, VDAX-New, VAEX, and VSTOXX are constructed from the market prices of options on the CAC 40 (France), the DAX (Germany), the AEX (Netherlands), and the DJ EURO STOXX 50 index, respectively. New VIX is based on S&P 500 implied volatility.

4.4.3 The Information Content of Implied Volatility

We start our analysis by assessing the percentage difference between the implied volatility index (σ_t^{IMP}) and the annualized realized volatility (σ_t), $\xi_t = \left(\sigma_{t-1}^{IMP} - \sigma_t\right)/\sigma_t$ which provides a standardized measure of daily forecast errors. We use the range-based volatility measure as a proxy for the realized volatility. As exhibited in Figures 4.3-4.7, forecast error is dominated by the periods of volatility overestimation in Euro area countries. There is also stronger tendency for an upward bias with respect to the volatility index for the Euro area countries than for the US market. Judging from the sign and magnitude of the errors in volatility expectations, the evidence suggests that because of upward bias, implied volatility is not a perfect forecast of future volatility.

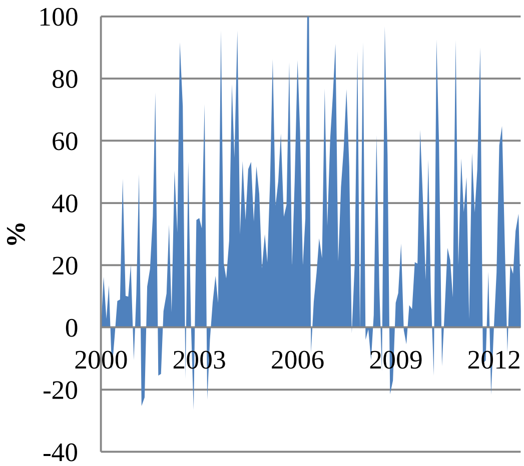

Figure 4.3. The percentage difference between implied and realized range-based volatility, France

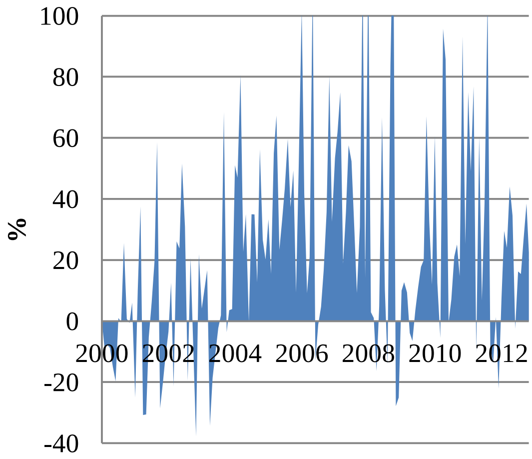

Figure 4.4. The percentage difference between implied and realized range-based volatility, Germany

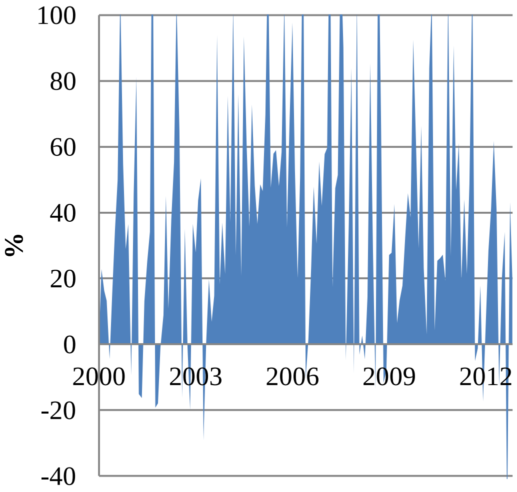

Figure 4.5. The percentage difference between implied and realized range-based volatility, the Netherlands

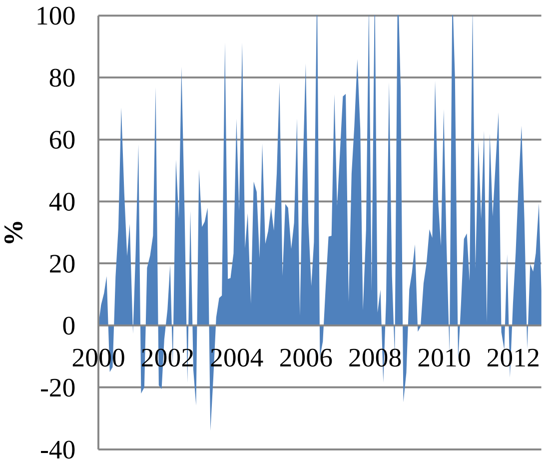

Figure 4.6. The percentage difference between implied and realized range-based volatility, the DJ EURO STOXX 500

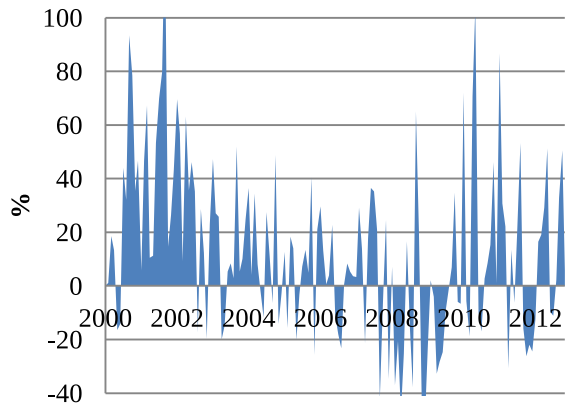

Figure 4.7. The percentage difference between implied and realized range-based volatility, U.S.

The forecasting ability of the implied volatility is typically assessed in the literature by an in-sample regression, also known as a Mincer-Zarnowitz regression (see Mincer and Zarnowitz, 1969). This approach requires the estimation of the coefficients of a regression of the target on a constant and a time series forecasts, i.e.

$$\sigma_t = \alpha_0 + \beta h_{t-1} + e_t \quad \forall t = 1, \ ..., T. \tag{4.14}$$

The null hypothesis of optimality of the forecast can be written as $H_0 : \alpha_0 = 0$ and $\beta = 1$. Given the latent nature of the target variable, the regression in (4.14) is unfeasible. Substituting the true variance by conditionally unbiased volatility, $\hat{\sigma}_t = \sigma_t + \eta_t$ with $E_{t-1}[\eta_t] = 0$, we can rewrite (4.14) as

$$\hat{\sigma}_t = \alpha_0 + \beta h_{t-1} + \varepsilon_t, \tag{4.15}$$

where the innovation are $\varepsilon_t = \eta_t + e_t$. Since $\hat{\sigma}_t$ is a conditionally unbiased estimator of the true variance then (4.15) yields unbiased estimates of α_0 and β. The Mincer-Zarnowitz regression allows to evaluate two different aspects of the volatility forecast. First, the Mincer-Zarnowitz regression allows to test the presence of systematic over- or under-predictions, that is whether the forecast is biased, by testing the joint hypothesis $H_0 : \alpha_0 = 0$ and $\beta = 1$. Second, being the R^2 of (4.15) an indicator of the correlation between the realization and the forecast it can be used as an evaluation criterion of the accuracy of the forecast.

We assess the forecasting ability between the implied volatility and the future realized volatility, which is range-based volatility in our analysis, by estimating the following specification:

$$\hat{\sigma}_t = \alpha_0 + \beta\sigma_{t-1}^{IMP} + \varepsilon_t. \qquad (4.16)$$

$\hat{\sigma}_t$ denotes the annualized range-based realized term volatility for period t and σ_{t-1}^{IMP} denotes the annualized implied volatility at the beginning of period $t-1$, being an ex-ante measure of t volatility. If the implied volatility contains information in forecasting the future volatility, then β should be significantly different from zero $(H_0 : \beta = 0)$. Moreover, if the implied volatility is an unbiased forecast of the realized range-based volatility, then the joint hypothesis that $\alpha_0 = 0$ and $\beta = 1$ cannot be rejected $(H_0 : \alpha_0 = 0 \text{ and } \beta = 1)$. R^2 captures the degree of variation in the ex-post realized range-based volatility explained by the forecast. Table 4.5 reports the regression results. The slope coefficient for the implied volatility, β, is significantly different from zero at 99% confidence interval for all realized range-based volatility measure , indicating that the implied volatility contains information in forecasting future realized range-based volatility. The value of β ranges from 0.806 for the Netherlands and 0.982 for Germany. The intercept α is significantly different from zero for Germany (-0.021). The *t*-statistics for the null hypothesis that $\beta = 1$ is rejected for France, the Netherlands, the DJ EURO STOXX 50, and the U.S. index. However, the null hypothesis that $\beta = 1$ cannot be rejected for Germany. In addition, the null hypothesis that the implied volatility is an unbiased estimator of future realized range-based volatility (H_0: $\alpha_0 = 0$ and $\beta = 1$) is strongly rejected for France, Germany, the Netherlands, the DJ EURO STOXX, and the U.S. index. This result is not surprising because the previous literature shows that the implied volatility measures generally are not unbiased as the implied volatility embeds a sizeable premium related to equity market volatility risk. The results are also consistent with the summary statistics in Tables 4.1 and 4.2 that show that the implied volatilities are on average much greater than the realized range-based volatilities. Overall, our results are consistent with the findings of Christensen and Prabhala

(1998). Regression (4.16) produced a β-estimate that was significantly greater than zero. The forecasting ability of the implied volatility as measured by the adjusted R^2 ranges from 0.666 for USA and 0.880 for the Netherlands. Our results are comparable with those of Shu and Zhang (2003) who analyse the forecast ability of implied volatility computed using the range based volatility estimator. They find that the adjusted R^2 is 0.3647 which is much lower than ours. This could be related to the fact that Shu and Zhang (2003) use the Black-Scholes model and Heston (1993) stochastic volatility option-pricing model, whereas our implied volatility is extracted from the model-free approach.

Table 4.5. Forecast regression of implied volatility

Sample Statistics	France	Germany	Netherlands	the DJ EURO STOXX 50	USA
α_0	-0.008 (-0.73)	-0.021 (-2.00)	-0.006 (-0.53)	-0.012 (-1.08)	0.017 (1.42)
β	0.865 (13.92)	0.982 (17.82)	0.806 (13.63)	0.879 (15.01)	0.825 (9.06)
Adj. R-sq	0.622	0.641	0.620	0.622	0.478
T statistic for $\beta = 1$	4.74	0.10	10.77	4.26	3.69
F test $\alpha_0 = 0$ and $\beta = 1$	42.24	15.15	53.99	40.70	2.62
DW	1.520 (0.001)	1.520 (0.001)	1.482 (0.000)	1.509 (0.001)	0.899 (0.000)

Notes: The heteroskedasticity and autocorrelation consistent t-statistics are reported below the regression coefficient. The implied volatility indices are based on the approach of model-free implied variance proposed by Demeterfi et al. (1999). VCAC, VDAX-New, VAEX, and VSTOXX are constructed from the market prices of options on the CAC 40 (France), the DAX (Germany), the AEX (Netherlands), and the DJ EURO STOXX 50 index, respectively. New VIX is based on S&P 500 implied volatility. The range-based volatility measure is used as a proxy for the realized volatility. Implied and realized range-based volatilities are annualized and given in decimal form. The sample period runs from January 3, 2000 to November 26, 2012, monthly frequency.

If the implied volatility fails to predict future realized range-based volatility, it may be due to the fact that the realized range-based volatility is totally unpredictable. For instance, it is a random process; the past information contains no information in forecasting future volatility. To eliminate this possibility, researchers typically run a regression between the historical volatility and the realized range-based volatility and see whether the realized range-based volatility is predictable. The model is

$$\hat{\sigma}_t = \alpha_0 + \gamma \sigma_{t-1}^{HIST} + \varepsilon_t . \qquad (4.17)$$

If the historical volatility contains information in forecasting future volatility, then γ should be significantly different from zero. Moreover, if the historical volatility is an unbiased forecast of the realized range-based volatility, then the joint hypothesis that $\alpha_0 = 0$ and $\gamma = 1$ cannot be rejected.

Table 4.6 reports our regression results. As we can see, the slope coefficient γ is significantly different from zero for all measurements of volatility, indicating that the historical volatility does contain information in forecasting the next period realized range-based volatility. However, such a forecast is downward biased, all slope coefficients are significantly less than one. The largest γ coefficient is for Germany (0.794). In contrast to the results of Table 4.5, we find that the historical volatility is lower (with the exception of the US) compared to the implied volatility in forecasting future range-based realized volatility. When we run a regression between historical range-based volatility and realized range-based volatility, the adjusted R^2 decreases. This result supports the conclusion that historical range-based volatility has less forecast ability than implied volatility in forecasting future range-based realized volatility. The result is not surprising because option traders are generally institutional traders and may have better information in forecasting future volatility, so the implied backed out from market option price is more closely correlated with future realized range-based volatility.

Table 4.6. Forecast regression of historical range-based volatility.

Sample Statistics	France	Germany	Netherlands	the DJ EURO STOXX 50	USA
α_0	0.038 (4.84)	0.038 (4.25)	0.033 (3.98)	0.039 (4.75)	0.032 (3.58)
γ	0.774 (15.20)	0.794 (15.45)	0.788 (13.39)	0.776 (14.36)	0.774 (10.52)
Adj. R-sq	0.599	0.632	0.619	0.602	0.601
T statistic for $\gamma = 1$	19.71	16.08	13.05	17.26	9.39
F test $\alpha_0 = 0$ and $\gamma = 1$	11.80	9.28	7.93	11.27	6.62
DW	2.058 (0.633)	2.048 (0.711)	2.017 (0.892)	1.972 (0.830)	1.908 (0.471)

Notes: The heteroskedasticity and the autocorrelation consistent t-statistics are reported below the regression coefficient. The implied volatility indices are based on the approach of model-free implied variance proposed by Demeterfi et al. (1999). VCAC, VDAX-New, VAEX, and VSTOXX are constructed from the market prices of options on the CAC 40 (France), the DAX (Germany), the AEX (Netherlands), and the DJ EURO STOXX 50 index, respectively. New VIX is based on S&P 500 implied volatility. The range-based volatility measure is used as a proxy for the realized volatility. Implied and realized range-based volatilities are annualized and given in decimal form. The sample period runs from January 3, 2000 to November 26, 2012, monthly frequency.

It is also interesting to examine the relative forecasting ability of the implied volatility and historical range-based volatility. Theoretically, if the two information sets both contain information in forecasting the realized range-based volatility, but one information set is the subset of the other information set, then the regression between them and the realized range-based volatility will lead to the slope coefficient of the first information set to be zero and the second to be one. There is support for the hypothesis that σ^{IMP} subsumes the information content in σ^{HIST} if $\beta > 0$ and $\gamma = 0$. The following regression is examined:

$$\hat{\sigma}_t = \alpha_0 + \beta\sigma_{t-1}^{IMP} + \gamma\sigma_{t-1}^{HIST} + \varepsilon_t . \tag{4.18}$$

If $\beta > \gamma$, then the implied volatility performs better than historical range-based volatility in forecasting realized range-based volatility, and vice versa.

Table 4.7. Forecast regression of implied and historical range-based volatility

Sample Statistics	France	Germany	Netherlands	the DJ EURO STOXX 50	USA
α_0	0.003 (0.26)	-0.001 (-0.08)	0.008 (0.69)	0.002 (0.15)	0.021 (2.66)
β	0.556 (3.99)	0.572 (3.02)	0.434 (3.58)	0.565 (4.09)	0.206 (1.81)
γ	0.306 (2.83)	0.357 (2.46)	0.388 (3.84)	0.304 (2.71)	0.633 (4.79)
Adj. R-sq	0.638	0.659	0.641	0.637	0.611
T statistic for $\beta = 1$	10.18	5.10	21.85	9.92	48.55
T statistic for $\gamma = 0$	40.90	19.54	36.48	38.51	7.71
F test $\alpha_0 = 0$, $\beta = 1$, and $\gamma = 0$	29.12	11.41	40.23	28.45	22.87
DW	1.771 (0.052	1.797 (0.094)	1.788 (0.060)	1.726 (0.018)	1.769 (0.041)

Notes: The heteroskedasticity and the autocorrelation consistent t-statistics are reported below the regression coefficient. The implied volatility indices are based on the approach of model-free implied variance proposed by Demeterfi et al. (1999). VCAC, VDAX-New, VAEX, and VSTOXX are constructed from the market prices of options on the CAC 40 (France), the DAX (Germany), the AEX (Netherlands), and the DJ EURO STOXX 50 index, respectively. New VIX is based on S&P 500 implied volatility. The range-based volatility measure is used as a proxy for the realized volatility. Implied and realized range-based volatilities are annualized and given in decimal form. The sample period runs from January 3, 2000 to November 26, 2012, monthly frequency.

When both the implied volatility and the historical range-based volatility are regressed to the realized range-based volatility (Table 4.7), the results are different from the univariate regression. The regression coefficients β and γ drop dramatically from their values in univariate regressions. The *t*-statistic for the null hypothesis $\beta = 0$ and $\gamma = 0$ is rejected at 99% confidence interval. The intercept α is not significantly different from zero. The result shows that the implied volatility dominates the historical range-based volatility in forecasting the future realized range-based volatility for the Euro area implied volatility indexes, which means that all information contained in past price has already been reflected in the option market. This can be regarded as evidence that option markets process information efficiently. This result is consistent with the literature. For example, Chiras and Manaster (1978), and Beckers (1981) also find that the implied volatility provides better estimates of future return volatility than standard deviations obtained from historical returns. Adding historical range-based volatility as an explanatory variable slightly increases the forecasting ability of the implied volatility. Comparing our results with those of Shu and Zhang (2003), our specification is better in forecasting the implied volatility when the implied volatility is extracted from the model-free specification. The results for the U.S. market are also of a particular interest. We find that the range-based volatility dominates the implied volatility in forecasting the future realized range-based volatility for U.S. This result can be explained by the higher regulatory requirements for the option markets in U.S.

Finally, the small sample properties of the predictive regressions are decidedly better when the return variation is measured in log volatilities as this eliminates the main positive outliers and renders the various series close to being Gaussian. As a consequence, many prior studies focus on this metric as well. For robustness and compatibility with earlier work, we therefore provide supplementary results for the predictive regressions targeting the future monthly log return volatility as well as the future monthly return variance. The results are consistent with previous findings and even provide stronger evidence of our hypotheses. The results of regressions in logs are presented in Tables 4.8-4.10.

Table 4.8. Forecast regression of log implied volatility.

Sample Statistics	France	Germany	Netherlands	DJ EURO STOXX 50	USA
α_0	-0.064 (-0.71)	0.052 (0.62)	-0.300 (-2.29)	-0.109 (-1.28)	-0.450 (-4.00)
β	1.1045 (21.30)	1.144 (23.61)	1.000 (12.36)	1.080 (21.37)	0.818 (15.38)
Adj. R-sq	0.696	0.685	0.636	0.697	0.577
T statistic for $\beta = 1$	4.06	8.82	0.00	2.51	11.79
F test $\alpha_0 = 0$ and $\beta = 1$	72.92	38.08	86.15	71.20	10.71
DW	1.506	1.374	1.403	1.526	1.005

Notes: The heteroskedasticity and the autocorrelation consistent t-statistics are reported below the regression coefficient. The implied volatility indices are based on the approach of model-free implied variance proposed by Demeterfi et al. (1999). VCAC, VDAX-New, VAEX, and VSTOXX are constructed from the market prices of options on the CAC 40 (France), the DAX (Germany), the AEX (Netherlands), and the DJ EURO STOXX 50 index, respectively. New VIX is based on S&P 500 implied volatility. The range-based volatility measure is used as a proxy for the realized volatility. Implied and realized range-based volatilities are annualized and given in decimal form. The sample period runs from January 3, 2000 to November 26, 2012, monthly frequency.

Table 4.9. Forecast regression of historical range-based volatility (in logs).

Sample Statistics	France	Germany	Netherlands	DJ EURO STOXX 50	USA
α_0	-0.321 (-4.37)	-0.298 (-4.36)	-0.344 (-4.37)	-0.329 (-4.41)	-0.407 (-4.59)
γ	0.831 (22.12)	0.837 (22.30)	0.827 (21.26)	0.822 (21.16)	0.803 (19.83)
Adj. R-sq	0.693	0.702	0.683	0.677	0.647
T statistic for $\gamma = 1$	20.18	18.90	19.88	21.04	23.66
F test $\alpha_0 = 0$ and $\gamma = 1$	10.11	9.71	9.99	10.52	11.98
DW	2.166	2.141	2.136	2.085	2.075

Notes: The heteroskedasticity and the autocorrelation consistent t-statistics are reported below the regression coefficient. The implied volatility indices are based on the approach of model-free implied variance proposed by Demeterfi et al. (1999). VCAC, VDAX-New, VAEX, and VSTOXX are constructed from the market prices of options on the CAC 40 (France), the DAX (Germany), the AEX (Netherlands), and the DJ EURO STOXX 50 index, respectively. New VIX is based on S&P 500 implied volatility. The range-based volatility measure is used as a proxy for the realized volatility. Implied and realized range-based volatilities are annualized and given in decimal form. The sample period runs from January 3, 2000 to November 26, 2012, monthly frequency.

Table 4.10. Forecast regression of implied and historical range-based volatility (in logs).

Sample Statistics	France	Germany	Netherlands	DJ EURO STOXX 50	USA
α_0	-0.127 (-1.41)	-0.095 (-1.06)	-0.273 (-3.24)	-0.146 (-1.73)	-0.314 (-3.50)
β	0.589 (4.06)	0.491 (3.29)	0.309 (2.01)	0.674 (4.63)	0.284 (2.84)
γ	0.418 (4.07)	0.507 (4.76)	0.600 (5.18)	0.334 (3.14)	0.577 (5.97)
Adj. R-sq	0.719	0.719	0.688	0.710	0.665
T statistic for $\beta = 1$	8.01	11.57	20.30	5.03	50.7
T statistic for $\gamma = 0$	32.17	21.42	11.87	39.05	19.16
F test $\alpha_0 = 0$, $\beta = 1$, and $\gamma = 0$	53.23	32.08	72.35	51.47	25.13
DW	1.888	1.894	1.993	1.799	1.849

Notes: The heteroskedasticity and the autocorrelation consistent t-statistics are reported below the regression coefficient. The implied volatility indices are based on the approach of model-free implied variance proposed by Demeterfi et al. (1999). VCAC, VDAX-New, VAEX, and VSTOXX are constructed from the market prices of options on the CAC 40 (France), the DAX (Germany), the AEX (Netherlands), and the DJ EURO STOXX 50 index, respectively. New VIX is based on S&P 500 implied volatility. The range-based volatility measure is used as a proxy for the realized volatility. Implied and realized range-based volatilities are annualized and given in decimal form. The sample period runs from January 3, 2000 to November 26, 2012, monthly frequency.

4.4.4 Volatility Transmission Mechanism from Volatility Measures

In this part we put a special emphasis on the characteristics of the international transmission. In particular, we want to analyse to what extend a movement from one market can explain the shock in another market. We also want to examine the relation of implied volatility and range-based volatility estimates.

The vector autoregressive analysis (VAR) developed by Sims (1980) gives estimates of unrestricted reduced form equations that have uniform sets of the lagged dependent variables of every equation as repressors. The VAR estimates a dynamic simultaneous equation system that helps us bring out the dynamic responses of markets to shocks in a particular market using the simultaneous responses of the estimated VAR system. Thus, we can assess the importance of a determined market to generate unexpected variations of returns to a particular market. In this manner, we can also observe the causal relation between implied volatility and range-based volatility measures.

The starting point for the analysis is the following *P*-th order, *K*-variable VAR specification

$$\mathbf{y}_t = \sum_{p=1}^{P} \mathbf{\Theta}_i \mathbf{y}_{t-p} + \boldsymbol{\varepsilon}_t \ , \qquad (4.19)$$

where $\mathbf{y}_t = (y_{1t}, y_{2t}, ..., y_{Kt})$ is a vector of *K* endogenous variables, $\mathbf{\Theta}_i$, $i = 1,..., P$, are $K \times K$ parameter matrices and $\boldsymbol{\varepsilon}_t \sim (\mathbf{0}, \mathbf{\Sigma})$ is vector of disturbances that are independently distributed over time; $t = 1,...,T$ is the time index and $k = 1,..., K$ is the variable index. For each of the indices considered (*FRA*, *GER*, *NETH*, *STOXX*, *US*), the VAR given by Equation (4.19) contains observations on the range-based volatility $(\sigma_{nt},\ n = 1,...,5)$ and the implied volatility $(\sigma_{nt}^{IMP},\ n = 1,...,5)$ measures, with *n* denoting the country index. Hence, with 5 indices and 2 variables, our VAR is made up of $K = 10$ variables, i.e., $\mathbf{y}_t = [\boldsymbol{\sigma}_t^{'}, \boldsymbol{\sigma}_t^{'IMP}]$, where $\boldsymbol{\sigma}_t$ and $\boldsymbol{\sigma}_t^{IMP}$ are 5×1 vectors with observation on the range-based volatility and the implied volatility measure for each of the 5 equity indices, respectively. For notational simplicity, both variables σ_{nt} and σ_{nt}^{IMP} in (4.19), are referred to as y_{it} and indexed by $i = 1,..., K = 10$ in the following.

The VAR model allows us to examine the decomposition of the forecast error variances and the pattern of impulse responses for the index volatility. The forecast error variance decomposition, which partitions the forecast error variance of each of the indices at a given horizon, may be considered as out-of-sample causality tests. This allows us to gauge the relative strength of impact from each index. To obtain additional insight into the mechanism of international equity indices volatility interactions, we trace out the impulse responses of each of the five indices with respect to innovations in a particular index. This is similar to a sensitivity analysis that provides the pattern of dynamic responses of each index to innovations of its own as well as to those from other indices.

As shown by Sims (1980), each autoregressive equation in the VAR is difficult to justify intuitively, and he therefore recommends tracing out the system's moving average representation. After successive substitution of lagged variable vectors on the right-hand-side of Equation (4.19), the moving average representation is obtained and its innovations can be made orthogonal as:

$$\mathbf{y}_t = \sum_{s=0}^{\infty} \boldsymbol{\gamma}_s \mathbf{z}_{t-s} , \qquad (4.20)$$

where $\boldsymbol{\gamma}_s$ is a matrix that collects the impulse responses of indices in *s* days to a shock of one standard deviation in the other equity index (see Hamilton, 1994). The orthogonalized innovation $\mathbf{z}$ is obtained from $\mathbf{z} = \mathbf{V}\boldsymbol{\varepsilon}$, where $\mathbf{V}$ is usually the inverse of a lower-triangular Cholesky decomposition of the innovation covariance matrix.

However, a different ordering of the series in the orthogonalization procedure can produce diverging results. To avoid this problem, we adopt the Generalized Impulse Response Function (GIRF) proposed by Pesaran and Shin (1998), which is invariant to the ordering of the variables. Nevertheless, the forecast error variance decompositions of a GIRF do not sum up to unity. Therefore, for variance decompositions, we adopt the practice of Hasbrouck (1995) by sequentially ordering each market first in the system to obtain the maximum share of its innovation, and then order last to obtain its minimum share. The average of the maximum and minimum shares becomes the final single decomposition share of the innovation.

To avoid over-parameterization, we choose one as the lag length in the VAR estimation of the indices' volatilities. Table 4.11 presents the empirical results of VAR estimates. We find that the implied volatility dominates the historical range-based volatility in forecasting future realized range-based volatility for France. We also find that the implied volatility of France has significant effect in forecasting future realized range-based volatility for Germany, the Netherlands, the DJ EURO STOXX 50, and U.S.

Table 4.11. First order VAR

Panel A					
Dependent Variable	France (range)	Germany (range)	Netherlands (range)	DJ EURO STOXX 50 (range)	USA (range)
France (range)	-0.579 (-1.53)	-0.962 (-2.13)	-1.070 (-2.60)	-0.951 (-2.24)	-0.141 (-0.37)
Germany (range)	0.277 (0.96)	0.572 (1.67)	0.216 (0.69)	0.214 (0.66)	-0.226 (-0.78)
Netherlands (range)	0.410 (1.36)	0.473 (1.31)	0.926 (2.82)	0.549 (1.62)	0.343 (1.12)
the DJ EURO STOXX 50 (range)	-0.116 (-0.30)	-0.016 (-0.03)	-0.079 (-0.19)	0.230 (0.53)	-0.464 (-1.18)
USA (range)	0.342 (2.55)	0.0123 (0.77)	0.273 (1.87)	0.244 (1.63)	0.800 (5.90)
France (vol. Index)	1.029 (2.52)	1.139 (2.34)	0.907 (2.05)	1.024 (2.24)	1.071 (2.59)
Germany (vol. Index)	-0.993 (-2.22)	-0.491 (-0.92)	-0.616 (-1.27)	-0.886 (-1.77)	-0.303 (-0.67)
Netherlands (vol. Index)	-0.306 (-1.22)	-0.235 (-0.78)	-0.181 (-0.66)	-0.326 (-1.15)	-0.088 (-0.34)
the DJ EURO STOXX 50 (vol. Index)	0.719 (1.45)	0.130 (0.22)	0.409 (0.76)	0.689 (1.24)	-0.192 (-0.38)
USA (vol. Index)	0.116 (0.87)	0.334 (2.09)	0.136 (0.94)	0.170 (1.13)	0.225 (1.67)
Const	0.009 (0.65)	-0.009 (0.53)	-0.001 (-0.09)	0.005 (0.29)	-0.006 (-0.45)

Table 4.11. First order VAR

Panel B					
Dependent Variable	France (vol. Index)	Germany (vol. Index)	Netherlands (vol. Index)	the DJ EURO STOXX 50 (vol. Index)	USA (vol. Index)
France (range)	0.065 (0.20)	-0.081 (-0.23)	-0.204 (-0.57)	0.068 (0.19)	0.213 (0.81)
Germany (range)	0.181 (0.72)	0.371 (1.37)	0.261 (0.96)	0.327 (1.21)	0.349 (1.76)
Netherlands (range)	0.213 (0.81)	0.219 (0.77)	0.540 (1.88)	0.151 (0.53)	0.078 (0.38)
the DJ EURO STOXX 50 (range)	-0.295 (-0.87)	-0.315 (-0.86)	-0.573 (-1.55)	-0.353 (-0.97)	-0.428 (-1.60)
USA (range)	0.080 (0.68)	0.054 (0.43)	0.176 (1.38)	0.088 (0.70)	0.070 (0.76)
France (vol. Index)	0.136 (0.38)	-0.094 (-0.25)	-0.057 (-0.15)	-0.227 (-0.59)	-0.042 (-0.15)
Germany (vol. Index)	-0.453 (-1.16)	-0.167 (-0.40)	-0.440 (-1.04)	-0.711 (-1.70)	-0.683 (-2.22)
Netherlands (vol. Index)	-0.006 (-0.03)	0.154 (0.66)	0.485 (2.03)	0.076 (0.32)	0.071 (0.41)
the DJ EURO STOXX 50 (vol. Index)	0.856 (1.99)	0.680 (1.47)	0.678 (1.44)	1.409 (3.05)	0.407 (1.20)
USA (vol. Index)	0.006 (0.05)	-0.031 (-0.25)	0.002 (0.02)	-0.036 (-0.28)	0.785 (8.51)
Const	0.049 (4.01)	0.049 (3.76)	0.039 (2.91)	0.051 (3.91)	0.036 (3.70)

Notes: The Table reports coefficient estimates for a monthly 1-lag VAR. The sample period runs from January 3, 2000 to November 26, 2012. Standard error are heteroskedastic-consistent (Robust-White). The VAR given contains observations on the range-based volatility $\left(\sigma_{nt},\ n=1,...,5\right)$ and the implied volatility $\left(\sigma_{nt}^{IMP},\ n=1,...,5\right)$ measures, with n denoting the country index.

Figures 4.8-4.11 illustrate the impulse responses in the realized range-based volatility due to a unit shock to the implied volatility for all indices. We find that the largest response in the realized range-based volatility occurs in response to the implied volatility shock for Germany. The impulse response curves for the range-based volatility exhibit a relative short-run dynamic effect from the shock to the implied volatility for France and Germany which disappears after approximately 8 days for France and 5 days for Germany. The impulse response functions for the range-based volatility from the shocks to the implied volatility are more persistent for the Netherlands and U.S. This is consistent with the results presented in Table 4.7. Specifically, in Table 4.7 we find that the coefficient on the range-based volatility is higher than the coefficient on the implied volatility for the US and the coefficient on the range-based volatility is close to the coefficient on the implied volatility for the Netherlands. This suggests the presence of the strong autoregressive persistence in the range-based volatility due to the own shocks for the U.S. market and the Netherlands. The impulse response curves for the range-based volatility in response to the implied volatility shocks reach their peaks after approximately 1 day and become negative at day 3 for France and Germany.

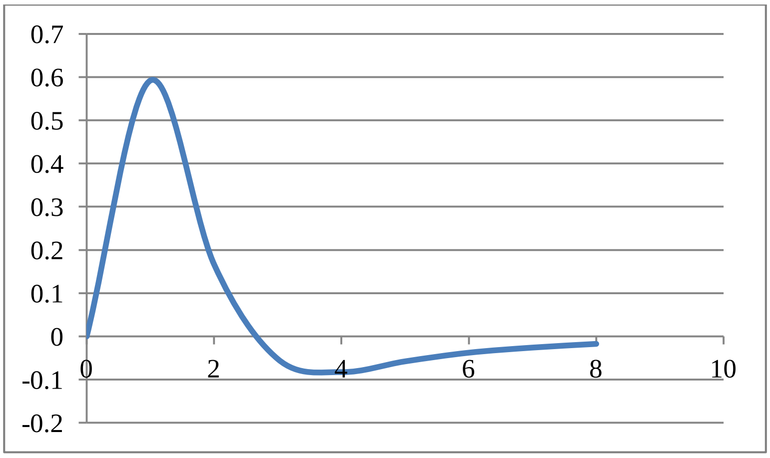

Figure 4.8. Response in realized range-based volatility due to one unit shock to implied volatility, France

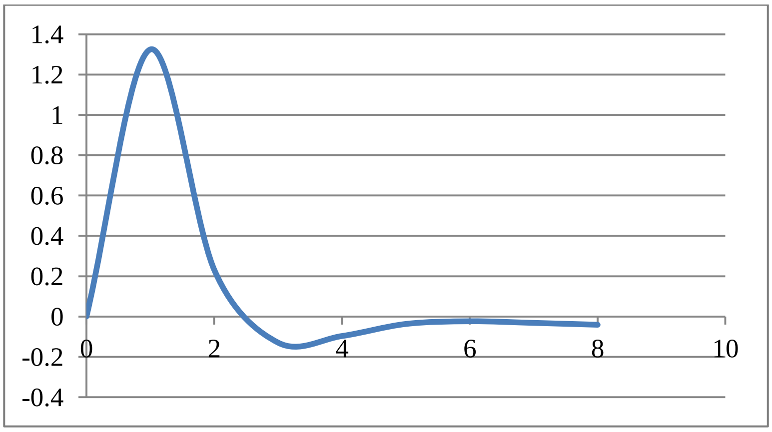

Figure 4.9. Response in realized range-based volatility due to one unit shock to implied volatility, Germany

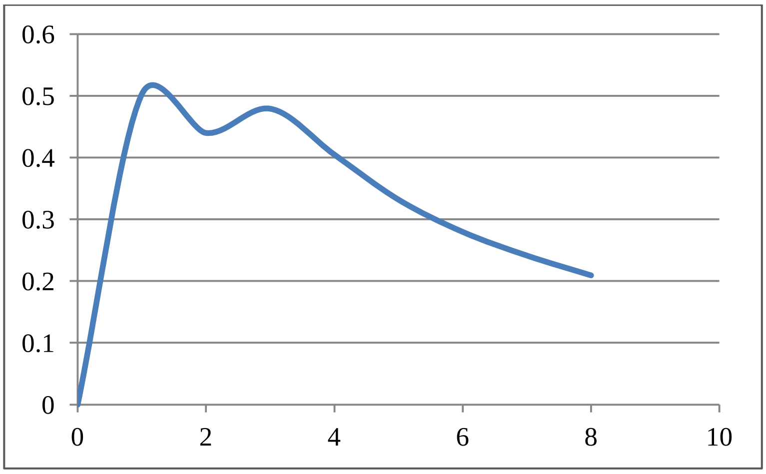

Figure 4.10. Response in realized range-based volatility due to one unit shock to implied volatility, the Netherlands

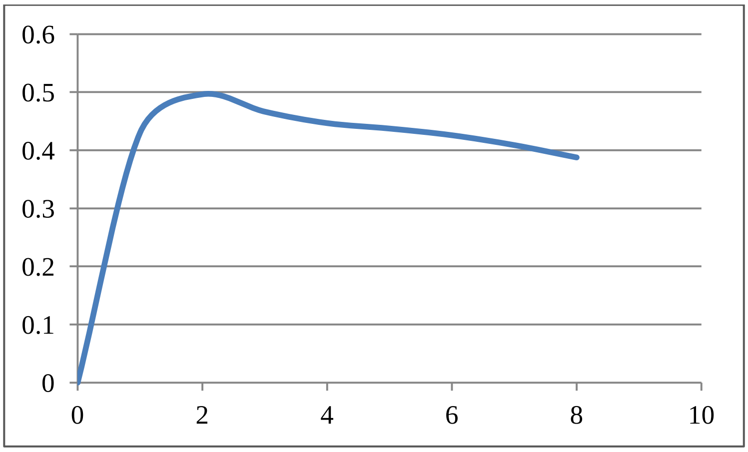

Figure 4.11. Response in realized range-based volatility due to one unit shock to implied volatility, USA

4.4.5 Variance Risk Premium

The model-free implied variance (squared) approximates the conditional expectation of the annualized return variance under the risk-neutral measure over the next 30 calendar days (Carr and Wu, 2006):

$$\sigma^{2\,IMP}_{t} \cong \mathrm{E}_t^{\mathrm{Q}}\left[\sigma_t^2\right]. \tag{4.21}$$

The notation is as before. We can also rewrite Equation (4.21) under the statistical measure P as:

$$\sigma^{2\,IMP}_{t} \cong \frac{\mathrm{E}_t^{\mathrm{P}}\left[M_{t,t+30}\sigma_t^2\right]}{\mathrm{E}_t^{\mathrm{P}}\left[M_{t,t+30}\right]} = \mathrm{E}_t^{\mathrm{P}}\left[\sigma_t^2\right] + Cov_t^{\mathrm{P}}\left(\frac{M_{t,t+30}}{\mathrm{E}_t^{\mathrm{P}}\left[M_{t,t+30}\right]}, \sigma_t^2\right), \tag{4.22}$$

where $M_{t,t+30}$ denotes a pricing kernel between times *t* and *T*. For traded assets, no-arbitrage guarantees the existence of at least one such pricing kernel (Duffie, 1992).

Equation (4.22) decomposes $\sigma^{2\,IMP}_{t}$ into two terms. The first term, $E_t^P\left[\sigma_t^2\right]$ represents the time-series conditional mean of the realized variance, and the second term captures the conditional covariance between the normalized pricing kernel and the realized variance. The negative of this covariance defines the time *t* conditional variance risk premium (VRP_t):

$$VRP_t \cong -Cov_t^{\mathrm{P}}\left(\frac{M_{t,t+30}}{\mathrm{E}_t^{\mathrm{P}}\left[M_{t,t+30}\right]}, \sigma_t^2\right) = \mathrm{E}_t^{\mathrm{P}}\left[\sigma_t^2\right] - \sigma^{2\,IMP}_{t}. \tag{4.23}$$

Taking unconditional expectations on both sides, we have

$$\mathrm{E}^{\mathrm{P}}\left[VRP_t\right] = \mathrm{E}_t^{\mathrm{P}}\left[\sigma_t^2 - \sigma^{2\,IMP}_{t}\right]. \tag{4.24}$$

Thus, we can estimate the average variance risk premium as the simple average of the differences between the realized return variance and the implied variance.

Our measure of variance risk premium is very close to that in Bollerslev et al. (2009) that measures the variance risk premium as ${\sigma^2}_t^{IMP} - \sigma_t^2$ rather than $\sigma_t^2 - {\sigma^2}_t^{IMP}$.

In Table 4.12 we report the average variance risk premium measures for the Euro area indices and the U.S. We use again the range-based volatility measure as a proxy for the realized return variance. Over our sample period, the mean variance risk premium is statistically significant for France, Germany, the Netherlands, and the DJ EURO STOXX 50 Index. The mean variance risk premium ranges from -0.0159 for the Netherlands and -0.0061 for Germany. Hence, the mean variance risk premium is strongly negative for European indices. This result suggests that investors are willing to pay a premium to hedge away upward movements in the return variance of the stock market. In other words, investors regard increases in market volatility as unfavourable shocks to the investment opportunity and demand a high premium for bearing such shocks. Another characteristic of the negative sign on the variance risk premium is that most of the time the premium covers the volatility risk for option sellers but it is sometimes undervalued. Particularly in the late 2008 (more precisely, after the Lehman shock), the realized volatility surged so rapidly and dramatically that the implied volatility levels failed to follow or cover the future realized volatility levels in Europe or the U.S., resulting in large positive realized variance risk premium.

From the perspective of a variance swap investment, the negative variance risk premium also implies that investors are willing to pay a high risk premium or endure an average loss when they are long variance swaps in order to receive compensation when the realized variance is high.

We can also think of the variance risk premium as the gain from the volatility arbitrage that is implemented by trading a delta neutral portfolio of an option. The objective is to take advantage of the differences between the implied volatility of the option, and a forecast of future realized volatility of the option's underlier. In volatility arbitrage, volatility rather than price is used as the unit of relative measure. Since in our notation the variance risk premium is $\sigma_t^2 - {\sigma^2}_t^{IMP}$, the gain from the volatility arbitrage is highly statistically positive for Euro area indices. In addition, there is no opportunity for the volatility arbitrage in the U.S. as the variance risk premium is not statistically different from zero for that market.

Table 4.12. The average variance risk premium

France	Germany	Netherlands	the DJ EURO STOXX 50	USA
-0.0119 (-8.30)	-0.0061 (-3.48)	-0.0159 (-11.24)	-0.0131 (-8.20)	-0.0009 (-0.39)

Notes: The Table reports the average variance risk premium measures for the Euro area indices and the U.S. We use the range-based volatility measure as a proxy for the realized return variance.

Dividing both sides of Equation (4.22) by ${\sigma^2}_t^{IMP}$, we can rewrite the decomposition in excess returns:

$$1 = \mathrm{E}_t^{\mathrm{P}}\left[\frac{\sigma_t^2}{{\sigma^2}_t^{IMP}}\right] + Cov_t^{\mathrm{P}}\left(\frac{M_{t,t+30}}{\mathrm{E}_t^{\mathrm{P}}\left[M_{t,t+30}\right]}, \frac{\sigma_t^2}{{\sigma^2}_t^{IMP}}\right). \qquad (4.25)$$

If we regard ${\sigma^2}_t^{IMP}$ as the forward cost of the investment in the static option position required to replicate the variance swap payoff, $\left(\sigma_t^2 / {\sigma^2}_t^{IMP} - 1\right)$ captures the excess return from going long the variance swap. The negative sign of the covariance term in Equation (4.26) represents the conditional variance risk premium in excess return terms:

$$VRPR_t \cong -Cov_t^{\mathrm{P}}\left(\frac{M_{t,t+30}}{\mathrm{E}_t^{\mathrm{P}}\left[M_{t,t+30}\right]}, \frac{\sigma_t^2}{{\sigma^2}_t^{IMP}}\right) = \mathrm{E}_t^{\mathrm{P}}\left[\frac{\sigma_t^2}{{\sigma^2}_t^{IMP}}\right] - 1. \qquad (4.26)$$

We can estimate the mean variance risk premium in excess return form through the sample average of the realized excess returns, $ER_{t,t+30} = \left(\sigma_t^2 / {\sigma^2}_t^{IMP} - 1\right)$. The estimation results are presented in Table 4.13. We find that the mean variance risk premium estimates are strongly negative and highly significant. Investors are willing to endure a negative excess return for being long variance swaps in order to hedge away upward movements in the return variance of the stock index.

Table 4.13. The sample average of the realized excess returns and the annualized information ratio

	France	Germany	Netherlands	the DJ EURO STOXX 50	USA
$ER_{t,t+30}$	-0.328 (-15.01)	-0.236 (-9.14)	-0.392 (-13.83)	-0.332 (-16.24)	-0.058 (-1.49)
IR	4.205	2.560	3.873	4.549	0.418

Notes: The Table estimates the sample average of the realized excess returns and the annualized information ratio using 30-day-apart non-overlapping data.

The average negative variance risk premium also suggests that shorting the 30-day variance swap and holding it to maturity generates an average excess return of 32.8% for France, 23.6% for Germany, 39.2% for the Netherlands, 33.2% for the DJ EURO STOXX 50 Index, and 5.8% for U.S. We compute the annualized information ratio using 30-day-apart non-overlapping data, $IR = -\sqrt{365/30}ER/S_{ER}$, where ER denotes the time series average of the excess return and S_{ER} denotes the standard deviation estimate of the excess return. The information ratio average estimate is highest for the Netherlands and lowest for the US. The information ratio average estimates indicate that shorting the 30-day variance swaps is very profitable on average.

To further check the historical behaviour of excess returns from the investment, we plot the time series of the excess returns in the Figures 4.12-4.16. The time series plots show that shorting the variance swaps provides a positive return 92% of the time for France, 81% for Germany, 94% for the Netherlands, 90% for the DJ EURO STOXX 50 Index, and 68% for the U.S. The occasionally negative realizations can be as large as 270% for the Netherlands.

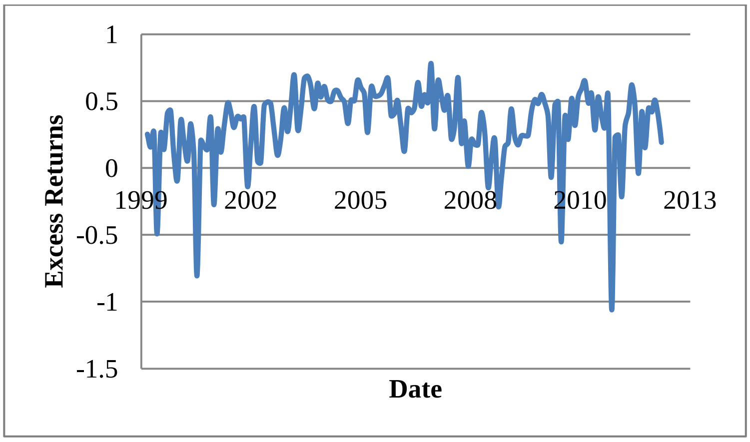

Figure 4.12. The time series of excess returns from shorting the 30-day variance swaps on the CAC 40 and holding the contract to maturity

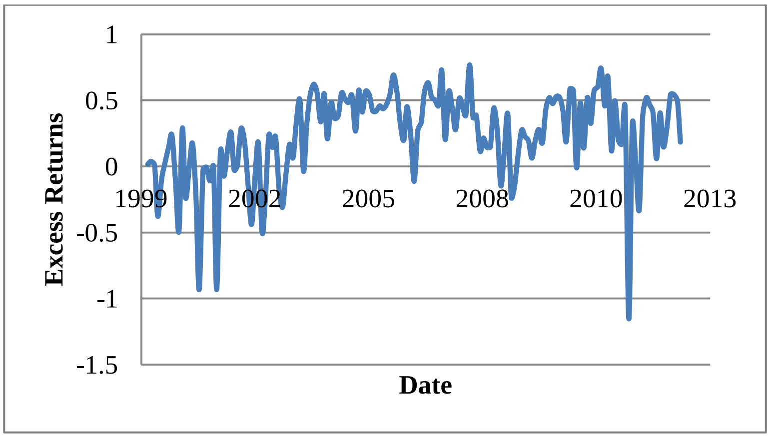

Figure 4.13. The time series of excess returns from shorting the 30-day variance swaps on the DAX and holding the contract to maturity

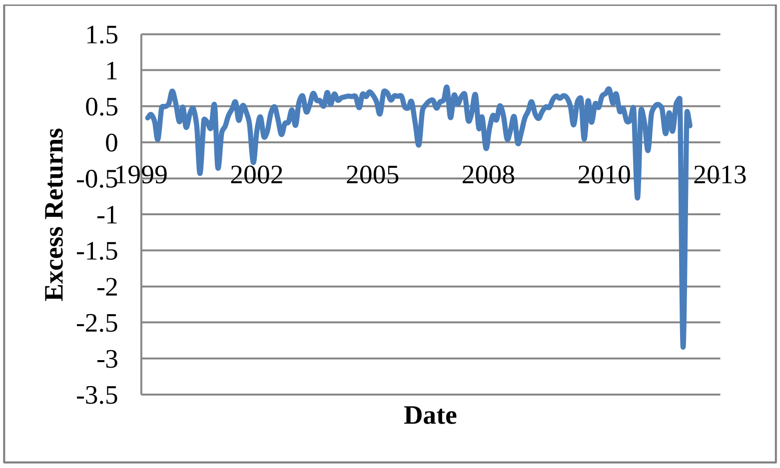

Figure 4.14. The time series of excess returns from shorting the 30-day variance swaps on the AEX and holding the contract to maturity

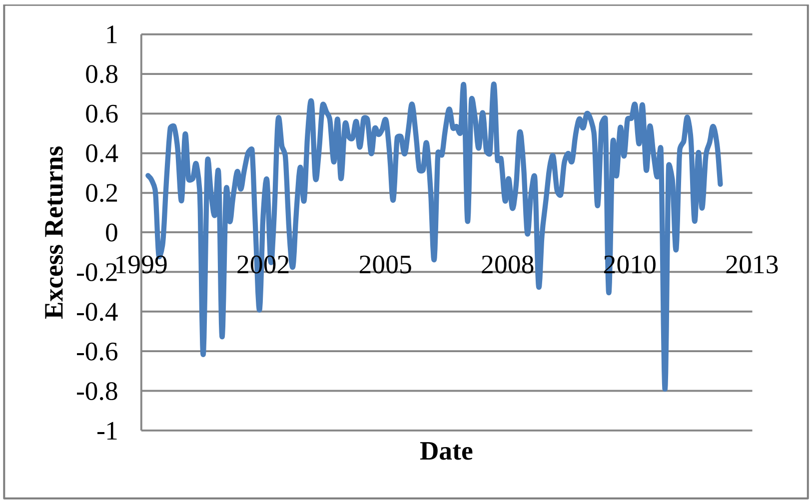

Figure 4.15. The time series of excess returns from shorting the 30-day variance swaps on the DJ EURO STOXX 50 Index and holding the contract to maturity

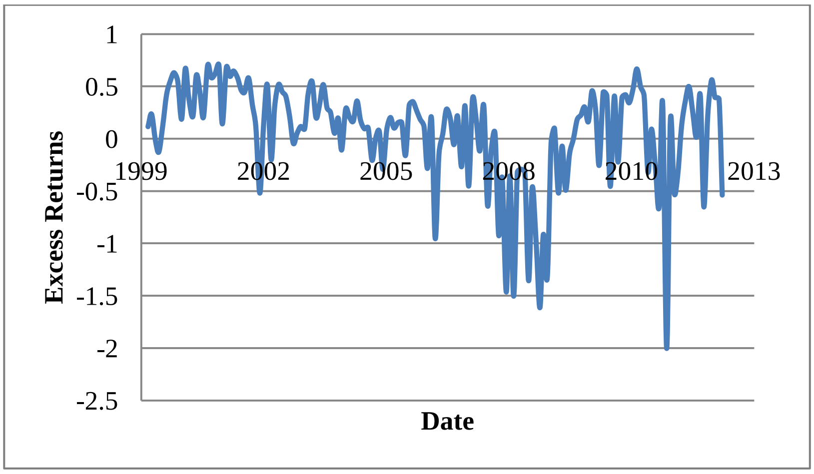

Figure 4.16. The time series of excess returns from shorting the 30-day variance swaps on the S&P 500 index and holding the contract to maturity.

4.5 Conclusions

In this chapter we assess the information content of the implied volatility by considering the implied volatility indices constructed based on the concept of model-free implied variance proposed by Demeterfi et al. (1999). In particular, using the range-based volatility estimator as a proxy for the realized variance we study the linkages between the range-based volatility and the implied volatility measure for the sample of five equity indices over the period January 3, 2000 to November 26, 2012. We assess the two-way relationships between the range-based volatility and the implied volatility, both within the index and accounting for spillovers between indices. Moreover, we study the evolution of spillovers between the range-based volatility and the implied volatility over time, identifying the net receivers and transmitters of shock and quantifying their magnitude using impulse response analysis. Finally, we consider the average variance risk premium estimate defined as the simple average of the differences between the realized return variance and the implied variance.

We find that the implied volatility does contain information in forecasting realized range-based volatility. The historical range-based volatility, on the other hand, has less explanatory power than the implied volatility in predicting realized range-based volatility. The univariate regression of historical volatility to the realized range-based volatility shows that historical range-based volatility has also information in predicting realized volatility, the regression of the historical range-based volatility and implied volatility simultaneously shows that the implied volatility dominates historical range-based volatility in forecasting realized range-based volatility, or that all the information contained in historical volatility has been reflected by the implied volatility, and the historical range-based volatility has no incremental forecasting ability. The results from the univariate regressions are also consistent with the existing option pricing literature which documents that stochastic volatility is priced with a negative market risk. The volatility implied from option prices is thus higher than their counterpart under the objective measure due to investor risk aversion. Our study shows that the option market processes information efficiently in the US market.

Chapter 5: Content Analysis of the IMF Article IV Staff Reports for Euro Area Countries

5.1 Introduction

With a few notable exceptions, economists worldwide failed to predict the emergence and gravity of the financial crisis that originated in the United States in 2007. Because governments worldwide rely on the IMF to provide a warning system to anticipate critical events (see statement of the G20 Leaders), it is crucial to investigate how the IMF failed to detect early signs of the crisis. The IMF is a multilateral organization that is statutorily mandated to provide an early warning to the member countries so that national authorities can take measures to mitigate the impact of a crisis. Despite this mandate, some have claimed that the IMF did not sound any alarm in the run-up to the current crisis, or that when raising concerns it did so in a muted or hedged manner (IEO, 2010). To illustrate this, in the summer of 2007, the IMF staff indicated that in the United States "core commercial and investment banks are in a sound financial position, and systemic risks appear low" (IMF, 2007:14). In addition, as late as April 2007, the opening sentence of the *Global Financial Stability Report (GFSR),* the IMF flagship on financial issues, noted, "Favorable global economic prospects, particularly strong momentum in the Euro area and in emerging markets led by China and India, continue to serve as a strong foundation for global financial stability. However, some market developments warrant attention, as underlying financial risks and conditions have shifted since September 2006 *GFSR*". In addition, Subramanian (2009) says that the failure of the IMF "was to preside over large capital flows to Eastern Europe despite the lessons that it should have learned from the experience of the Asian financial crisis in the late 1990s. These flows to Eastern Europe were, in some cases, so large that it did not require hindsight to see the problems that they would lead to. Warnings about the unsustainability of these flows should have been loud and insistent. And they were not.” Others have claimed that the IMF issued warnings but that they were not heeded.

The primary purpose of this article is to evaluate these differing views and establish whether the IMF Reports foresaw the crisis and warned people about it. Moreover, if so, how explicit were those warnings? At the empirical level, we address these issues by analysing the tone of the IMF Article IV Staff Reports and Executive Board Assessment for the euro area countries using computerized textual analysis algorithm DICTION 5.0. Under Article IV of the IMF’s *Articles of Association*, the IMF holds bilateral discussions with members, usually every year. A staff team member visits the country, collects economic and financial information, and discusses with officials the country’s economic development and policies. The team returns to headquarters and the staff prepare a report. This Staff Report forms the basis for discussion by the Executive Board. The views of the Executive Board are summarized in a Public Information Notice (PIN) that is attached to the Article IV report.

The main contribution of this study is an evaluation of the effectiveness of the IMF external surveillance in the run-up to the current credit crises. In addition, in contrast to previous studies, this study is the first to apply content-analysis methodology in order to analyse the IMF Reports. According to Li (2006), very few studies examine the texts of publicly available documents; instead, the majority of the analysis has been on the quantitative variables contained in the reports.

5.2 Background

5.2.1 Was the Miracle a ‘Mirage’? EMU Fiscal Policies

In 2007, the last year before the onset of the economic and financial crisis, the public finances in the euro area were in their strongest position for decades. This result owed more than was appreciated at the time to favourable economic conditions. With the onset of the crisis in 2008, GDP growth fell dramatically and turned negative by the end of the year, leading to a marked deterioration in the public finances. In 2009, a year of deep recession followed, with growth shrinking by 4.09 % on average in EU16 (compared to an expansion of 2.77 % in 2007) (Table 5.1). In detail, the highest negative real GDP growth rates in 2009 were in Finland (-7.76 %), Slovenia (-7.33 %), and Ireland (-7.10 %). Furthermore, the greatest drop in the growth rate, by 15.24 % from 2007 to 2009, was in the Slovak Republic. In 2007, general government deficits corresponded to less than 1 % of GDP in EU16. Debt has also deteriorated strongly. In 2007, the euro area debt corresponded to 66 % of GDP (European Commission, 2010).

Table 5.1. Real GDP growth in euro area

Country	2007	2008	2009
Austria	3.547	2.048	-3.613
Belgium	2.843	0.832	-3.006
Cyprus	5.133	3.619	-1.742
Finland	4.944	1.206	-7.762
France	2.26	0.32	-2.186
Germany	2.517	1.248	-4.973
Greece	4.472	2.015	-1.963
Ireland	6.024	-3.036	-7.096
Italy	1.482	-1.319	-5.038
Luxembourg	6.474	0.032	-4.224
Malta	3.831	2.14	-1.93
Netherlands	3.613	1.996	-3.983
Portugal	1.872	0.043	-2.678
Slovak Republic	10.579	6.17	-4.66
Slovenia	6.796	3.493	-7.331
Spain	3.563	0.858	-3.639
Euro area	2.768	0.648	-4.085

Notes: Source: International Monetary Fund, World Economic Outlook Database, April 2010

However, until the outbreak of the financial crisis in August 2007, the mid-2000s was a period of strong economic performance throughout the euro area. Economic growth was generally robust; inflation generally low; the real interest rate equal to GDP growth; international trade and especially financial flows expanded; and the euro area members experienced widespread progress and a notable absence of crises. For instance, looking at average GDP growth rates in 2000-2007, the winners were Ireland, Greece, and Finland (with average growth rates above 3 %); and there were only three EMU countries that had below 2 % GDP growth rates (Italy, Germany, and Portugal) (Mathieu and Sterdyniak, 2010). However, this apparently favourable equilibrium was underpinned by certain trends that appeared increasingly unsustainable as time went by. In particular, before the crisis started, the euro area was characterised by rising imbalances between two groups of countries implementing two unstable macroeconomic strategies (Mathieu and Sterdyniak, 2010). Some virtuous northern countries (Germany, Austria, and the Netherlands) experienced competitiveness gains and accumulated huge external surpluses. In contrast, some southern countries accumulated huge external deficits under unbalanced high growth strategies driven by strong negative real interest rates (see Deroose et al., 2004; Mathieu and Sterdyniak, 2007). In 2007, several countries ran substantial current account surpluses: Germany (7.9 % of GDP), Finland (4.9%), Belgium (3.5%), and Austria (3.3%), whereas some others ran large deficits: Ireland (-5.3 % of GDP), Portugal (-8.5 %), Spain (-9.6%), and Greece (-12.5%) (WEO, 2009). In addition, average general government gross debt across the euro area remained above the target level over the long period preceding the crisis (Table 5.2). During the period from 2004 to 2007, Italy recorded the highest debt ratio, at over 100 % of GDP. Debt ratios for Belgium, Germany, Greece, France, Malta, and Portugal were above the target 60 % of GDP in 2007.

Furthermore, in 2007 there were substantial inflation differentials in the euro area. Countries running higher inflation were mainly those catching up, with higher output growth and low initial price levels, due to the Balassa-Samuelson effect (Greece, Ireland, Spain, and Portugal). However, Italy and the Netherlands also had relatively high inflation rates. Even when accounting for the Balassa-Samuelson effect, which may explain 1 percentage point of inflation in Greece, 0.7 in Portugal and 0.5 in Spain (for a discussion, see ECB, 2003), prices seem to have risen too rapidly in these three countries and this led to price competitiveness losses.

Inflation was extremely low in Germany, which prevented other countries from restoring their price competitiveness. In 2007, inflation disparities remained large in the euro area: inflation stood at 1.6% in the three countries with the lowest inflation and at 2.9% in the countries with the highest inflation. The single monetary policy was contractionary for Germany and Italy but expansionary for Ireland, Greece and Spain where companies and households had a strong incentive to borrow and invest, which boosted domestic GDP growth and inflation. While Austria, Germany, and the Netherlands succeeded in supporting domestic GDP growth through positive net exports contribution, Spain and France suffered from a negative external contribution. Fixed exchange rates and rigid inflation rates induced persistent exchange rates misalignment periods.

Table 5.2. General government debt (general government gross debt, % of GDP)

Country	2004	2005	2006	2007
Austria	63.8	63.5	61.8	59.1
Belgium	94.2	92.1	88.2	84.9
Cyprus	70.2	69.1	64.8	59.8
Finland	44.1	41.3	39.2	35.4
France	64.9	66.4	63.6	64.2
Germany	65.6	67.8	67.6	65.0
Greece	98.6	98.0	95.3	94.5
Ireland	29.5	27.4	25.1	25.4
Italy	103.8	105.8	106.5	104.0
Luxembourg	6.3	6.1	6.6	6.8
Malta	72.6	70.4	64.2	62.6
Netherlands	52.4	52.3	47.9	45.4
Portugal	58.3	63.6	64.7	63.6
Slovak Republic	41.4	34.2	30.4	29.4
Slovenia	27.6	27.5	27.2	24.1
Spain	46.2	43.0	39.7	36.2
Euro area	69.6	70.2	68.5	66.4

Notes: Source: Eurostat

To sum up, the crisis has shown that the divergent growth patterns in the EMU and growing macroeconomic imbalances should have been seen as contingent budgetary risks. In particular, the countries that suffered the greatest deterioration in their public finances between 2007 and 2009 had typically experienced increasing external imbalances and booming credit in the run-up to the crisis, while the countries that suffered the smallest deterioration generally had displayed stable or falling macro-financial risks.

5.2.2 Case Studies – Fiscal Policy and External Imbalance

An examination of experience in some selected EMU countries provides insights into the role that macroeconomic fiscal policy and microeconomic incentives have played in the building-up of competitiveness imbalances and in their winding down as well.

While tax revenue has shrunk in many euro area countries in the recent economic downturn, the revenue collapse in Spain and Ireland has been much more pronounced. We are looking more closely at those two countries.

Spain

Since the mid-1990s, Spain has been growing at an average rate of almost 4 % per year. Such an exceptionally long expansionary period of the Spanish economy was driven by a succession of credit-led impulses, demographic shocks, and adjustment processes. The combination of low real interest rates and dynamic demography resulted in a significant increase in the indebtedness of households and firms and stimulated a large asset boom, especially in housing. A sharp increase in house prices came hand-in-hand with an unprecedented increase in the number of new dwellings built each year. While the number of new residences had hovered at around a quarter of a million between the mid- 1970s and the mid-1990s, the figure rose to three quarters of a million by 2006. Equity markets also boomed in Spain during the last decade. The index of the Spanish stock exchange (IBEX 35) increased by 380 % from around 3500 points in 1995 to above 12000 points in 2006 (European Commission, 2010). The asset boom in Spain resulted in a change in the GDP composition towards investment in dwellings, whereas corporate profits soared. Within this context, the total tax burden rose from 32.75% of GDP in 1995 to above 37% in 2007 without relying on significant tax increases. Over the period of 1995-2007, the Spanish economy recorded a steady appreciation of the real effective exchange rate. This resulted from persistent and positive inflation and wage differentials with the euro area, combined with an also persistent but negative productivity differential. The combination of the steady appreciation of the real effective exchange rate, the reduction of the risk premia and an increase in population, were supportive of a demand-based growth model that was highly rich in taxes. In effect, while exports, which have low tax content, were not growing as fast as the whole economy, private consumption and the boom in the housing market pushed indirect taxes up. Moreover, the economic boom raised profits, especially those linked to real estate and financial operations, and consequently revenues from corporate taxes (European Commission, 2010).

Ireland

During the late 1990s, the so-called ‘Celtic Tiger’ identified a remarkable period in Irish economic history, with very rapid output growth (of approximately 7% per annum), far in excess of historical averages, and the attainment of effective full employment (around 4%). Moreover, there were further important structural shifts in the nature of the Irish economy. These developments were linked to the building up of macroeconomic imbalances and weaknesses for the Irish fiscal policy during the 2003-2006 credit and housing boom, which led to a remarkable collapse in tax revenues in 2008-2009. From the late 1990s and until 2007, the general government balance was in surplus. The improvement in the structural balance between 2003 and 2006 was due to the very significant windfall revenues produced by the housing boom and tax-rich economic activity more generally. There has been more and more dependence on the corporation tax, stamp duties and capital gains tax, which rose from about 8 % in 1987 to 30 % at the peak of house prices in 2006 (Honohan, 2009). The tax take from stamp duty on the purchase of property was accounting for about 17 % of all tax revenues. When one adds on the income tax paid by construction workers and VAT collected on property sales, the industry was contributing about one-fifth of all government tax revenue (Connor, Flavin, and O’Kelly, 2010). Furthermore, expenditure growth between 2003 and 2006 exceeded that of nominal GDP (European Commission, 2009). It was particularly buoyant in the areas of social transfers, the public sector wage bill, and public investment. Overall, despite improvements in the structural balance, fiscal policy was insufficiently *leaning against the wind*; this accelerated the deterioration of competitiveness. Looking at fiscal policy from a more microeconomic perspective, a favourable tax treatment of housing in Ireland is likely to have contributed to the expansion of the housing market.

Such a situation cannot be considered a stable and sustainable macroeconomic environment. However, what was the IMF saying in response to this series of circumstances?

5.2.3 The Role of the IMF

The IMF is a multilateral organization that is statutorily mandated to prevent crisis, or at least provide an early warning to the membership so that country authorities can take measures to mitigate the impact of a crisis. A key tool at its disposal is 'surveillance', or the process of monitoring and consultation with each of its member countries.[1] Despite this goal, whether the policy surveillance reports are truly informative remains an open empirical question.

Researchers find various characteristics found in intergovernmental agencies' reports interesting. For instance, one line of research focuses on studying the accuracy of information (Batchelor, 2001). Consistent with the intergovernmental agency intention, external policy reports provide relevant information for economists, researchers, and the general public. Some experts assert that the IMF is uniquely placed to provide information of a high quality and depth beyond what other institutions can offer (Lombardi and Woods, 2008). One obvious reason for its unique position is that the IMF has access to a truly universal membership of 185 governments, all of which are mandated, as a requirement of membership, to consult regularly with the organization.

On the other hand, the external policy reports produced by the IMF may not be as informative as intended for several reasons. First, some studies show that data published by the IMF may be inaccurate (Pellechio and Cady, 2005). Discrepancies stem principally from differences in the objectives underpinning these publications. Data may differ for reasons such as adaptations made to suit country-specific analysis and more recent data revisions in Staff Reports. Moreover, according to *the Fund's Transparency Policy*, "members should retain the ability to propose deletions of highly-sensitive material contained in country documents and country policy intentions documents that have been issued to the

[1] According to the 2007 Surveillance Decision, the primary goal of the IMF country surveillance is identifying potential risks to the economy's domestic and external stability that would call for adjustments to that country's economic or financial policies. A key part of the IMF surveillance process is the regular round of consultations by IMF staff with governments, central bank officials and other organizations in individual member countries. These consultations take place under Article IV of the Fund's *Articles of Association*.

Board prior to publication".[2] Aiyar (2010) points out that the IMF is subject to political pressures since its shareholders are governments, and so it often cannot say aloud what it really thinks. For instance, when a particular sector of the economy (e.g. the construction sector in Ireland), is a huge source of revenue, the government does not have the political will to dampen the sector. Finally, various recent independent evaluations have noticed the persistence of technical and organizational weaknesses that impair the IMF's ability to integrate macroeconomic and financial sector analyses, and to draw credible risk indicators from them (Bossone, 2008b). Another concern with IMF reports is that their policy advices may be ambiguous. Policy advices may also include substantial generic language and immaterial detail without much information content. As a result, such deficiencies in the data practices of IMF staff could pose a reputational risk to the Fund as its published data come under increased external scrutiny.

Another problem for any international body is the tendency of the general public and national authorities to resist warnings of vulnerability during good times (See Kindleberger and Aliber, 2005 for the historical evidence on this issue). One of the reasons is the so called "This time is different syndrome", which leads to over-optimism and induces risky investment.[3] Connor, Flavin, and O'Kelly (2010) show how the risk appetite of investors was growing during the period of 2004 to 2007 in Ireland. For instance, there were more mortgages for higher loan-to-value: the percentage of mortgages for greater than 95% of the property value increased from 6% to 16% in the period 2004 to 2007. Furthermore, the maturity of the mortgages lengthened, with the percentage of loans having a maturity of greater than 30 years jumping from 10% to 35% in the period 2004 to 2007.

[2] According to *the IMF Transparency Policy*, the criteria for deletion of highly sensitive material "need to strike the right balance between preserving candor and providing adequate safeguards against possible adverse consequences of publications". In addition, there is no clear definition given for the criterion of high market sensitivity. Finally, directors agree that the determination of what constitutes highly market-sensitive information will continue to have to be made on a case-by-case basis.

[3] The essence of the "This time is different syndrome" is rooted in the firmly-held belief that financial crises are something that happen to other people in other countries at other times, crises do not happen here and now to us (Reinhart and Rogoff, 2010).

Therefore, in order to differentiate between alternative hypotheses and establish whether the IMF Article IV Reports foresaw the crisis and warned about it, we need to employ the content analysis algorithm, which is aimed at quantifying qualitative aspects of a text.

5.3 Measuring Communication

5.3.1 'Anecdotal' Evidence

The IMF holds annual consultations – called Article IV consultations – and should have been able to flag Ireland's unsustainable macroeconomic path. Strikingly, it failed to do so. To illustrate, the *2005 Article IV Consultation-Staff Report* suggested, "impressive performance is due in significant measure to sound economic policies, including prudent fiscal policies…" Furthermore, the *2006 Article IV Consultation-Staff Report* points "to the need for further increases in public spending to achieve social goals". Finally, the 2007 report was submitted in June 2007. By then the subprime mortgage crisis was already at an advanced stage in the United States. As Federal Reserve Chairman Ben Bernanke put it in speech at the Economic Club of New York on October 15, 2007, "the rate of serious delinquencies had risen, notably for subprime mortgages with adjustable rates, reaching nearly 16 percent in August; roughly triple the low in mid-2005." The writing was on the wall for other prudent countries too, but the IMF report on Ireland could not see it. The overall assessment of the IMF report was that, "Ireland continued impressive economic performance". Coming from the regulator, these statements certainly could not be interpreted as encouraging Ireland to correct the unsustainable macroeconomic and fiscal policies.

Another way of assessing the efficiency of the IMF surveillance policy is simply to count the number of risk factors identified by the IMF. Based on the researchers' calculations detailed in Table 5.3, note that the number of risks identified by the staff is relatively low during the boom times (on average 2-3), and that this number increases to seven factors in the Article IV consultation report of 2009. This latter number more accurately signals the true risk exposure of the Irish economy but comes too late to provide any policy guidance.

Table 5.3. Concerns/risk factors identified by IMF

	1999	2000	2001	2002	2003	2004	2005	2006	2007	2009
Inappropriate fiscal stance	X		X	X			X		X	X
Inflexibility of NWAs	X	X	X							
Declining competitiveness				X	X			X	X	X
House price overvaluation					X		X			X
Unwinding of construction boom						X		X	X	X
Unbalanced growth								X		X
Vulnerability to external shocks	X							X		X
Vulnerability of banking system	X						X			X
Total score	4	1	2	2	2	1	3	4	3	7

Notes: The Table shows the risk factors identified by the IMF Staff based on the textual analysis.

Some might argue, however, that all of these ways of analysing the official reports are limited by the subjectivity associated with human readers. They are also time consuming and, hence, inefficient. Therefore, to assess the content and tone of the fiscal policy reports, we employed a computerized textual-analysis software, namely DICTION 5.0.

5.3.2 Data and Summary Statistics

This article studies the information content of the IMF Article IV country Staff Reports and Executive Board Assessment produced by the IMF. Specifically, we explore variations in the tone of the fiscal policy reports for the euro area countries and examine whether the information content of various reports changes over time, especially during the pre-crisis period of 2005-2007.

Applying DICTION 5.0 methodology, we construct a *WARNING* tone measure from the Hardship dictionary. Appendix C lists the DICTION 5.0 classification. Hardship indicates natural disasters, hostile actions, censurable human behaviour, unsavoury political outcomes, and human fears. The dictionary Hardship is composed of the words Bias, Deficit, Deteriorate, Distress, Risk, Shock, Weakness and so on; these are words we wish to capture in the IMF reports.

To apply the content analysis algorithm, we first download all the IMF Article IV Staff Reports for the euro area countries for the period 1999-2009. Under Article IV of the IMF's *Articles of Association*, the IMF holds bilateral discussions with members, usually every year. A staff team member visits the country, collects economic and financial information, and discusses with officials the country's economic development and policies. On return to the headquarters, the staffs prepare a report, which forms the basis for discussion by the Executive Board. The views of the Executive Board are summarized in a Public Information Notice (PIN), which is attached to the Article IV report. Since the primary task of the current study is to evaluate the qualitative aspects of the IMF Reports, we exclude figures, graphs, appendixes, references, and footnotes. Many Staff Reports also include Selected Issues on various economic aspects. To be consistent across the sample, we also exclude Selected Issues sections for our study. Public Information Notices are also excluded from the analysis of the Staff Reports and analysed in a separate category. For Executive Board Assessment Documents, we follow a similar process by concentrating only on the textual part of the document.

Tone measures with respect to U.S.

In order to establish that DICTION 5.0 *WARNING* scores are meaningful, we first look at what happens to this measure in the case of U.S. Staff Reports during the period of 2005-2009. First, we perform *t*-tests to determine whether the mean score for the *WARNING* measure is statistically different from the U.S. score for a particular year (Table 5.4). We use the population mean of 5.86 and standard deviation of 4.64, which are based on 122 runs of the DICTION 5.0 software in a variety of news stories that are related to financial issues (e.g., tax returns, market predictions, trends in stocks and bonds, tax law, etc.) and obtained from on-line publications such as *Forbes*, *The San Francisco Chronicle* and the Daily News Bulletin.

Table 5.4. Significance of *WARNING* for U.S. relative to the population mean

Year	Score	*Z*-score	*t*-test	*p*-value
2005	4.750	-0.240	2.6423	0.0093
2006	7.210	0.290	-3.2136	0.0017
2007	6.630	0.160	-1.8330	0.0693
2008	6.090	0.050	-0.5475	0.5850
2009	13.94	1.750	-19.2342	0.0000

Notes: Staff Reports, 2005-2009

The most striking result for the United States is a high *WARNING* level in the 2009 Staff Report. It is almost two standard deviations above the financial news population mean and shows that economic outcomes were disappointing in the U.S. during the year 2009. Furthermore, the *WARNING* score for the U.S. in 2009 is highly statistically different from the population mean value as indicated by *t*-tests. We also observe a score statistically significant above the mean score of *WARNING* for the U.S. in 2006. This may be associated with the first fall in GDP growth during the last few years.

Tone measures with respect to the euro area countries.

Next, we perform the IMF Article IV Staff Reports and Executive Board Assessments' tone measures sample mean comparison tests (Table 5.5). We find that the average *WARNING* measure for Executive Board Assessments is highly statistically different from the *WARNING* scores of Staff Reports. The result is consistent with Bossone's (2008a) statement that the IMF board devotes much of its time to discussing staff country reports and to issuing recommendations (to which member countries do not attach particular importance), but that the board fails to exploit its potential as a collegial body of corporate governance to ensure the highest quality of surveillance and seek full cooperation from member countries. Furthermore, Bossone (2009b) states, "the board's lack of autonomy and limited authoritativeness reduces its efficacy and power of influence. Swamped by a heavy routine, board members do not invest much time beyond including the information reported to them by the Staff, nor do they systematically integrate the information available to their offices to inform board discussions or to control management conduct thoroughly. The board tends to fall prey to ‘tunnel visions’ shaped around staff-management views, depriving the institution of the internal dialectics and checks and balances that a resident board in continuous session should be able to ensure”. In addition, Bossone (2008b) adds that on financial sector surveillance policy in particular, over recent years the board has been unable to provide adequate oversight and strategic direction. As a result, surveillance policy reviews are weaker than what they might be under an effective board.

Table 5.5. Executive Board Assessment and Staff Report tone measures sample mean comparison test, euro area

Year	Mean: Staff Reports	Mean: Executive Board Assessments	*t*-test	*p*-value
1999	4.480	3.669	0.9665	0.3448
2000	5.644	2.481	4.6893	0.0001
2001	5.352	3.178	2.5647	0.0167
2002	6.923	3.814	2.4581	0.0216
2003	6.111	3.430	2.7135	0.0117
2004	4.584	3.000	2.3867	0.0249
2005	5.404	3.662	2.4122	0.0235
2006	7.110	3.897	3.4015	0.0027
2007	6.309	2.975	3.6082	0.0013
2008	7.146	2.758	4.2968	0.0006
2009	9.423	4.395	6.1878	0.0000
Total	**6.155**	**3.399**	**10.4324**	**0.0000**

Notes: The Table performs the IMF Article IV Staff Reports and Executive Board Assessments' tone measures sample mean comparison tests.

Tables 5.6 reports summary statistics of the *WARNING* measure across the euro area countries for the period 2005-2007 for the IMF Article IV Staff Reports and Executive Board Assessments. In Table 6.6, we also report the *t*-test results to determine whether the country average *WARNING* score over the period of 2005-2007 is statistically different from the EMU sample mean. Interestingly, for the Staff Reports, we observe the highest *WARNING* value for Greece, Luxembourg, Malta, Portugal, and Slovenia (Figure 5.1). The *WARNING* scores of the Staff Reports for these countries are almost one standard deviation above the euro area sample average. In contrast, the lowest *WARNING* measures are for Spain and Belgium. These are more than one standard deviation below the mean value. Also, note that the *WARNING* measures are insignificantly different from the EMU average for Finland, France, Germany, Ireland, the Netherlands and Slovakia. Interestingly, in the crisis, the more severely hit countries (e.g., Ireland, Finland

and Spain) were not distinguished by the warning tone of the IMF Article IV Staff Reports. This result, however, is consistent with the evaluation of the *IMF Interactions with Member Countries*, which covered the period 2001–08. The evaluation found that IMF interactions were more effective with low-income countries and with other emerging economies than they were with advanced and emerging economies.[4]

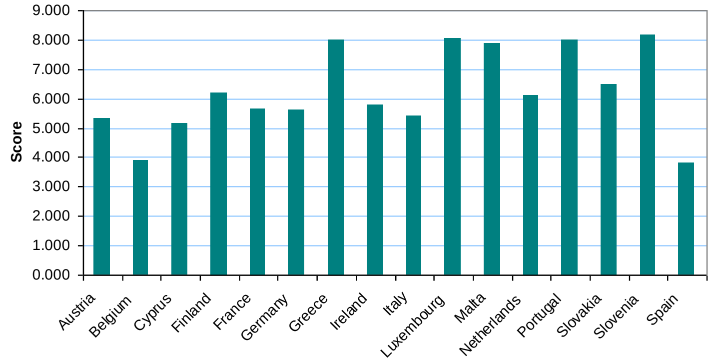

Figure 5.1. Average *WARNING* scores for the euro area: Staff Reports, 2005-2007

For the Executive Board Assessments (Table 5.6), we observe the highest *WARNING* value for Portugal. The *WARNING* score of the Executive Board Assessment for Portugal is 1.5 standard deviations above the sample mean. Furthermore, the *WARNING* scores of the Executive Board Assessment Reports for the Netherlands and Luxembourg are almost one standard deviation above the sample average value. In contrast, the lowest *WARNING* measures are for Cyprus, Germany, and Ireland. These values are also significant in econometric terms. In contrast, as indicated by the *t*-test, countries' average *WARNING* scores over the period of 1999-2009 are insignificantly different from the euro area average value for Austria, Belgium, Greece, Italy, Slovakia, Slovenia, and Spain.

[4] Interactions, in this context, are defined to include exchanges of information, analysis, and views between IMF officials and country authorities. Interactions take place in the context of the policy challenges faced by countries, and the relationships established between the IMF and its 185 member countries (IEO, 2008).

Table 5.6. *WARNING* measure for EMU: Staff Reports and Executive Board Assessments, 2005-2007

Country	Staff Reports			Executive Board Assessment		
	N	Mean	Std dev	*N*	Mean	Std dev
Austria	2	5.330	0.948	2	3.300	0.863
Belgium	3	3.897	2.651	3	2.967	1.056
Cyprus	2	5.155	1.379	1	1.750	n/a
Finland	2	6.195	0.983	2	2.520	0.735
France	3	5.670	1.618	3	2.520	0.751
Germany	3	5.603	1.358	3	1.813	1.379
Greece	3	8.013	3.018	3	3.603	3.408
Ireland	3	5.790	0.891	3	1.770	0.417
Italy	2	5.405	1.082	2	3.640	0.552
Luxembourg	1	8.030	n/a	1	5.170	n/a
Malta	2	7.865	2.256	2	4.215	1.648
Netherlands	3	6.093	2.683	3	5.127	3.312
Portugal	3	8.010	1.913	3	6.583	2.036
Slovakia	2	6.470	0.311	2	3.970	2.517
Slovenia	3	8.163	1.448	3	3.877	0.990
Spain	2	3.805	0.983	2	3.025	0.672
Total	**39**	**6.210**	**2.019**	**38**	**3.501**	**1.959**

Notes: The Table reports the summary statistics of *WARNING* measure across the euro area countries for the period 2005-2007 for the IMF Article IV Staff Reports and Executive Board Assessments. Table also presents the *t*-test results to determine whether the country average *WARNING* score over the period of 2005-2007 is statistically different from the EMU sample mean.

Finally, we construct a *t*-test to assess whether the mean *WARNING* score in a particular year is statistically different from the previous year mean *WARNING* score for the euro area average scores. Results for the Staff Reports and the Executive Board Assessments are presented in Table 5.7. Table 5.7 shows that for the Staff Reports, the mean *WARNING* value for the euro area in years 2009 and 2006 is significantly different from the average tone the previous year.

The *WARNING* score for year 2006 is also 2.2 standard deviations above the sample mean. This indicates that on average for the euro area there were some warning signals in the Article IV Staff Reports in the year 2006. In the case of the Executive Board Assessments, over the period 2000-2009, the mean *WARNING* score in each year is not statistically different from the previous year's level for the euro area.

Table 5.7. Did *WARNING* score change over time for EMU?

	Staff Reports			Executive Board Assessments		
Year	*N*	Mean	Std deviation	*N*	Mean	Std deviation
1999	9	4.480	1.138	14	3.669	2.331
2000	14	5.644	2.447	15	2.481	0.891
2001	13	5.352	2.459	14	3.178	1.932
2002	13	6.923	3.893	13	3.814	2.375
2003	14	6.111	2.853	14	3.430	2.351
2004	13	4.584	2.055	14	3.000	1.346
2005	14	5.404	1.538	13	3.662	2.182
2006	11	7.110	2.235	12	3.897	2.288
2007	14	6.309	2.076	13	2.975	1.338
2008	9	7.146	2.809	9	2.758	1.223
2009	10	9.423	1.877	11	4.395	1.844
Total	**134**	**6.155**	**2.661**	**164**	**3.399**	**1.889**

Notes: The Table shows whether the *WARNING* score changed over time for the Staff Reports and Executive Board Assessments, 1999-2009.

To sum up, in the run-up to the current credit crisis, we detect the presence of warning signs in the Article IV Staff Reports only for Slovenia, Luxembourg, Greece, and Malta. On average for the Staff Reports, over the period 2005-2007 there are insignificant differences between the EMU sample mean and the Staff Reports' yearly averages. Furthermore, we find the presence of a significantly different level of tone from the average tone the previous year for the IMF Article IV Staff Reports in 2006. Finally, there is a systematic bias of *WARNING* scores for Executive Board Assessments versus *WARNING* scores for the Staff Reports. Hence, we further focus on the results for the Staff Reports.

Spain

We now turn to the specific analysis of Spain and Ireland. Table 5.8 reports the results of the *t*-test to determine whether the Staff Report level of *WARNING* for Spain in a particular year is statistically different from the euro area average in that year. We find that the level of *WARNING* for Spain is statistically different from the euro area averages over the period 1999-2009 (where data are available).

Table 5.8. Significance of *WARNING* for Spain relative to the euro area average

Year	Score	*Z*-score	*t*-test	*p*-value
1999	n/a	n/a	n/a	n/a
2000	4.820	-0.502	5.8075	0.0000
2001	2.760	-1.276	14.7689	0.0000
2002	4.650	-0.566	6.6470	0.0000
2003	2.530	-1.362	15.7694	0.0000
2004	5.000	-0.434	5.0245	0.0000
2005	n/a	n/a	n/a	n/a
2006	4.500	-0.622	7.1996	0.0000
2007	3.110	-1.144	13.2463	0.0000
2008	3.730	-0.911	10.5492	0.0000
2009	n/a	n/a	n/a	n/a
Total	**3.888**	**-0.852**	**9.8619**	**0.0000**

Notes: The Table shows the results of the *t*-test to determine whether the Staff Report level of *WARNING* for Spain in a particular year is statistically different from the euro area average in that year over the period 1999-2009.

Ireland

Finally, we also analyse the *WARNING* variable for Ireland. First, we plot the euro area average *WARNING* scores together with Ireland's *WARNING* level of Staff Reports over the period 1999-2009 (Figure 5.2).

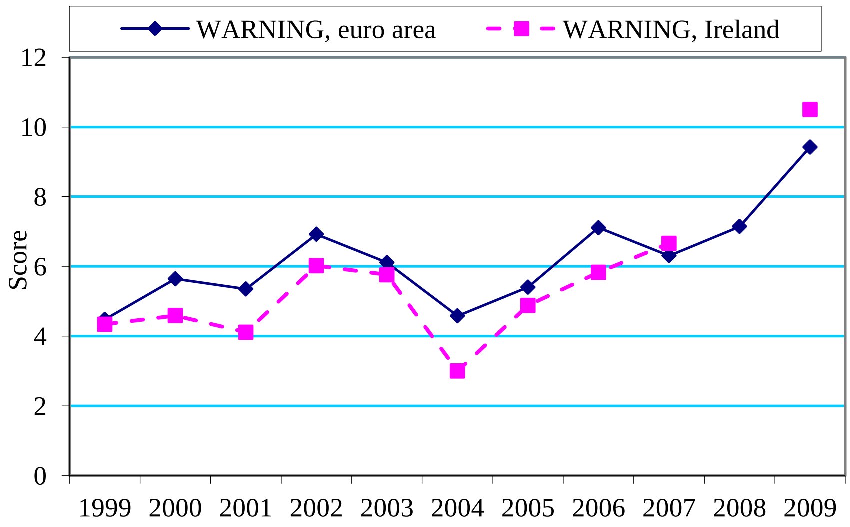

Figure 5.2. *WARNING* scores for the euro area and Ireland: Staff Reports, 1999-2009

In addition, Table 5.9 reports the results of the *t*-test to determine whether the Staff Report level of *WARNING* for Ireland in a particular year is statistically different from the euro area average in that year. Figure 5.2 shows that the level of *WARNING* of Article IV Staff Reports for Ireland is below the EMU mean over the period 1999-2007. In addition, the level of *WARNING* for Ireland is not statistically different from the euro area averages for the years 2002, 2003, and 2006. This result is consistent with the so-called *irrational exuberance*, which developed during the *Celtic Tiger* period in Ireland.[5] Also note the highly significantly differences from zero change in the *WARNING* score for Ireland in 2009. This indeed might indicate the "true" level of tone during the pre-crisis period.

[5] Connor, Flavin, and O'Kelly (2010) define irrational exuberance as an anomaly of intermittent periods of aggregate over-confidence and over-optimism in security markets, which leads to over-inflated asset prices and excessive aggregate risk-taking.

Table 5.9. Significance of *WARNING* for Ireland relative to the euro area average

Year	Score	*Z*-score	*t*-test	*p*-value
1999	4.340	-0.682	7.8956	0.0000
2000	4.590	-0.588	6.8080	0.0000
2001	4.110	-0.769	8.8961	0.0000
2002	6.020	-0.051	0.5873	0.5580
2003	5.760	-0.148	1.7618	0.0804
2004	3.000	-1.186	13.7248	0.0000
2005	4.880	-0.479	5.5465	0.0000
2006	5.830	-0.122	1.4138	0.1598
2007	6.660	0.190	-2.1968	0.0298
2008	n/a	n/a	n/a	n/a
2009	10.500	1.633	-18.9015	0.0000
Total	**5.569**	**-0.220**	**2.5492**	**0.0119**

Notes: The Table assesses whether the *WARNING* score for Ireland in a particular year is statistically different from the previous year average *WARNING* level for the euro area. Staff Reports, 1999-2009

Finally, we directly assess the relationship between the tone of the IMF Article IV Staff Reports and economic conditions, adjusting for a nation's wealth. In particular, we estimate the following specification:

$$WARNING_{it} = \alpha_i + \boldsymbol{\beta Econ}_{it+1} + \lambda\Delta GDP_{it} + \boldsymbol{\gamma}(\boldsymbol{Econ}_{it+1}\times\Delta GDP_{it}) + c_i + \mu_t + \varepsilon_{it},$$

$$\boldsymbol{Econ}_{it+1} = \{CA_{it+1}, GGSB_{it+1}, OG_{it+1}\}, \qquad (5.1)$$

where $WARNING_{it}$ represents our $WARNING_{it}$ measure extracted from DICTION 5.0, i indexes countries, t indexes time, and α_i is constant. ΔGDP_{it} is the difference in the GDP at market prices at year t. $\boldsymbol{Econ}_{it+1}$ is a set of economic variables at year (t+1). $\boldsymbol{Econ}_{it+1}$ includes the current account balance, the general government structural balance, and the output gap in percent of potential GDP. In addition, we include an interaction term between ΔGDP_{it} and the $\boldsymbol{Econ}_{it+1}$ variables, in order to establish whether the co-variation pattern exists between the change in the GDP at year t and the countries' economic conditions at year (t+1). The $\boldsymbol{\beta}$-vector represents the vector of the coefficients on the leading economic indicators. The $\boldsymbol{\beta}$-vector captures the effectiveness of the IMF Article IV Staff Reports. $\boldsymbol{\gamma}$ captures the cross-effects related of the wealth/poverty of the countries. Furthermore, c_i denotes the unobserved country-specific time-invariant variable and μ_t represents the time dummy variable in period t to capture common shocks affecting all countries simultaneously. ε_{it} is the error term, a white noise error with mean zero. All of the measures of countries' economic variables are taken from *Eurostat* database.

Table 5.10 shows the baseline results. We find that the coefficients on the current account balance and on the general government structural balance are statistically different from zero. In addition, β_1 and β_2 are negative, suggesting that lower current account balance and general government structural balance at year (t+1) are associated with higher *WARNING* score at year t for euro area countries on average. This result is consistent with the primary role of the IMF to provide an early warning to the membership so that country authorities can take actions to mitigate the impact of a crisis. Therefore, higher *WARNING* scores at time t leads to corrective actions by the governments and, as a result, to improved economic conditions. The coefficient on the output gap is not statistically different from zero. This suggests that there were no actions taken influencing the output gap at year (t+1) after the warnings by the IMF staff. The coefficient on the difference in the GDP is not statistically different from zero, indicating that the change in the GDP at year t does not influence the level of the *WARNING* at that year. This result suggests that the level of *WARNING* is a forward-looking measure. Finally, we find a statistically significant coefficient on the interaction terms between ΔGDP_{it} and the general government structural balance at year (t+1). γ_2 is also significantly positive. This result suggests that a higher *WARNING* level is associated with lower GDP change at year t and a lower government balance at year (t+1). In addition, a higher *WARNING* level is also associated with higher GDP change at year t and higher government balance at year (t+1). This finding is again consistent with the evaluation of the *IMF Interactions with Member Countries*, which concluded that IMF interactions were more effective with low-income countries than they were with advanced economies.

Table 5.10. The relationship between *WARNING* scores and countries' economic conditions

Dependent = *WARNING*	
Constant (α)	5.385 (8.01)
Current Account Balance (t+1) (β_1)	-0.184 (-1.95)
General Government Structural Balance (t+1) (β_2)	-0.409 (-2.25)
Output Gap (t+1) (β_3)	0.142 (0.91)
ΔGDP (t) (λ)	$5.03*10^{-6}$ (0.17)
Current Account Balance (t+1) × ΔGDP (t) (γ_1)	$6.83*10^{-8}$ (0.02)
General Government Structural Balance (t+1) × ΔGDP (t) (γ_2)	$1.51*10^{-5}$ (1.98)
Output Gap (t+1) × ΔGDP (t) (γ_3)	$-4.66*10^{-6}$ (-0.66)

Notes: Estimation is by OLS with country and time fixed effects. *P*-values are reported in parenthesis.

5.4 Solutions and Recommendations

To sum up, the current study finds that the average *WARNING* measure for Executive Board Assessments for the euro area countries is highly statistically different from the *WARNING* scores of the Article IV Staff Reports. This result is consistent with the fact that the IMF board devotes much of its time to discussing staff country reports and to issuing recommendations, but it fails to exploit its potential as a collegial body of corporate governance to ensure the highest quality surveillance and to seek full cooperation from member countries (Bossone, 2008). Furthermore, in the run-up to the current credit crises, average *WARNING* levels of Staff Reports for Slovenia, Luxembourg, Greece, and Malta are one standard deviation above the EMU sample mean; and for Spain and Belgium, they are one standard deviation below the mean value. Moreover, these deviations from the mean value are significant in econometric terms. Also, the average *WARNING* measures are insignificantly different from the EMU average for Finland, France, Germany, Ireland, the Netherlands, and Slovakia. Interestingly, in the crisis, the more severely hit countries (e.g., Ireland, Finland, and Spain) were not distinguished by the warning tone of the IMF Article IV Staff Reports. This result, however, is consistent with the evaluation of the *IMF Interactions with Member Countries*, which found that the IMF interactions were more effective with low-income countries and with other emerging economies than they were with advanced and large emerging economies. The econometric specification analysing the relationship between the tone of the IMF Article IV Staff Reports and economic conditions supports that finding. We find a significantly positive coefficient on the interaction term between the change in the *GDP* at year *t* and the general government structural balance at year (*t*+1). In addition, on average for the Staff Reports over the period 2005-2007, there are insignificant differences between the EMU sample mean and Staff Reports' yearly averages. Furthermore, the *t*-test shows that the 2009 euro area Staff Reports' average score is significantly different from the sample average as well as from the average EMU tone the previous year.

Results for Ireland are of particular interest. The level of *WARNING* in the Article IV Staff Reports for Ireland is below the EMU mean over the period 1999-2007. In addition, the level of *WARNING* for Ireland is not statistically different from the euro area averages for the years 2002, 2003, and 2006. There is also a highly statistically significant increase in the level of *WARNING* for Ireland above the euro area mean value in 2009. In addition, the *WARNING* value for Ireland in 2009 is also significantly different from the average EMU tone the previous year.

Finally, as a robustness check, we estimate an econometric model assessing the relationship between the tone of the IMF Article IV Staff Reports and countries' economic conditions. We find significantly negative coefficients on the current account balance and the general government structural balance with the *WARNING* score, which suggests that higher warnings by the IMF staff lead to improved economic conditions.

This study provides only an initial step in uncovering the variation in tone measures of the inter-governmental agencies' reports. For instance, we have focused on the IMF Article IV Staff Reports. It could also be interesting to investigate the variation in tone measures of the EU and the OECD reports. Another question is how various economic indicators (e.g. output gap, current account etc.) influence the variation in the tone scores. A further extension could also verify whether the variation in tone of the IMF is consistent with those of other intergovernmental agencies' reports. We defer these important questions to future research.

5.5 Conclusion

The main policy recommendation is to improve the informativeness and the discriminating content of the IMF surveillance. This may be implemented by improving the quality of the international dimensions of the IMF's work. The language of surveillance needs to be unambiguous. Greater clarity is needed to differentiate between critical risks and vulnerabilities. Another recommendation is to develop knowledge-based products to enhance the IMF's ability to engage government authorities in its surveillance activities. The effectiveness of core IMF activities may be improved through the adoption of a more strategic and standard-based approach for staff interactions with the authorities on country assessments. However, even good surveillance is not enough if countries do not follow up with good policies. The international community needs a legitimate and effective leading governing body that is politically responsible if it is to ensure international financial stability and coordinate international financial policy activities; this is the only hope for nation states to try to govern transnational financial phenomena. A reformed International Monetary and Financial Committee of the IMF would be the appropriate entity to play this role (Bossone, 2009a). It should do so in the context of IMF reform needed to strengthen the board and make management accountable.

Concluding Remarks

Recent developments in financial markets such as for instance the bursting of the IT bubble, the US subprime mortgage crisis and Europe's ongoing sovereign debt crisis, exemplify the importance of adequate risk measurement and risk management techniques that adapt more rapidly to changing market circumstances than traditional methods do. This thesis focuses on the use of range-based risk estimators for financial markets.

Chapter 2 assesses the daily risk dynamics and inter-market linkages of four European stock markets using daily range data. We compare the conditional autoregressive range model of Engle and Gallo (2006) in which the realized range has a gamma distribution to a new formulation in which intraday returns are normally distributed and realized range has a Feller distribution. The two models give similar estimates of autoregressive range dynamics, but the gamma-distribution-based model better captures the leptokurtotic feature observed in daily range data. There are also some spillover effects. The previous day's realized range in other European markets positively influences the next day's expected range. These spillover effects are not uniform across the markets; the strongest spillover comes from the previous day's realized range of the US market index. We also compare the pre-crisis and European financial crisis subperiods of our sample. In all four markets, average daily range increased sharply during the crisis period, and the contemporaneous correlations between the markets increased in most cases. Spillover effects between European markets did not seem to change, but the influence of yesterday's US market range on realized range in European markets increased.

In Chapter 3 we create a new range-based beta measure which uses the information on the daily opening, closing, high, and low prices. We also combine our new estimation methodology with a non-parametric approach for modelling the changes in beta. We demonstrate that our approach yields competitive estimates of firm-level betas compared with traditional methods. Extensive simulation study shows the range based beta estimates are slightly downward biased which is consistent with the fact that the range of the discretely sampled process is strictly less than the range of the underlying diffusion. The range-based correlation estimates are biased, even for moderately small values of the true correlation. Applying bias correction for the range-based correlations for DAX constituents we observe improvement in the range-based correlation estimates.

In Chapter 4 we assess the information content of implied volatility by considering implied volatility indices constructed based on the concept of model-free implied variance proposed by Demeterfi et al. (1999). In particular, using the range-based volatility estimator as a proxy for the realized variance we study the linkages between the range-based volatility and the implied volatility measure for the sample of five equity indices over the period January 3, 2000 to November 26, 2012. We assess the two-way relationships between the range-based volatility and the implied volatility, both within the index and accounting for spillovers between indices. Moreover, we study the evolution of spillovers between the range-based volatility and the implied volatility over time, identifying the net receivers and transmitters of shock and quantifying their magnitude using impulse response analysis. Finally, we consider average variance risk premium estimate defined as the simple average of the differences between the realized return variance and the implied variance.

In Chapter 4 using Mincer-Zarnowitz and encompassing regressions we find that the implied volatility does contain information in forecasting realized range-based volatility. The historical range-based volatility, on the other hand, has less explanatory power than the implied volatility in predicting realized range-based volatility. The univariate regression of historical volatility to the realized range-based volatility shows that the historical range-based volatility also has information in predicting the realized volatility, the regression of the historical range-based volatility and the implied volatility simultaneously shows that the implied volatility dominates historical range-based volatility in forecasting the realized range-based volatility, or that all the information contained in historical volatility has been reflected by the implied volatility, and the historical range-based volatility has no incremental forecasting ability. The results from the univariate regressions are also consistent with the existing option pricing literature which documents that the stochastic volatility is priced with a negative market of risk. The volatility implied from option prices is thus higher than its counterpart under the objective measure due to investor risk aversion. Our study shows that the option market processes information efficiently in the US market.

Finally, Chapter 5 considers financial market risk from a different perspective. Chapter 5 analyses the tone and information content of the two external policy reports of the International Monetary Fund (IMF), the IMF Article IV Staff Reports and Executive Board Assessments, for Euro area countries. Our results show a number of interesting facts. First, the average *WARNING* measure for the Executive Board Assessments for the euro area countries is highly statistically different from *WARNING* scores of the Article IV Staff Reports. Second, in the run-up to the current credit crises, average *WARNING* levels of Staff Reports for Slovenia, Luxembourg, Greece, and Malta are one standard deviation above the EMU sample mean; whereas for Spain and Belgium, they are one standard deviation below the mean value. Also, the average *WARNING* measures are insignificantly different from the EMU average for Finland, France, Germany, Ireland, the Netherlands, and Slovakia. The econometric specification analysing the relationship between the tone of the IMF Article IV Staff Reports and the economic conditions supports that finding. We find a significantly positive coefficient on the interaction term between the change in the *GDP* at year t and the general government structural balance at year (t+1). In addition, on average for Staff Reports over the period 2005-2007, there are insignificant differences between the EMU sample mean and Staff Reports' yearly averages. Furthermore, the t-test shows that the 2009 euro area Staff Reports' average score is significantly different from the sample average as well as from the average EMU tone the previous year.

Several interesting directions for future research emerge from our study. First, it will be interesting to study the theoretical properties of the range-based volatility, correlation, and beta estimates. Second, alternative estimators based on intraday highs and lows could be explored. Third, the range-based beta framework can be extended to multiple risk factors, where factor betas can be estimated based on the range-base variance and covariance. In addition, it would be interesting to consider multivariate range-based volatility model, in the spirit of DCC of Engle (2002). Finally, we can consider high frequency range-based correlation and beta estimates, which are based on the intraday opening, close, high, and low prices.

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Appendices

Appendix A

Rogers and Zhou (2008) Theorem 1:

Consider all cross-quadratic functional, by which we mean linear combination of the terms H_PH_M, H_PL_M, L_PH_M, L_PL_M, H_PS_M, L_PS_M, S_PH_M, S_PL_M, S_PS_M, where $H_{Pt} \equiv \max_{0\le t\le 1} P_\tau$, $H_{Mt} \equiv \max_{0\le \tau\le 1} M_\tau$, $L_{Pt} \equiv \min_{0\le t\le 1} P_\tau$, $L_M \equiv \min_{0\le \tau\le 1} M(\tau)$, $S_P = P(1)$, and $S_{Mt} = M(1)$. *Consider the estimator:*

$$\hat{\rho}_{PMt} \equiv \hat{\rho}_{PMt}\left(H_{Pt}, L_{Pt}, S_{Pt}, H_{Mt}, L_{Mt}, S_{Mt}\right)$$

Among the cross-quadratic functional, the correlation is a function of the high, low and final log-prices of the two assets which satisfy the unbiasedness condition

$$E_{\rho_{PMt}}\left[\hat{\rho}_{PMt}\right] = \rho \qquad (\rho = -1,0,1),$$

(10)

the one whose variance $E_0\left[\hat{\rho}^2{}_{PMt}\right]$ *is minimal when* $\rho = 0$ *is*

$$\hat{\rho}_{PMt} = \frac{1}{2} S_{Pt} S_{Mt} + \frac{1}{2(1-2b)}\left(H_{Pt} + L_{Pt} - S_{Pt}\right)\left(H_{Mt} + L_{Mt} - S_{Mt}\right). \qquad \text{(A.1)}$$

The constant b is equal to $2\log 2 - 1 \cong 0.386294$ *and the minimized variance is* $E_0\left[\hat{\rho}^2{}_{PMt}\right] = 1/2$.

Rogers and Zhou (2008) Proof of Theorem 1.

The goal is to make an unbiased estimator of ρ by forming linear combinations of the nine possible cross terms. $Z_{HH}=H_PH_M$, $Z_{HL}=H_PL_M$, $Z_{LH}=L_PH_M$, $Z_{LL}=L_PL_M$, $Z_{HS}=H_PS_M$, $Z_{LS}=L_PS_M$, $Z_{SH}=S_PH_M$, $Z_{SL}=S_PL_M$, and $Z_{SS}=S_PS_M$. Now, the means of these products *are known for the cases* ρ = -1, 0, 1 and the paper by Rogers and Shepp (2006) establishes that

$$EZ_{HH} = f(\rho)$$

$$\equiv \cos\alpha \int_0^\infty d\nu \frac{\cosh \nu\alpha}{\sinh \nu\pi/2} \tanh \nu\gamma,$$

where $\rho = \sin\alpha$, $\alpha \in (-\pi/2, \pi/2)$ and $2\gamma = \alpha + \pi/2$. Table 1 summarizes the situation. We seek a linear combination of $\hat{\rho}_{PM}$ of the nine cross products with the following properties:

(i) $E_{\rho_{PM}}[\hat{\rho}_{PM}] = \rho$ for ρ = -1, 0, 1;

(ii) when ρ = 0, the variance of $\hat{\rho}$ is minimal.

Table A.1. Means of the components of *Z*

	$\rho=-1$	$\rho=0$	$\rho=1$	ρ
EZ_{HH}	b	$2/\pi$	1	$f(\rho)$
EZ_{HlL}	-1	$-2/\pi$	$-b$	$-f(-\rho)$
EZ_{LH}	-1	$-2/\pi$	$-b$	$-f(-\rho)$
EZ_{LL}	b	$2/\pi$	1	$f(\rho)$
EZ_{HS}	-1/2	0	1/2	$\rho/2$
EZ_{LS}	-1/2	0	1/2	$\rho/2$
EZ_{SH}	-1/2	0	1/2	$\rho/2$
EZ_{SL}	-1/2	0	1/2	$\rho/2$
EZ_{SS}	-1	0	1	ρ

In order to find a minimum-variance linear combination, we need to know the covariance of $Z \equiv (Z_{HH}, Z_{HL}, Z_{LH}, Z_{LL}, Z_{HS}, Z_{LS}, Z_{SH}, Z_{SL}, Z_{SS})$ when ρ = 0. In this case , the two Brownian motions are independent and the entries of the covariance matrix can be computed from the entries of Table 1. For example, $E_0[Z_{HH}Z_{SL}] = E_1[Z_{HS}] \cdot E_1[Z_{HL}] = -b/2$. Routine but tedious calculations lead to the following covariance matrix:

$$V=\begin{pmatrix} 1 & -b & -b & b^2 & 1/2 & -b/2 & 1/2 & -b/2 & 1/4 \\ -b & 1 & b^2 & -b & 1/2 & -b/2 & -b/2 & 1/2 & 1/4 \\ -b & b^2 & 1 & -b & -b/2 & 1/2 & 1/2 & -b/2 & 1/4 \\ b^2 & -b & -b & 1 & -b/2 & 1/2 & -b/2 & 1/2 & 1/4 \\ 1/2 & 1/2 & -b/2 & -b/2 & 1 & -b & 1/4 & 1/4 & 1/2 \\ -b/2 & -b/2 & 1/2 & 1/2 & -b & 1 & 1/4 & 1/4 & 1/2 \\ 1/2 & -b/2 & 1/2 & -b/2 & 1/4 & 1/4 & 1 & -b & 1/2 \\ -b/2 & 1/2 & -b/2 & 1/2 & 1/4 & 1/4 & -b & 1 & 1/2 \\ 1/4 & 1/4 & 1/4 & 1/4 & 1/2 & 1/2 & 1/2 & 1/2 & 1 \end{pmatrix}.$$

Writing $m=(1,-b,-b,1,1/2,1/2,1/2,1/2,1)^T$, $y=(1,-1,-1,1,0,0,0,0,0)^T$, objective now is to choose a 9-vector ϖ of weights to minimize $\omega V\omega$ subject to the constraints that $\omega y=0$ and $\omega m=1$. This optimization problem is easily solved: we find that the solution takes the form

$$\omega=\alpha V^{-1}m+\beta V^{-1}y,$$

where α,β are determined by

$$\begin{pmatrix} mV^{-1}m & mV^{-1}y \\ yV^{-1}m & yV^{-1}y \end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}.$$

Lengthy but routine calculations lead to the final form (2), as claimed, and the value $E_0\left[\hat{\rho}^2{}_{PM}\right]=1/2$ is calculated from the explicit forms of V, m, and y. ■

Appendix B

Garman Klass (1991) Lemma (Estimator Invariance Properties)*:* Let Θ be a parameter space. Let $\mathbf{X}$ = (X , X ,…, X_n) be a vector of (not necessarily independent) observations whose joint density $f_\Theta(\mathbf{X})$ depends on an unknown parameter $\theta \in \Theta$ to be estimated. Let T: $\mathbf{R^n} \to \mathbf{R^n}$ be a fixed measure-preserving transformation. Suppose that, for all $\theta \in \Theta$ and all $\mathbf{X}$ in the support of $f_\Theta(\mathbf{X})$,

$$f_\Theta(T\mathbf{X}) = f_\Theta(\mathbf{X}). \quad \text{(B.1)}$$

Let $D(\mathbf{X})$ be any decision rule which estimates θ. Let $L(\theta, D(\mathbf{X}))$ be any loss function such that $L(\theta, D(\mathbf{X}))$ is a convex function for each fixed $\theta \in \Theta$. Defining $T^j \equiv TT^{j-1}$, where T^0 is the identity operator, let A_k be an averaging operator which maps decision rules into decision rules according to the prescription

$$A_k(D(\mathbf{X})) = \frac{1}{k}\sum_{j=1}^{k} D\left(T^{j-1}\mathbf{X}\right). \quad \text{(B.2)}$$

Then, for all $\theta \in \Theta$,

$$E_\theta L(\theta, A_k(D(\mathbf{X}))) \le E_\theta L(\theta D(\mathbf{X})). \quad \text{(B.3)}$$

Proof*:* [Deleted for brevity. Use the convexity, take expectations.]

Garman and Klass (1991) model assumes that a diffusion process governs security prices:

$$P(t) = \varphi(B(t)). \quad \text{(B.4)}$$

Here P is the security price, t is time, φ is a monotonic, time-independent transformation, and *B(t)* is a diffusion process with the differential representation

$$dB = \sigma dz, \quad \text{(B.5)}$$

where dz is the standard Gauss-Wiener process and σ is an unknown to be estimated. This formulation is sufficiently general to cover the usual hypothesis of the geometric-Brownian motion of stock prices, as well as some of the proposed alternatives to the geometric hypothesis (e.g. Cox and Ross, 1975).

Garman and Klass (1991) adopt notation as follows:

σ^2 = unknown constant variance (volatility) of price change;

f = fraction of the day (interval [0, 1]) that trading is closed;

$C_0 = B(0)$, previous closing price;

$O_1 = B(f)$, today's opening price;

$H_1 = \max B(t), \quad f \le t \le 1,$ today's high;

$L_1 = \min B(t), \quad f \le t \le 1,$ today's low;

$C_1 = B(1)$, today's close;

$u = H_1 – O_1$, the normalized high;

$d = L_1 – O_1$, the normalized low;

$c = C_1 – O_1$, the normalized close;

$g(u, d, c;\ \sigma^2)$ = the joint density of (u, d, c) given σ^2 and $f = 0$.

To simplify the initial analysis, Garman and Klass (1991) suppose $f = 0$, that is, trading is open throughout the interval [0, 1]. Next, consider estimators of the form $D\ (u, d, c)$, that is, decision rules which are functions only of the quantities u, d, and c. Garman and Klass (1991) restrict attention to these normalized values because the process $B(t)$ renews itself everywhere, including at $t = 0$, and so only the increments from the level $O_1(= C_0)$ are relevant. According to the lemma (Estimator Invariance Properties) above, any minimum-squared-error estimator $D\ (u, d, c)$ should inherit the invariance properties of the joint density of (u, d, c). Two such invariance properties may be quickly recounted: for all $\sigma^2 > 0$ and all $d \le c \le u, \ d \le 0 \le u$,

$$g\left(u, d, u; \sigma^2\right) = g\left(-d, -u, -c; \sigma^2\right) \tag{B.6}$$

and

$$g\left(u, d, u; \sigma^2\right) = g\left(u - c, d - c, -c; \sigma^2\right). \tag{B.7}$$

The first condition represents price symmetry: for the Brownian motion of form (B.5), $B(t)$ and $–B(t)$ have the same distribution. Whenever $B(t)$ generates the realization (u, d, c), $–B(t)$ generates (u, d, c). The second condition represents time symmetry: $B(t)$ and $B(1 – t) – B(1)$ have identical distributions. Whenever $B(t)$ produces (u, d, c), $B(1 – t) – B(1)$ produces $(u – c, d – c, –c)$. By the lemma

(Estimator Invariance Properties), then, any decision rule *D* (*u*, *d*, *c*) may be replaced by an alternative decision rule which preserves the invariance properties (B.6) and (B.7) without increasing the expected (convex) loss associated with the estimator. Therefore, we seek decision rules which satisfy

$$D(u,d,c) = D(-d,-u,-c) \quad \text{(B.8)}$$

and

$$D(u,d,c) = D(u-c,d-c,-c). \quad \text{(B.9)}$$

Next, Garman and Klass (1991) observe that a scale-invariance property should hold in the volatility parameter space: for any $\lambda > 0$,

$$g(u,d,c;\sigma^2) = g(\lambda u,\lambda d,\lambda c;\lambda^2\sigma^2). \quad \text{(B.10)}$$

In consequence of (B.10), we now restrict our attention to scale-invariant decision rules for which

$$D(\lambda u,\lambda d,\lambda c) = \lambda^2 D(u,d,c),\ \lambda > 0. \quad \text{(B.11)}$$

Garman and Klass (1991) adopt the regularity condition that the decision rules considered must be analytic in a neighborhood of the origin, condition (B.11) implies that the decision rule *D* (*u, d, c)* must be quadratic in its arguments. (Proof of this is given in Garman Klass (1991) Appendix B.) Thus we have

$$D(u,d,c) = a_{200}u^2 + a_{020}d^2 + a_{002}c^2 + a_{110}ud + a_{101}uc + a_{011}dc. \quad \text{(B.12)}$$

Scale invariance and analyticity are combined to reduce the search for a method of estimating σ^2 from an infinite dimensional problem to a six-dimensional one. Applying the symmetry property (B.8) to equation (B.12), we have the implications $a_{200} = a_{020}$ and $a_{101} = a_{011}$. By virtue of property (B.9), we have the additional constraint $2a_{200} + a_{110} + 2a_{101} = 0$, hence we have

$$D(u,d,c) = a_{200}(u^2 + d^2) + a_{002}c^2 - 2(a_{200} + a_{101})ud + a_{101}c(u+d). \quad \text{(B.13)}$$

Insisting that *D* (*u*, *d*, *c*) be unbiased, that is, $E[D(u,d,c)] = \sigma^2$, leads to one further reduction. Since $E[u^2] = E[d^2] = E[c^2] = E[c(u+d)] = \sigma^2$ and $E[ud] = (1 - 2\log 2)\sigma^2$, we may restrict attention further to the two-parameter family of decision rules *D*(.) of the form

$$D(u,d,c) = a_1(u-d)^2 + a_2\{c(u+d) - 2ud\} + \{1 - 4(a_1 + a_2)\log_e 2 + a_2\}c^2. \quad \text{(B.14)}$$

To minimize this quantity, note that, for any random variables X, Y, and Z, the quantity $V(a_1, a_2) \equiv E\left[(a_1 X + a_2 Y + Z)^2\right]$ is minimized by a_1 and a_2 which satisfy the first-order conditions

$$E[(a_1X + a_2Y + Z)\,X\,] = E[(a_1X + a_2Y + Z)Y] = 0. \quad \text{(B.15)}$$

Solving the above for a_1 and a_2, we have

$$a_1^* = \frac{E[XY]E[YZ] - E[Y^2]E[XZ]}{E[X^2]E[Y^2] - (E[XY])^2} \quad \text{(B.16)}$$

and

$$a_1^* = \frac{E[XY]E[XZ] - E[Y^2]E[YZ]}{E[X^2]E[Y^2] - (E[XY])^2}. \quad \text{(B.17)}$$

In the problem at hand,

$$X = (u-d)^2 - (4\log_e 2)c^2,$$

$$X = (u+d)^2 - 2ud + (1 - 4\log_e 2)c^2, \quad \text{(B.18)}$$

$$Z = c^2.$$

Analysis via generating functions (Appendix C in Garman and Klass, 1991) reveals the following fourth moments:

$$E[u^4] = E[d^4] = E[c^4] = 3\sigma^4,$$

$$E[u^2c^2] = E[d^2c^2] = 2\sigma^4,$$

$$E[u^3c] = E[d^3c] = 2.25\sigma^4,$$

$$E[uc^3] = E[dc^3] = 1.5\sigma^4,$$

$$E[udc] = E[u^2dc] = \left\{\frac{9}{4} - 2\log_e 2 - \frac{7}{8}\varsigma(3)\right\}\sigma^4 = -0.1881\sigma^4,$$

$$E[u^2d^2] = [3 - 4\log_e 2]\sigma^4 = 0.227\sigma^4,$$

$$E[udc^2] = \left[2 - 2\log_e 2 - \frac{7}{8}\varsigma(3)\right]\sigma^4 = -0.4381\sigma^4,$$

$$E[ud^3] = E[u^3d] = \left[3 - 3\log_e 2 - \frac{9}{8}\varsigma(3)\right]\sigma^4 = -0.4381\sigma^4,$$

where $\varsigma(3) = \sum_{k=1}^{\infty} 1/k^3 = 1.2021$ is Riemann's zeta function. Substituting the above moments into (B.16) and (B.17) via (B.18), we find that $a_1^* = 0.511$ and

$a_2^* = -0.019$. Employing these values in (B.14) yields the **best analytic scale invariant estimator:**

$$\hat{\sigma}_4^2 \equiv 0.511(u-d)^2 - 0.019\{c(u+d) - 2ud\} - 0.383c^2. \qquad \text{(B.19)}$$

Garman and Klass (1991) find that $Eff\left(\hat{\sigma}_3^2\right) \cong 7.4$.

Appendix C. DICTION 5.0 thematic categories and sub-categories

Major categories				
Activity	Optimism	Certainty	Realism	Commonality
Sub-features				
Numerical terms	Satisfaction	Cognition	Concreteness	Liberation
Ambivalence	Inspiration	Passivity	Past concern	Denial
Self-reference	Blame	Spatial terms	Centrality	Motion
Tenacity	**Hardship**	Familiarity	Rapport	
Leveling terms	Aggression	Temporal terms	Cooperation	
Collectives	Accomplishment	Present concern	Diversity	
Praise	Communication	Human interest	Exclusion	