Signal Accuracy
and
Informational Cascades

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Abstract

In an observational learning environment, rational agents with incomplete information may mimic the actions of their predecessors even when their own signal suggests the opposite. This herding behavior may lead society to an inefficient outcome if the signals of the early movers happen to be incorrect.

This paper analyzes the effect of signal accuracy on the probability of an inefficient informational cascade. The literature so far has suggested that an increase in signal accuracy leads to a decline in the probability of inefficient herding, because the first movers are more likely to make the correct choice. Indeed, the results in Bikhchandani, Hirshleifer and Welch (1992) support this proposition. Here we show that this is not the case in general. We present simulations which demonstrate that even a small departure from symmetry in signal accuracy may lead to non-monotonic results. An increase in signal accuracy may result in a higher likelihood of an inefficient cascade.

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I. Introduction

Seminal papers by Banerjee (1992), Bikhchandani, Hirshleifer and Welch (1992) and Welch (1992) show that it may be optimal for a rational agent with incomplete information to follow the actions of his predecessors even when his own private signal suggests the opposite. In an observational learning environment this herding behavior may lead society to common actions, possibly resulting in sudden booms and crashes. If the early movers' signals happen to be incorrect the followers will be mislead, yielding an inefficient outcome. This paper analyzes the effect of signal accuracy on the probability of an inefficient informational cascade.[1]

Herding may have dramatic consequences depending on the market we study[2]. Herding in the labor market may result in a prolonged period of unemployment of an individual if he initially turns out to be unlucky in a few job interviews. Herding among portfolio managers may result in an inefficient allocation of pension fund assets. Herding in R&D projects may result in delays in finding the cure for a fatal disease. In financial markets a sudden crash can have severe macroeconomic consequences.

The analysis of the factors that affect the likelihood of an inefficient cascades may be of interest in helping to reduce the probability of such events. For instance, the

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[1]In an informational cascade every subsequent agent makes the same choice independent of his private signal. Therefore private information is no longer conveyed to the market and social learning ceases.

While Banarjee (1992) and Bikhchandani, Hirshleifer and Welch (1992) have a predetermined sequence of moves in agents' decisions, Chamley and Gale (1994) endogenize the timing of moves and show that herding will eventually arise with probability one, resulting in either a boom or a collapse.

[2]There are a wide variety of markets where herding may arise. Among others, see Avery and Zemsky (1998), Chamley (2003), Chari and Kehoe (2003), Devenow and Welch (1996), Nelson (2002) and Scharfstein and Stein(1990) for analysis of herd behavior in financial markets, Neeman and Orosel (1999) for analysis in auctions, Morton and Williams (1999) for herding in a political economy framework and Choi, Dassiou and Gettings (2000), Kennedy (2002) and De Vany and Lee (2001) for herding among firms.

Securities Act of 1933 and the Securities Exchange Act of 1934 were enacted in hopes of preventing catastrophic crashes like Black Thursday in 1929. Among other regulations these acts require periodic reporting of financial information concerning publically traded securities in order to improve signal accuracy. Depending on the market under consideration, signal accuracy may be affected by a variety of factors, such as changes in accounting standards, technological advancement in information dissemination and informative advertising. In this paper we would like to study the effects of an improvement in signal accuracy on the probability of an inefficient cascade.

The literature so far has suggested that an increase in signal accuracy leads to a decline in the probability of inefficient herding because the first movers are more likely to make the correct choice. Indeed, Bikhchandani, Hirshleifer and Welch (1992) (henceforth BHW) clearly support this proposition. We show this not to be the case in general.

In BHW the agent receives a signal about the true value of the project, either good or bad. The signal is correct with probability p. Agents take the decision to invest or not. In the BHW framework an increase in signal accuracy always leads to a decrease in the probability of inefficient herding. In this paper, we consider the case where signals do not have symmetric accuracy.

In general, the good signal and the bad signal do not necessarily need to be of the same accuracy. For instance, a good job candidate may come to a job interview on time with a 95 % probability and a bad candidate may be on time with an 85% probability. As long as the probabilities are different, promptness may be a useful signal of candidate quality. If the candidate is good, he will be prompt and send the good signal with a 95% chance. The accuracy of the good signal is 95%. If the candidate is bad, he will be late and send a bad signal with a 15% chance. The accuracy of the bad signal is 15%. In this example the signals do not have symmetric accuracy. In the symmetric case, one forces the probability of the bad candidate sending the correct signal (hence being late) to be 95%, which is quite restrictive. We show that even small departures from symmetry may lead to non-monotonic results. In some cases an increase in signal accuracy may result in a higher likelihood of an inefficient cascade.

II. Symmetric Signal Accuracy

In BHW the value of the project is either high or low with even prior probabilities. The gain to adopting is either 1 in the High (H) state or 0 in the Low (L) state and the cost of adopting is ½. Each risk-neutral agent receives a private conditionally independent signal about the value of the investment project. An individual's signal is either h or ℓ. The signal is correct with probability p. For presentation purposes it will be convenient to add ½ to each of these payoffs, converting the BHW problem into an equivalent payoff matrix. The agent faces two investment projects:[3] The risky project yields either 1 in the High state or 0 in the Low state. The safe project yields a safe return of ½ in either state. The payoff matrix is then given by:

Table 1

	Risky Project	Safe Project
High State	1	½
Low State	0	½

ex ante Prob(High)=0.5

If the risky project is rejected, the safe project is adopted. There is a predetermined sequence moves and agents observe the actions of those ahead of them[4]. Agents follow

[3]Vives (1996) shows that a framework to help explain the observation of incorrect herds in a social learning setting needs to have two ingredients: Indivisibilities in terms of the discrete action space and signals of bounded precision.

[4]See Pastine (2005) for the effects of signal accuracy in an endogenous-timing framework. See Smith and Sørensen (1999) for a model which generates herding without the perfect observability assumption.

Bayes' Rule in their learning process. Following BHW, when an agent is indifferent between the two projects he is assumed to randomize, choosing each project with 50% probability.[5]

The above described scenario is equivalent to the following: There are two urns; H and L. Each urn has some balls marked *h* and some balls marked ℓ. Urn L has a higher percentage of balls marked ℓ than urn H. In the BHW framework the percentile of correct balls in each urn, the signal accuracy *p*, is symmetric. That is, the percentage of *h* balls in the H urn is equal to the percentage of ℓ balls in the L urn. Nature draws one urn with equal probabilities. Then all agents privately draw one ball each with replacement from the same urn.

The agent's problem is to determine which urn the ball comes from. The early movers' actions will reflect their private information, allowing followers to infer their private signal. At some point this public information will overwhelm the informational content of a single private agent. After that point all following agents will take the same action regardless of their private signals. The probabilistic nature of the individual signals implies that incorrect cascade may form. If most of the early agents happened to receive an ℓ signal, all newcomers may choose Urn L even when the correct Urn is H. In other words, the society may settle on the safe project even though the true value of the risky project is high. The probability of an L cascade when the true state is H is referred to as the probability of an inefficient negative cascade. Likewise, the society may settle in the risky project even though the true value of the risky project is low. The probability of an H cascade when the true state is L is referred to as the probability of an inefficient positive cascade. The inefficient cascade probability is simply given by the inefficient

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[5] In laboratory experiments Anderson and Holt (1997) find that in situations where the subject is theoretically indifferent he typically goes with his own signal rather than randomizing. We have also run all the simulations under this assumption and the results we discuss are qualitatively unchanged.

positive cascade probability and the inefficient negative cascade probability weighted by the *ex ante* probabilities of states H and L.

Figure 1 summarizes our replication of BHW's results on signal accuracy.[6] An increase in signal accuracy always leads to a decrease in the probability of inefficient herding since the early movers are more likely to take the correct action.

[6] In BHW there is a closed-form solution for the probability of inefficient cascades due to the recursive nature of the symmetric signal accuracy framework. When we depart from symmetry the recursive nature breaks down. All simulations in the paper are done with 10 million runs per data point. In all cases the 99% confidence intervals are less than the width of the symbols used to represent data points. We have created a Windows program which can be used to easily simulate a wide variety of BHW-based herding models. The software is self contained, requiring no additional programs, and can be downloaded from:

http://www.ucd.ie/economic/staff/ipastine/herding.htm

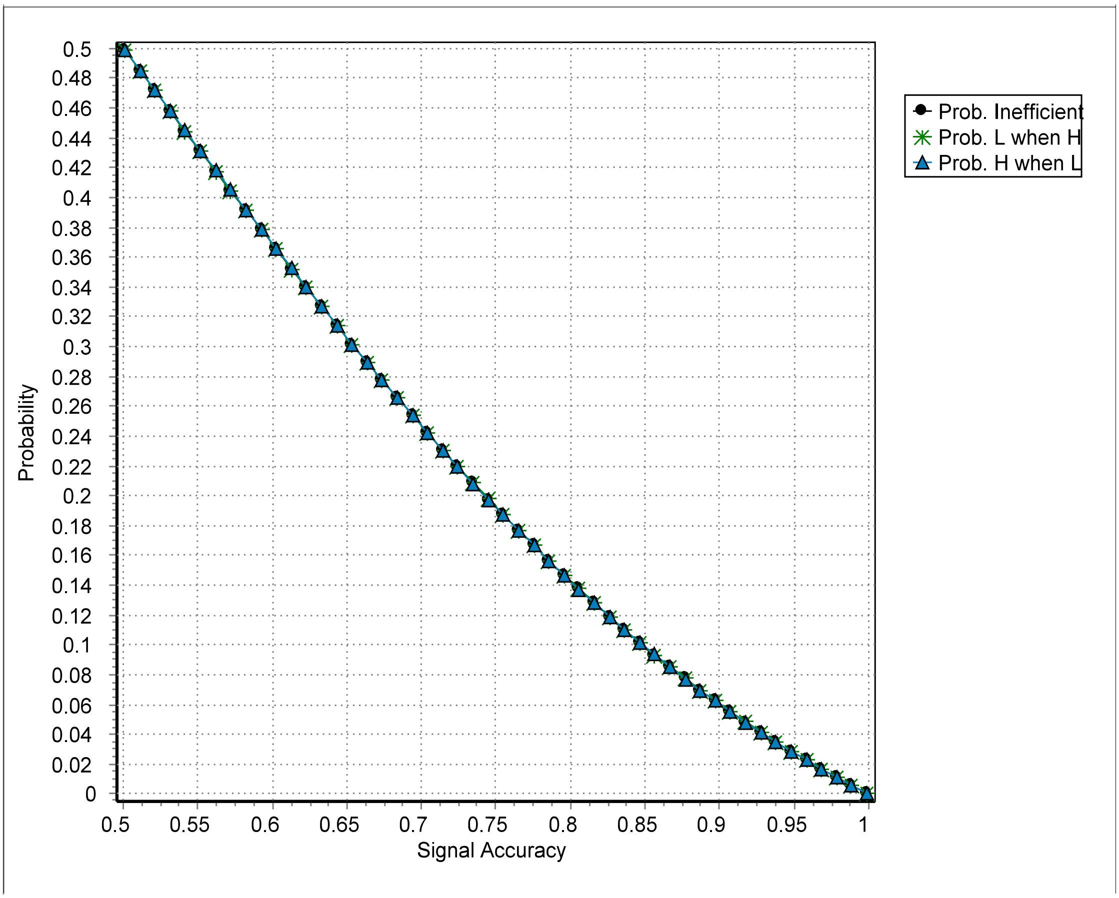

Figure 1

We aim to show that this monotonicity result is not general. Even a small departure away from symmetry may lead to the violation of this result. The BHW framework is symmetric because: i) The signal has the same accuracy in both states. There are exactly the same percentile of correct balls in each urn. ii) The *ex ante* probabilities of high and low project values are even.

It will be useful to notice that here the probability of an inefficient cascade is equal to the probability of an inefficient positive cascade. It is also equal to the probability of an inefficient negative cascade. This is due to the symmetry of the framework.

III. Asymmetric Signal Accuracy

In the urn setting, asymmetric signal accuracy translates into asymmetric percentile of correct balls in each urn.[7] p_h refers to the percentile of *h* balls in Urn H. It is equal to the probability of receiving signal *h* conditional on H, Prob(*h*|H). p_ℓ refers to the percentile of ℓ balls in Urn L. It is equal to the probability of receiving signal ℓ conditional on L, Prob(ℓ|L). For the signals to be informative it must be the case that $p_\ell \neq 1\text{-}p_h$. By appropriately labeling the signals $p_\ell + p_h > 1$ without loss of generality.

3.1. Even State Probabilities

Figure 2 reports the simulation results for signal accuracy of *h* fixed at 70% and varying the accuracy of signal ℓ. Fixing the accuracy of the *h* signal at different levels does not change the spirit of the results. The payoff matrix and the *ex ante* probabilities of states H and L are as in Table 1.

[7] To the best of our knowledge, the first examination of asymmetric signal accuracies in a herding context was the laboratory experiments of Anderson and Holt (1997).

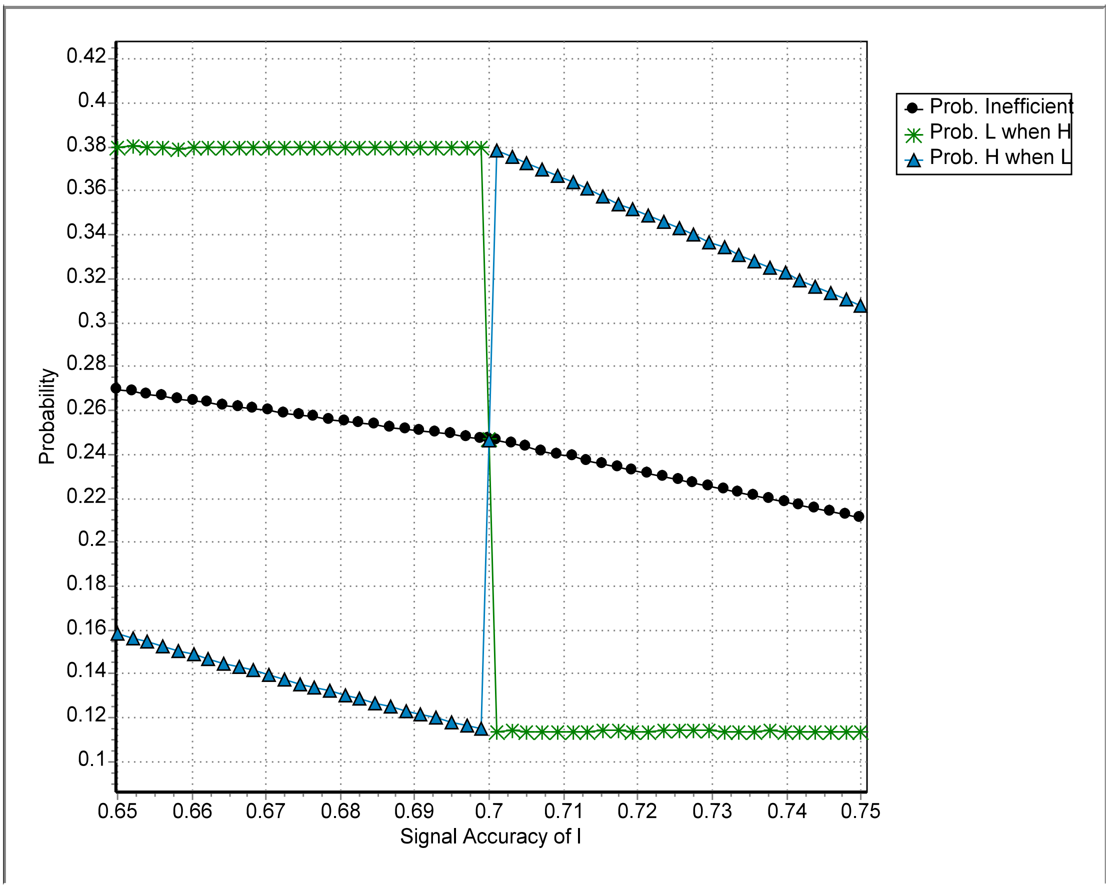

Figure 2

Figure 2 reports simulation results right around the point of symmetry. The three plots in the graph are the probability of a inefficient positive cascade ("Prob. H when L"), the probability of a inefficient negative cascade ("Prob. L when H") and the *ex ante* probability of an inefficient cascade ("Prob. Inefficient") which is the average of the two former weighted by the *ex ante* probabilities of the states.

In this example, the probability of an inefficient cascade is monotonic in signal accuracy. However, the probability of an inefficient positive cascade is not monotonic. The probability of an H cascade when the true state is L decreases with an improvement in signal accuracy until the point of symmetry. But then it jumps up from 0.12 to 0.38. It then continues to decrease with an increase in accuracy. The probability of an L cascade when the true state is H also shows jumpy behavior. At the point of symmetry it drops down from 0.38 to 0.12.

To understand what lies behind the jumps in the positive and negative inefficient cascade probabilities around the point of symmetry first notice the following: When p_h and p_ℓ are symmetric, the second agent will either go with his own signal or he will be just indifferent between the two actions. The first mover will follow his own signal, so the second agent will be able to infer his signal from his action. If the first and second signals are different, the second signal simply cancels out the first since they have equal accuracy. When there is asymmetry, however slight, signal h and signal ℓ do not cancel each other out because they have different weights in the updating process. Therefore herding can start earlier when signal accuracies are not symmetric.

When p_ℓ is just below p_h, the second agent always herds when the first agent chooses L.[8] Hence the probability of an inefficient L cascade is high. When p_ℓ is just above p_h, the second agent always herds when the first agent chooses H. Hence the probability of an inefficient H cascade is high. Right at the point of symmetry the negative and positive cascade probabilities are equal. Hence, we observe the inefficient positive cascade probability jumping up and the inefficient negative cascade probability jumping down.

Numerically, examine the case where the signal accuracy of ℓ is just below the signal accuracy of h, $p_h=0.7$ and $p_\ell=0.699$. If the first mover chooses the risky investment, indicating that he has received signal h, the second agent will follow his own signal. He will choose the risky project if he receives signal h and he will choose the safe project if

[8]The less accurate signal has a higher weight in the updating process. Staring out with even *ex ante* probabilities, when an agent receives signal h, he updates his belief that the project value is high from 0.5 to Prob(H|h)=$p_h/(1+p_h-p_\ell)$. When the agent receives signal ℓ, he updates his belief that the project value is low from 0.5 to Prob(L|ℓ)=$p_\ell/(1+p_\ell-p_h)$. As long as $p_\ell<p_h$, Prob(L|ℓ)> Prob(H|h).

he receives signal ℓ. This is because the probability that the true value of the risky project is low conditional on an h signal and then an ℓ signal is greater than 0.5, it is given by:

$$\text{Prob}(L \mid h \; and \; \ell) = \frac{p_\ell \; \text{Prob}(L \mid h)}{p_\ell \text{Prob}(L \mid h) + (1 - p_h)(1 - \text{Prob}(L \mid h))} \approx 0.500474 > 0.5$$

where

$$\text{Prob}(L \mid h) = \frac{(1/2)(1 - p_\ell)}{(1/2)(1 - p_\ell) + (1/2)p_h} \approx 0.3007$$

However if the first agent picks L, the second agent will already herd. He will choose the safe project even if he receives signal h. The probability that the true value of the risky project is high conditional on an ℓ signal and then an h signal is less than 0.5. It is given by:

$$\text{Prob}(H \mid \ell \; and \; h) = \frac{p_h \; \text{Prob}(H \mid \ell)}{p_h \; \text{Prob}(H \mid \ell) + (1 - p_\ell)(1 - \text{Prob}(H \mid \ell))} = 0.4995 < 0.5$$

where

$$\text{Prob}(H \mid \ell) = \frac{(1/2)(1 - p_h)}{(1/2)(1 - p_h) + (1/2)p_\ell} \approx 0.3003$$

When the accuracy of ℓ falls below the accuracy of *h,* the second agent mimics the first mover if the first mover picks L. He does not go against his own signal if the first mover picks H. Therefore the probability of an incorrect L cascade is high.

When the signal accuracy of ℓ is just above the signal accuracy of h, $p_h = 0.7$ and p_ℓ=7.001, we have the opposite situation. If the first mover chooses the safe investment, the second agent will follow his own signal. However if the first agent picks H, the second agent will choose the risky project even if he receives signal ℓ. The probability that the true value of the risky project is high conditional on an h and then an ℓ signal is 0.5004778. A positive information cascade starts right away with the second agent if the first agent receives signal h. Therefore the probability of an incorrect H cascade is high.

At the point of symmetry, the probability of an incorrect H cascade is equal to the probability of an incorrect L cascade. This is why we observe the jumps in Figure 2.

While our primary purpose here is to analyze the *ex ante* probability of an inefficient cascade, it is worth noting that in many markets the interest is in the probability of either inefficient positive or negative cascades. In many situations analyzed using herding models there are important externalities from the market to society at large. Bank panics, capital flight and stock market crashes have external consequences which may induce a social planner to place a greater weight on inefficient negative cascades rather than on inefficient positive cascades. In other markets the party designing the structure of the market may not have an incentive to weigh all market participants equally. In the IPO market, for example, the features of the market are not controlled by a central planner, but rather by the firms offering companies for public sale. These companies may try to increase the probability of an H outcome, whether efficient or not.

3.2. Inefficient Cascade Probability

We have now established the main building blocks for understanding why an increase in signal accuracy can lead to an increase in the probability of inefficient herding. Since the inefficient cascade probability is given by the inefficient positive cascade probability and the inefficient negative cascade probability weighted by the *ex ante* probabilities of state H and state L, the inefficient cascade probability itself may be non-monotonic in signal accuracy when we have uneven *ex ante* probabilities. Here is a new payoff matrix:

Table 2

	Risky Project	Safe Project
High State	2	1/2
Low State	0	1/2

ex ante Prob(High)= 0.25

The expected value from the risky project is still equal to the expected value from the safe project. But now the risky project is riskier than before. If we constrain the accuracy of signal h to be equal to the accuracy of signal ℓ, we still get the same monotonicity result as in BHW with equal negative and positive incorrect cascade probabilities. Now let us fix the signal accuracy of h, but vary the signal accuracy of ℓ. Figure 3 summarizes the simulation results for p_h=0.7. Once again, fixing the accuracy of the h signal at other levels does not change the qualitative results.

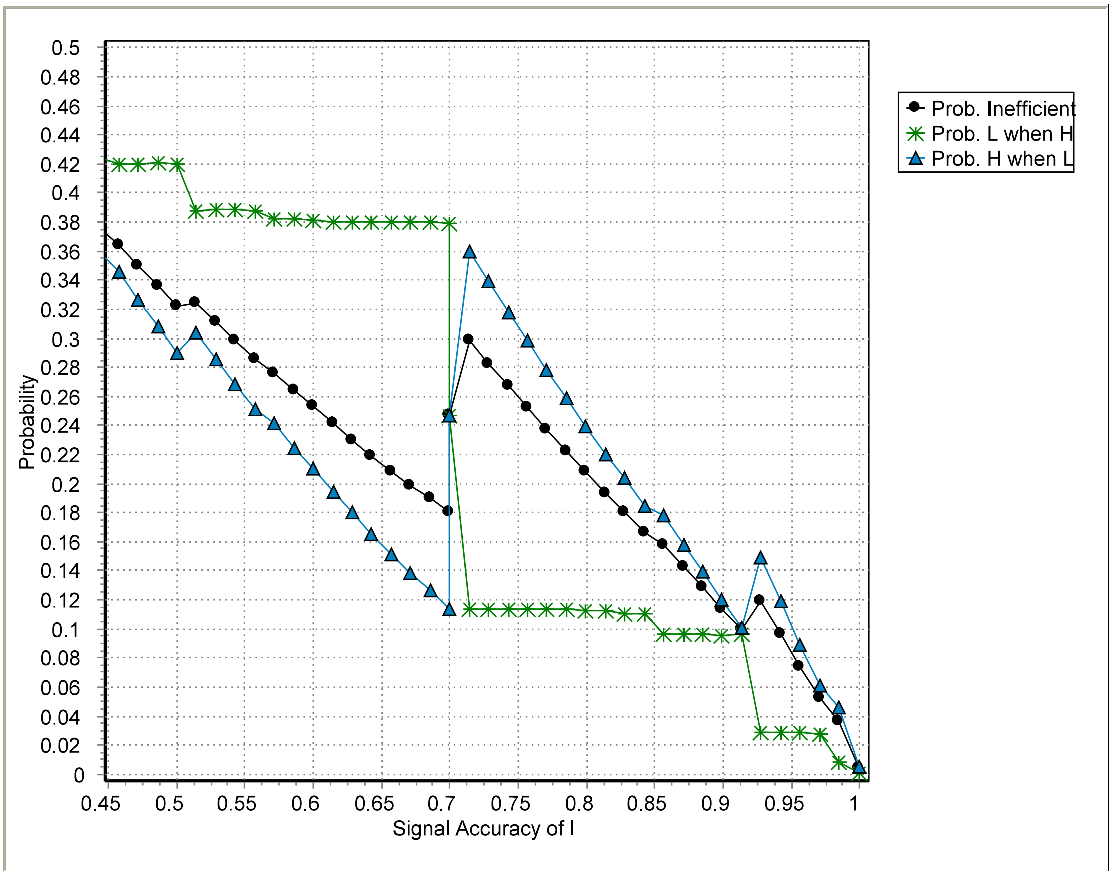

Figure 3

The probability of an inefficient cascade is clearly non-monotonic in signal accuracy. It jumps up at three levels of signal accuracy: At the 0.505 level, at 0.7 (the point of symmetry), and at the 0.9275 level. Before explaining the particularities of these levels of accuracy, let us gain some intuition into the jaggedness of the plots. The nonmonotonicity of the probabilities presents itself as plots with sudden jumps up and down rather than as differentiable graphs. This is due to the binary nature of the problem the agent faces. The agent decides whether to follow his own signal or to go against his own signal. As the signal accuracy improves in a continuous scale the expected value of each of these options changes continuously, but the agent's decision switches from one to the other in a discrete jump.

At the point of symmetry we have the same incentives as in the previous case. However, since the *ex ante* probability of L is now 0.75, the positive cascade probability has a higher weight in the *ex ante* inefficient cascade probability. At the point of symmetry the probability of an inefficient cascade jumps up from 0.18 to 0.30.

Let's now examine the jump at 0.505. Set signal p_ℓ at 0.5 – just below 0.505. Imagine four agents in the following sequence of actions: H,L,H,H. At this level of signal accuracy none of these agents herd. Their actions do reflect their private signals. Having observed this sequence, it is optimal for the fifth agent not to herd. He will follow his own signal. But when we set the accuracy at 0.51 – just above 0.505 – having observed the same sequence (and once again at this level of accuracy the actions of the four agents do reflect their private signals) it is optimal for the fifth agent to herd to H. Therefore, just past the 0.505 level, the probability of an inefficient H cascade jumps up. And the probability of an inefficient L cascade jumps down. The weighted average, the probability of an inefficient cascade, jumps up from 0.32 to 0.335. There are of course many alternative sequences of signals one can observe before herding starts. The discontinuities in the probabilities arise at points where small changes in parameters switch agents in some sequence from one action to the other. The size of the discontinuity is then related to the likelihood of that sequence.[9]

The third jump up in the probability of an inefficient cascade is at the accuracy level 0.927. Imagine the first two agents take the sequence of actions: L,L (with these parameters one could infer their private signals from their actions). The third agent will go with his own signal if the signal accuracy of ℓ is just above 0.927. But he will herd to L if the accuracy is just below 0.927. Hence as the signal accuracy improves from 0.927, the probability of an incorrect L cascade jumps down. At the same time the probability of an incorrect H cascades jumps up. The net effect on the probability of an inefficient cascade is a jump up.

3.3. Changing Both Signal Accuracies

One can also analyze the effect of increasing the accuracy of the signal in both the good and the bad states at the same time. This would be in the same spirit as the analysis performed in BHW where there is a single parameter which represents both signal accuracies, and hence they both changed together. As always, in the symmetric case the effect of an increase in signal accuracy on the probability of an inefficient cascade is negative. However, for an asymmetric model this is not always the case. Figure 4 gives the simulation results for payoff Table 2, where initially there are asymmetric signal accuracies of p_h=0.5 and p_ℓ = 0.8 and then both accuracies are changed together.

[9]This suggests that the results would be stronger in models where there are relatively few pre-herding sequences that arise in practice. This feature is typical of models with exogenous timing. Typically in models with endogenous timing, such as Chamley and Gale (1994), large numbers of agents invest before herding commences so the likelihood of any particular sequence of decisions will be small. Nevertheless, Pastine (2005) shows that similar non-monotonicty results can arise in these frameworks as well.

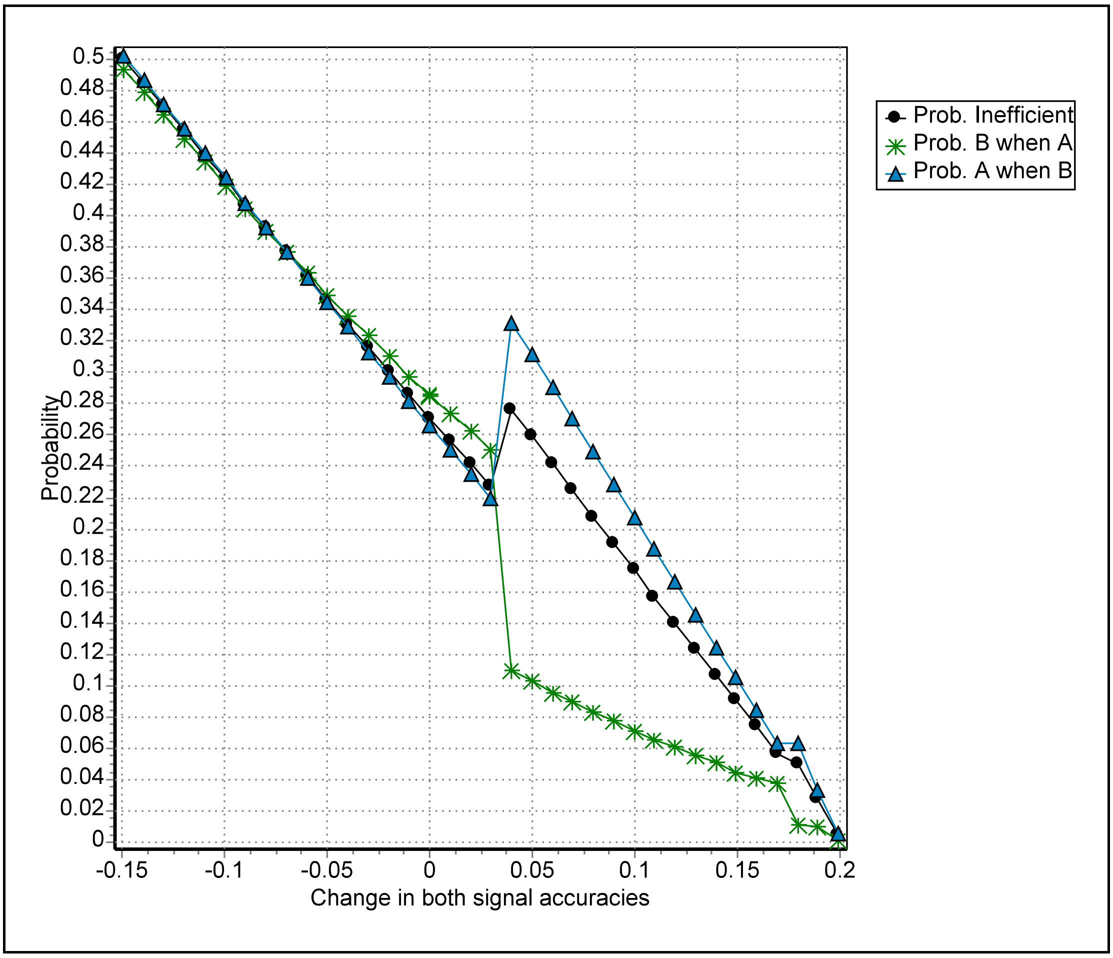

Figure 4

The probability of an inefficient cascade jumps up from 0.23 to 0.28 as the signal accuracy of both signals increase past p_h=0.504 and p_l = 0.804. The possibility of an increase in the probability of an inefficient cascade due to an increase in signal accuracy is not an artifact of increasing the accuracy of only one signal. With asymmetry, when the accuracy of both signals are increased as in BHW, the monotonicity of the probability of an inefficient cascade may break down.

3.4. An Example and Intuition

Let us give an example from the labor market. There is a job vacancy. The safe alternative is to hire an adjunct professor with a payoff of ½. The risky alternative is to hire a tenure-track professor with either a 2 or a 0 payoff (as in Table 2). The candidate for the tenure-track position presents himself in private office meetings to each of the hiring committee members (each committee member draws one signal from the same urn). Committee members then vote sequentially with their hiring decisions.[10]

A good job candidate has a high probability of successfully presenting himself in an office meeting. A bad candidate has a lower probability of successfully presenting himself. Now imagine that graduate schools stop training bad candidates for presentation skills. This new policy leads to a decline in the probability of bad candidates successfully presenting themselves (the accuracy of the bad signal goes up). Nevertheless, this might lead to an increase in the probability of the bad candidate getting the job due to an inefficient positive cascade within the hiring committee. See Figure 3 for p_h fixed and p_ℓ varying. An increase in p_ℓ can lead to an increase in the probability of an inefficient positive cascade (hiring a bad candidate).

There are two forces at work. When schools stop training bad candidates for presentation skills, the informational value of a good presentation goes up. Of course observing a good presentation by a bad candidate is now less likely. But if the first committee member to speak happens to have seen a good presentation, herding may start early since all following voters would put more informational weight on that good report. This leads to the increase in the probability of hiring a bad candidate. The second effect may overwhelm the first depending on the initial levels of signal accuracy.

IV. Conclusion

In a social learning environment, herding may lead society to settle in an inefficient alternative. A social planner would like to reduce the probability of an inefficient outcome, such as the collapse of financial markets, misallocation of pension funds, or the

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[10]See Hung and Plott (2001) for information cascades in sequential voting.

widespread adoption of dubious medical practices. In some cases the designer of the system might be on one side of the market. In an IPO market the seller would be interested in reducing the probability of a negative cascade. A firm introducing a new technology would like to induce a positive cascade, even if the technology is not a superior alternative. It is therefore useful to understand how social policy or private agents may to be able to manipulate herds. This paper is an effort toward learning more about how to influence the probabilistic outcome of social learning.

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