How do students deal with difficulties in mathematics?

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Abstract

We report on a study carried out by the Mathematics Department at the National University of Ireland Maynooth to determine why students do or do not engage with mathematics support. Initial interviews were conducted with students who had failed first year. This paper gives preliminary findings from interviews with a second group of students who had passed first year. Students were chosen who had similar mathematical backgrounds to the first group and who had all engaged with mathematics. The students' mathematical backgrounds do not appear to be the only major factor in determining engagement. We found that both groups experienced similar difficulties and problems. However, the second group had several different strategies or coping mechanisms to enable them to get through. We compare the two groups and will discuss some of these coping mechanisms in detail.

1. Introduction

This paper forms part of a study at National University of Ireland Maynooth (NUIM) into why students do or do not avail of the many mathematical supports in place if they experience difficulties. A previous statistical analysis showed that students who engaged with these supports were more likely to succeed than those who did not [1]. An earlier part of this study focused on a group of students who had not engaged with mathematics. This paper will focus on a group of students who had engaged with mathematics and will compare and contrast these students with the original group. The students have similar mathematical backgrounds and consequently the main aim of this study is to discover the real reasons students do not engage with mathematics.

Other authors have found that the fear of showing a lack of knowledge or ability negatively impacts on students' willingness to ask questions [2]. In [3] we found that factors important in examining non-engagement with mathematics included the demoralising effect of failing first semester examinations, the anonymity of large classes, and to a lesser extent the lack of awareness of support services. Many of these factors were also identified in a study of students at Loughborough University [4]. As a result of these findings [3] we decided to conduct a study of students with similar mathematical backgrounds to the first group but had engaged with mathematics. A preliminary analysis suggests that although these students encounter many of the same difficulties as the group of students who had not engaged, the majority have some plan or strategy to overcome these difficulties. They mention friends and working with peers whereas the non-engagers rarely mentioned these supports. The engaging students appear to be very aware of their own learning style and what works for them. The main focus of this paper will be the varying strategies used by these students.

2. Methodology

Thirty-nine students who were repeating first year mathematics modules were identified in September 2009. These students were contacted and asked to take part in this study. Twelve students agreed to fill out a short questionnaire and seven of these agreed to be interviewed. Coincidentally, the seven students who agreed to do interviews were all students who had not actively engaged with mathematics or mathematics support. The supports consist of small-group tutorials and homeworks which are corrected and handed back in the tutorial every week. In addition, a diagnostic test is administered to all incoming first years. Students who score 20 or less out of a possible 60 marks are deemed to have failed the test and they are then registered for online courses which have been designed to help students with weak mathematical backgrounds. The Department of Mathematics also runs a very successful Mathematics Support Centre (MSC). The first group of students generally had poor attendance at lectures and tutorials, poor submission rates with respect to assignments and had rarely attended the MSC. The group comprised of four male Science students, two female Finance students and one (mature) female Arts student. Mathematics is a compulsory subject in first year for Science and Finance students.

In February 2010, we decided to compare the first group with students who had engaged with mathematics. We considered students who had attended the MSC at least five times and had passed their first year exams. We also split these students into two categories which corresponded to the mathematical background of the first group. The first category comprised of students who had taken Leaving Certificate (LC) higher level mathematics or who had scored an A grade in ordinary level (OL) combined with a pass in the diagnostic test. The second category comprised of students with a LC grade of B or lower in OL mathematics and a fail on the diagnostic test. The Leaving Certificate is the final exam at the end of second level education in Ireland, all students take mathematics. Ten students were contacted, nine students responded and were interviewed. They also filled in a short questionnaire. They generally had good attendance at lectures and tutorials and had submitted the majority of assignments. The group consisted of two male Arts students (one a mature student), two female Arts students, two male Science students (one a mature student) and three female Science students.

The interviews were conducted by the first author. Each interview lasted for approximately forty minutes. The questions were open-ended and concerned the student's mathematical education prior to enrolling at NUIM as well as their experiences of mathematics in the first year of their degree. They were questioned on their experiences of lectures, tutorials, assignments and the MSC. The interviews were the same for both groups but with the addition of a detailed section on the services provided by the MSC for the second group.

The interviews were transcribed by the first author. All three authors coded the transcriptions using Grounded Theory [5] and the codes were then compared. Pseudonyms were used to protect the students' identities.

3. Results

A preliminary analysis of the data has been carried out to date. It was apparent from the initial analysis of the first group [3] that they had almost exclusively not engaged with mathematics. Analysis showed that students were often not aware they had a problem or were unwilling to admit it (to themselves or others) until it was too late. Students were also reluctant to ask for help and feared embarrassment. Mathematical background is an important factor in determining grades but some students from our first group had a good LC grade and had passed the diagnostic test. We would not have labelled them as "at-risk" students. However, it was apparent that other factors also played a major part. Subsequently we will present some of the other possible reasons why the second group engaged and succeeded with mathematics.

3.1 Similarities

It was apparent that both groups encounter similar difficulties and problems. The main category to emerge from the coding of the first group of interviews was that of fear or embarrassment. A more detailed analysis can be found in [3]. The fear category appeared to comprise of four separate but overlapping concepts, fear of failure, fear of the unknown, fear of being singled out and fear of showing a lack of knowledge or ability. Here, Jonathan from our first group discusses the MSC, an example of fear of showing a lack of knowledge or ability:

"I was actually really embarrassed and intimidated about going and saying listen guys I struggle horribly with maths."

Similarly, Janice from our second group shows the same kind of fear when discussing asking a lecturer a question:

"You'd be afraid to go to a lecturer, they're doing this for ages and they've got their PhDs…and you're going in and asking them about the domain and range…they're looking at you as if to say, "Ah come on now, we did this in the first lecture!"

3.2 Differences and Coping Mechanisms

Similarities amongst both groups are to be expected as both groups had similar mathematical backgrounds. Similarly, we would expect differences because students in the second group engaged and succeeded at mathematics.

We will outline how the attitudes and experiences of the first and second groups contrasted. This will give us some insight into the different mindset of the two groups. Then we will outline the main strategies or coping mechanisms that emerged from the analysis of the second group.

Two basic issues on which the groups differed were awareness of supports and attitudes towards supports. Interestingly all the students in our second group were much more aware of the supports available to them from the college and their peers. Students from our second group discuss their awareness of the MSC:

"Ciarán (MSC manager) came in…our first or second lecture and told us about it…the opening hours and I took them down straight away and then after the first week I said I had better go" (Janice)

"They were telling us the MSC was there before you actually started the course" (Susan)

Students in the first group did not seem to be as aware of the MSC. It may be the case that these students were already not engaging at the start of semester one. Craig from our first group speaking about the MSC drop-in centre:

"I was vaguely aware of it…I thought it was an extra class…I would have thought it was one hundred students just being lectured again"

Another interesting difference is in the attitude of our two groups. The first group are more negative towards certain aspects of mathematical support. To explain, we can see how students in both groups interpret the marking system of assignments. Students submit five questions and one is corrected at random. A typical response from the first group:

"Just randomly picking just one question and us not knowing is unfair" (Bob)

Whereas a student from our second group can see positives in this system:

"It is good that it shows you that you have to do every question, it makes you do the whole lot of them cause you might not do one and you get zero for it." (Liz)

Overall it is quite clear that the two groups have contrasting attitudes on several issues. The main difference is that the second group all seem to have some plan or strategy whereas our study of the first group revealed that the majority of them did not seem to have a coherent strategy in their approach to engaging with mathematics.

We will now give a breakdown of the main coping mechanisms that have emerged from the analysis to date. The first type of strategy is exemplified by three of our students, David, Joe and Andrew, who are all mature students. That strategy is to avail of almost all possible supports. Here Joe is asked about why he attends an extra tutorial available to his class group:

"I was coming in with such a low level. I was just gonna give myself every chance. Anything that was going I was gonna use it"

Interestingly, Andrew talks about how he knows when he had enough resources or supports to get him through. He does not allow himself to be overwhelmed and chooses to not avail of some supports. Here he discusses an online refresher course that covers basic topics:

> *"I did use that…it refreshed all the basic rules…for someone like me who had had a break from it (mathematics) for a long time…Towards the end of the semester, when we were a lot busier I found it more of an inconvenience than a help"*

Perhaps the reason for the mature attitude these students have to engagement is due to the fact that they are older and more experienced than a student who has recently left school. David discusses the possible advantage of being a little older coming to university when asked about asking questions in lectures:

> *"…maybe not in lectures cause I think people feel it's a bit more intimidating cause they can say, "…am I the only one thinking this?". But I do think being a little older has helped…I think maybe if I was 18, I would have been a bit more reluctant"*

What was also apparent was that some of our students favoured indirect strategies over direct strategies such as going to the MSC with a friend and allowing the friend or group of friends to ask the questions while they absorb the information. In addition to this students placed high value on the support of their peers. What is apparent is that they seek help from the sources they are comfortable with. It is also interesting that these students had similar fear and embarrassment issues to the first group but with this indirect approach they managed to circumvent it and get help. Andrea contrasts one-to-one help and group work in the MSC:

> *"If I ever went over on my own I hated it…felt kind of stupid, even though everyone's really nice…when I was in a group it was brilliant cause everyone would work out different pieces or something. I'd usually go with 4 or 5 people…when the tutor came over, they'd just explain it to everyone"*

The final type of strategy is a realisation that it was not absolutely necessary for them to attend fully or to understand fully all of the material to progress. This came as a surprise to us considering these students had engaged and succeeded with mathematics. This strategy was adopted by some of our group but by Shaun in particular. Shaun repeatedly expressed a preference for one-to-one tuition. He initially attended all his classes but soon began to realise that he was not gaining any extra understanding through that:

> *"I just realised all I was getting out of the lectures was notes and all I was ever doing was looking over the notes when I came out of it and I got the same understanding of it"*

Shaun instead would get the notes from a friend, work on them himself and then go to the MSC and ask for help there. On the issue of understanding Shaun talks about how he feels he has not enough time to go back and correct his understanding. He realises he does not need a full understanding to get through the year though. Here he talks about why some of his peers are failing and he is progressing he says:

> *"I find that a lot of people that are failing, they are going to the lectures and trying their best but they're just not 'smart studying', exam studying"*

It is clear that students who engage do not have a uniform strategy. Several different strategies have emerged and some of those were unexpected. Investigations into these are ongoing.

4. Further Work

We have already designed interventions as a result of our findings from the first group. These interventions were trialled in 2009/10 and are in operation again this year. They include a mentoring programme where students are offered the chance to come and talk to a sympathetic faculty member. We are also monitoring engagement and intervening with students who do not engage. We also designed a document for incoming first years explaining the differences between mathematics at second level and third level and the structures and supports in place to help them through.

We are also analysing 470 questionnaires completed by first year mathematics students last year. Students filled out a questionnaire at the beginning of semester one and the end of semester two. The questionnaire contained sections on confidence in one's mathematical ability, their perception of how useful mathematics was and also a section on study methods. We hope that with further analysis of both our transcripts and these questionnaires that we can design and implement more interventions that will help with the problem of student engagement.

5. Conclusion

From the preliminary research it is apparent that mathematical background is not the only major factor in predicting whether a student will engage and succeed with mathematics. From our analysis of the second group there is clearly some motivation, some desire that pushes them through and forces them to engage to some extent. We hope that further coding and analysis in conjunction with re-examining the first group will reveal exactly what this motivation is.

References

1. Mac an Bhaird, C., Morgan, T. and O'Shea, A. (2009) The impact of the mathematics support centre on the grades of first year students at the National University of Ireland Maynooth. *Teaching Mathematics and its Applications*, Vol. 28 (No. 3): 117-122.

2. Ryan, A.M., Pintrich, P.R., and Midgley, C. (2001) Avoiding seeking help in the classroom: Who and why? *Educational Psychological Review*, Vol. 13 (No. 2): 93-114.

3. Grehan, M., Mac an Bhaird, C. and O'Shea, A. (2010) Why do students not avail of mathematics support? A case study of first year students at the National University of Ireland Maynooth. *Proceedings of the British Congress for Mathematics Education (BCME) 2010*, Vol. 30 (No. 1): 254-258.

4. Symonds, R., Lawson, D., and Robinson, C. (2008) Promoting student engagement with mathematics support. *Teaching Mathematics and its Applications*, Vol. 27 (No. 3): 140-149.

5. Strauss, A. and Corbin, J. (1998) *Basics of Qualitative Research: Techniques and Procedures for Developing Grounded Theory*. Sage Publications: London.

Assessment for learning: Using Moodle quizzes in mathematics

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Abstract

The introduction of online, interactive computer marked assessments within a distance taught foundational mathematics module is discussed. The aim of the assessments is to encourage and support learning. The style of assessment used is described together with details of their development and how they are integrated within the module teaching strategy.

Data showing how the assessments were used by students on the first presentation of the module are given, together with some student feedback and issues encountered. Although the uptake of these assessments was not as great as hoped, the student experience has been generally positive.

1. Introduction

The Open University (OU) is the UK's leading provider of distance education, with students generally studying at home using provided resources and being supported by an allocated part-time tutor. In February 2010 the OU launched a new foundational mathematics module *MU123: Discovering Mathematics* to replace the previous offering. This 30 credit module introduces topics such as basic algebra, geometry, trigonometry, exponentials and statistics. It is offered twice per year with approximately 1,600 students per presentation.

A blended teaching approach is taken, with printed texts supplemented by interactive resources on the University's Virtual Learning Environment (VLE). The OU VLE is based on a customised version of the open source Moodle system [1]. The online resources include short video screencasts explaining some of the examples in the texts; interactive applications for investigating statistics, graphs and geometry; and extensive online assessments, both formative and summative.

In this paper, the rationale for including these online assessments is described, together with the particular style of assessment used and their development. Data demonstrating student usage during the first presentation of the module is presented, along with unsolicited comments from student discussion forums.

2. Online interactive computer marked assignments

2.1 Rationale

The main form of assessment within mathematics at the OU is the tutor marked assignment (TMA). A student (typically) hand writes solutions to posed problems which are then posted directly to their tutor for marking. The marking usually includes extensive individualised feedback and teaching comments. The script is returned to the student by post, via the University. Whilst providing excellent feedback and teaching, the delays inherent in this system can reduce the effectiveness of that feedback [2]. The use of electronic submission techniques, which have been adopted by much of the University and trialled in mathematics [3], can reduce these delays, yet teaching by this method still falls short of an interactive dialogue.

In many modules, TMAs have been supplemented with computer marked assignments (CMAs) where all students are given the same questions, answers are entered via a web-site and results are given after a submission deadline. Again, this does not permit immediate feedback to the student. Such feedback would be particularly valuable to distance learning students as it enables them to swiftly highlight any misunderstanding or misconceptions before they become too ingrained.

There are a variety of systems available that can support such interactions, both commercial and open source. In addition, many of these systems support the generation of randomised variants of individual questions, which both enables the student to practise as many examples as they feel they need, and allows different students to be given different questions to reduce opportunity for plagiarism in summative assignments. In recent years, the OU has started to make use of a number of these systems, in particular OpenMark [4], a system developed at the OU and now available as open source software, and the quiz system of the Moodle VLE, which is currently also maintained by the OU.

The aims of introducing such VLE-based assessments in *Discovering Mathematics* were to enable more effective learning by providing students with immediate feedback (including specific feedback to common errors) and providing students with sufficient opportunity to practise.

2.2 Implementation

A formative "Practice Quiz" was developed for each of the fourteen study units of the module. These are designed to enable students to check and consolidate their understanding of the content of the unit as they progress. To help students engage with the learning of mathematics as they use these quizzes, they are permitted up to three attempts at each question. After each incorrect attempt the student is given a graduated hint and in many cases a specific comment appropriate to their incorrect answer. The first hint given is a reference to an appropriate location in the associated study text, the second being more detailed guidance on how to attempt the question, possibly reminding them of a formula or technique that is required. After the final attempt, or indeed once the correct answer has been given, a full worked solution to the question is provided. This type of question behaviour is known as "Interactive with multiple tries" and has been developed within the Moodle quiz engine by the OU. It will be made available to the Moodle community as part of Moodle version 2.1.

A variety of different question types are used within the Practice Quizzes, including

- multiple choice: where one answer is to be selected from several possibilities;
- multiple response: where several answers are to be selected from a list of options;
- matching: where several answers are to be selected from a set of drop-down lists;
- numerical: where free-form numerical input is matched against target ranges;
- drag-and-drop: where items of text need to be selected and moved on the screen to their correct location within a statement or sentence;
- short answer: where free-form text is matched against answer templates.

The final type is mainly used for the input of fractions in a linear syntax.

All the questions offered are randomised, so that a student can attempt each quiz more than once (for practice and reinforcement), and will (probably) get a different set of questions each time. Currently, the OU VLE is unable to generate randomised questions on-the-fly, but does support the selection of a random question from a pre-generated pool. This randomised selection is used within the Practice Quizzes. Indeed, this manner of operation has quality control advantages. Each possible variant of a question can be reviewed before they are released to students, to ensure no degeneracy, special case or otherwise unwanted element of the question was accidentally introduced by the randomisation. In general there are between 5 and 20 variants of each question. The generation of these questions was facilitated by the use of a tool developed by one of the authors of this paper. The tool is described in the following section.

Alongside the Practice Quizzes are five summative computer assignments (denoted in the module as iCMAs: interactive computer marked assignments) which generally cover more than one study unit. These supplement the five TMAs of the module, and constitute 12% of the final module grade. The summative assignments aim to encourage students to make use of the Practice Quizzes: students are informed that the questions which appear in the iCMAs will be of a similar style to those of the Practice Quizzes, and encouraged to use the Practice Quizzes to familiarise themselves with the mathematical material, style of questions and the assessment system itself before taking a summative assessment.

The iCMAs also increase the number of points within the module at which a student must submit an assignment. This is aimed at helping students keep to a reasonable study timetable throughout the module. The iCMAs focus on small calculations, methods or the testing of definitions and concepts, thus allowing TMAs to concentrate on assessing problems requiring longer arguments, or skills that are less easy to assess automatically.

The feedback on summative iCMAs is delayed until after the assessment cut-off date, and students are only permitted one attempt at each. This model of operation was chosen for this module, but is not required by the VLE. Several other OU modules use a model closer to the Practice Quizzes where the student loses one third of the available marks for a question for each incorrect attempt. Questions within iCMAs are also randomised, and care was taken to ensure all the variants of each question are of equal difficulty. Randomising the questions in this way enables the same assessment questions to be used from one presentation to the next, saving staff costs in the long term.

The questions themselves, for both Practice Quizzes and iCMAs, were suggested by those preparing the corresponding study text of the module, then modified to fit the assessment system and implemented by the authors. During the development process the questions were subjected to extensive testing, both reviewing hard-copy of each of the possible question variants, and online testing of draft questions by a number of expert volunteers. The volunteers also tested for readability and accessibility across a range of computers and browsers.

In total, approximately 2500 questions were generated in support of this module. An example question showing the hint given after a second incorrect attempt and specific feedback to a common error is shown in Figure 1.

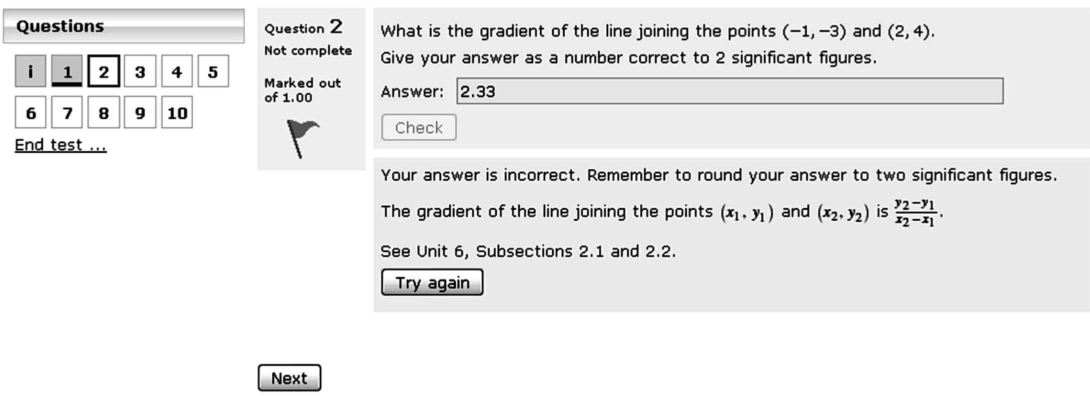

Figure 1: An example numerical question showing feedback to an incorrect response.

Links to access each Practice Quiz and iCMA are embedded in the Study Calendar on the module VLE site.

All the attempts at Practice Quizzes and summative iCMAs are recorded by the VLE and are available to a student's own tutor. In this way a tutor can, if they so wish, monitor a student's progress on the module and be proactive in offering advice and support.

2.3 iCMAtool

A program called iCMAtool was developed to generate variants of each question. The person writing each question uses an offline editor to produce a question template that consists mainly of standard Latex markup but also contains a code section that defines the variable parts of the template. (Here, Maple was used to process this section, but the other options are available.) The template is processed by iCMAtool to produce several outputs. One output is a Latex file of questions that enables offline reviewing of every question variant. Another output is a Moodle Quiz XML file [5] file for uploading to the VLE. A third output is a printable PDF booklet that can be sent to students without internet access.

An example Latex question template is shown in Figure 2.

```
\begin{numerical}
 \title{Simple multiplication}
\begin{code}
 a=op(N,[2,3,4,5,6])
 ans=2*a
\end{code}
\begin{questiontext}
What is $2\times \var{a}$?
\end{questiontext}
\begin{answer}[100]
\val{\var{ans}}
  \fb{$2\times \var{a}=\var{ans}$.}
\end{answer}
\end{numerical}
```

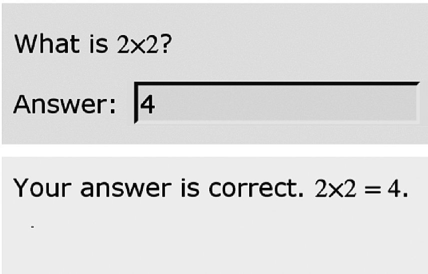

Figure 2: An example Latex question template, together with one of the questions generated.

3. Student usage and feedback

The marks obtained by students on each of the 5 summative assessments (iCMAs) and 14 practice assessments (PQs) are shown in Figure 3. Note that only scores where the student has explicitly finished the assessment by "submitting" their answers are included, and for each quiz only the best mark obtained by each student from their (possibly) multiple attempts has been included. It can be seen that the marks achieved are high: these assessment were viewed as trying to encourage and help the students with their study rather than discriminate among them, which was considered to be the function of the TMAs. As expected, the marks for the summative assignments decrease as the module progresses and the material becomes more difficult.

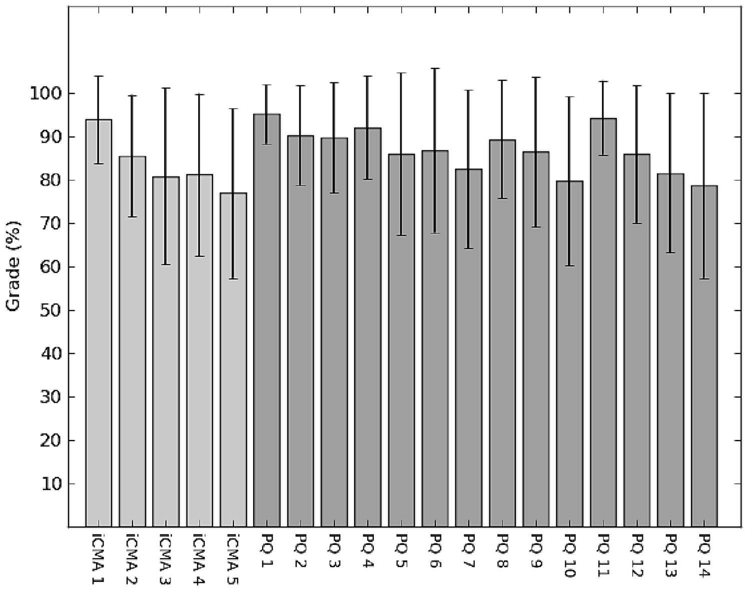

Figure 3: The mean mark obtained on each summative and practice assessment, and the standard deviation.

The number of attempts at each assessment is shown in Figure 4, which differentiates between the first and subsequent attempts of Practice Quizzes and those assessments which were not formally completed by "submitting" the assignment. The number of students registered at the start of this presentation of the module was 1561. As usual, the number of attempts at iCMAs decreases reflecting a retention rate that is typical for a module at this level with no entry requirements. For each summative iCMA there are also a number of students who did not "submit" their attempt, and hence the result would not be counted towards their final module result.

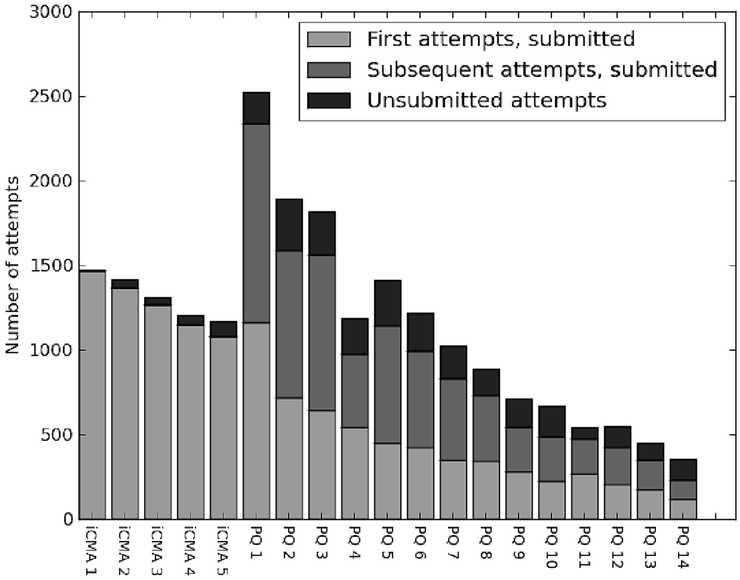

Figure 4: The number of attempts at each summative and practice assessment.

The number of individual students making use of the Practice Quizzes (as indicated by the number of first attempts) disappointingly decays rapidly over time. It is interesting to note that even at the start of the module, not all students attempted the first quiz. However, there were a large number of repeat attempts. The number of repeat attempts also generally decreases over time, although there is a marked increase on quiz 5, which corresponds to the point at which algebra is introduced.

Figure 5 charts the total number of students who made one or more attempts at each practice quiz. This also declines rapidly indicating that, perhaps, only 10 variants of each question are needed to satisfy the demand of students repeating the quiz. However, the chart also demonstrates that one student attempted one practice quiz 38 times.

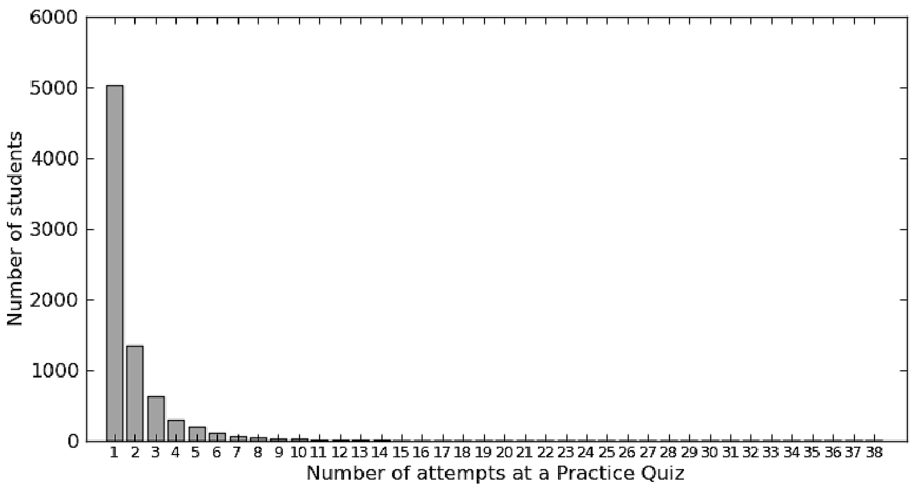

Figure 5: The number of students repeating attempts at Practice Quizzes.

Even though the quizzes were underutilised most of the comments made on the module online discussion forums were positive, for example:

> "The practice quizzes are a great way to build up knowledge."

"It's a great way to consolidate."

"I already submitted my iCMA ,I found that the practice quiz helped a lot."

"I like the practice quizzes, you can do lots of them until (hopefully) the lights come on."

"When I've finished a Unit I do the Practice Quiz over and over again until I get a really high score."

"I had a lot of trouble with this Unit too but found that if I kept doing the practice quizzes and when I got an answer wrong, working through the feedback on that question, it was a great help. I think I did the Practice Quizes about 8 times before I felt reasonably happy with It:)"

Some comments were more negative, reflecting the need to be online to attempt the quizzes and the low weighting given by the module assessment strategy:

"The practise quizzes are great - just wish I could do them when I'm sat under a tree in the forest waiting for my little girl to finish school."

"I don't enjoy the icma's at all and always do much better on the tma's. . . . I just think that they account for such a small percentage of your final mark that they really aren't worth worrying about."

4. Conclusions

Although the uptake of these assessments was less than hoped for, the student experience has been generally positive. The changes introduced in *Discovering Mathematics* have led to the module having a higher retention rate than its predecessor, although it is hard to attribute this to any specific change made.

There were a small number of technical issues relating to the online assessments that arose during this first presentation of the module. Some students were unable to view some of the font characters used within some questions, due to the use of old versions of web browsers. Some students also included superfluous text within answers to numerical questions (such as details of the rounding used), despite the quiz introduction and the individual questions giving instructions to the contrary. This led to some correct answers being marked as wrong. It is hoped this can be addressed with planned future improvements to the question types available. The range of question types currently available within the University also precludes the use of free-form algebraic text entry, but it is hope this will be addressed by the adoption of STACK [6].

In addition, there are certain groups of students who do not have internet access (for example, offender learners and submariners) hence who cannot access the quizzes. The ability to generate printed booklets from the Latex question source enables some support to be given to these groups, but alternatives are also under consideration.

The seeming success of using these assessments to help students learn is leading to their incorporation into further courses that are being developed.

References

1. http://moodle.org (accessed 28 January 2011).

2. Gibbs, G. and Simpson, C. (2004) Conditions under which assessment supports students' learning. *Learning and Teaching in Higher Education*, Vol. 1: 3-31.

3. Jordan, C., Arrowsmith, G., Lowe, T. and Mestel, B. (2010) Electronic marking in mathematics – the marker and student perspectives. *MSOR Connections* , Vol. 10 (No. 1): 43-47.

4. Butcher, P.G. (2006) "OpenMark Examples". http://www.open.ac.uk/openmarkexamples (accessed 28 January 2011).

5. Moodle XML format. http://docs.moodle.org/en/Moodle_XML_format (accessed 28 January 2011).

6. Sangwin, C.J. (2007). Assessing Elementary Algebra with STACK. International Journal of Mathematical Education in Science and Technology, Vol. 38 (No. 8): 987-1002.

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ASPs: Snakes or Ladders for Mathematics?

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Abstract

We review our experience at the University of Portsmouth over the past academic year, following a shift from home-grown to mass-produced electronic resources from an educational publisher, WileyPLUS. We present the results of a survey on the way mathematics students taking calculus units in their 1st and 2nd year viewed the shift towards WileyPLUS, and the opinions of staff involved in tutoring and supporting them. We discuss whether a personalised approach to teaching and learning can be maintained in a world of global education. Should the shift towards WileyPLUS become total, remain partial or simply be reversed? To what extent should we continue to integrate other resources, particularly assessment questions (e.g. from MapleTA, WebCT/Blackboard or PRS) with WileyPLUS?

Several other publishers of mathematics textbooks are providing comprehensive packages of interactive resources online. Application Services Providers (ASPs) create, store and deliver from their own server. Their resources include an electronic version of the textbook or e-book, supplementary materials, worked solutions, study guides, applets, formative assessment with extensive feedback and even summative assessment. Lecturers can customise these materials, most significantly by choosing questions for self-assessment tests and exams. Inevitably there are constraints on the extent to which they can modify or add to these materials.

Have ASPs improved the teaching and learning in our large classes of around 150 students? What about smaller classes? The stark choice between open source collaboration and commercial provision often polarises individual academics and even whole institutions. We argue that the ideal is a financially sound, hybrid model, which allows for greater customisation, sustainability, extension and interoperability than is currently available from either commercial providers or open source initiatives.

1. Introduction

Cleopatra, the last Pharoah of Ancient Egypt, allegedly killed herself by means of an asp bite on August 12th 30 BC. Over two thousand years later our question is whether the adoption of commercial teaching and learning software products from Applications Services Providers (ASPs) is a suicidal step. Academics cherish their independence of thought and individuality of their teaching, but are they being killed off by the emergence of integrated resources which seem to offer a complete on-line learning package?

In some ways the situation is not new. Lecturers have always had to make decisions about the adoption of standard mathematics textbooks, which chapters to include and how rigidly to follow the printed page. Some write their own in-house notes and ignore textbooks other than for reference or further reading; others develop supplementary materials such as their own notes to provide a digestible overview or worksheets to provide more problems and examples.

While the mathematics textbook has remained the mainstay of learning, some lecturers have invested major effort in developing their own e-teaching, e-learning or e-assessment resources to support their courses. Funded projects, both large and small, emerged in which academics collaborated on the production of electronic resources with or without associated paper resources. The sums of money spent on these projects have often been significant and the products variable in their impact and longevity. A major problem has always been the maintenance and ongoing development of resources, especially electronic resources, when the funding has dried up. The commercialisation of mathematical resources, which started up on public money, is unusual and the majority of "cottage industries" are doomed to long-term obsolescence. It is left to the goodwill and altruism of individual academics, who rarely have the time necessary for production and maintenance of high quality online resources.

Commercial publishers sometimes release new editions of mathematics textbooks every year and often every two or three years. Lecturers who adopt the textbooks are, in some sense, handing over control of their academic content and could even be regarded as committing intellectual suicide. Control can be maintained relatively easily by providing additional printed material to breathe life into a course. As commercial publishers develop increasingly sophisticated online resources to support or even replace textbooks, is the control of content being lost further? Do academics need to adapt their teaching again? ASPs create and store interactive resources for online delivery from their own server. Their commercial products are available from the publishers of several prominent mathematics textbooks: MyMathLab for Pearson (http://www.mymathlab.com), WebAssign for Thompson Learning/Cengage (http://www.webassign.net), MathPortal for W.H. Freeman and WileyPLUS for Wiley (http://wileyplus.com). Are ASPs to be regarded as a threat or an opportunity, snakes or ladders? We present our own views and those of our students based upon our initial experience of WileyPLUS.

2. Context: Staff and Students

The number of mathematics students at the University of Portsmouth increased dramatically between 2002 and 2010 with a greater than five-fold increase from under 30 to over 160 entering each year (Figure 1). In 2010/11 the total number of students exceeded 400 for the first time and the increase in total numbers is set to continue. During this period there were negligible increases in staff numbers and significant changes in staffing due to retirements. Mathematics often achieved top-4 National Student Survey and league table positions for both teaching and applied mathematics research. The entry grades of students also improved as the reputation of the department grew. We developed substantial use of Question Mark Perception for assessment during this period of growth in order to maintain the quality of teaching and learning for increasing numbers.

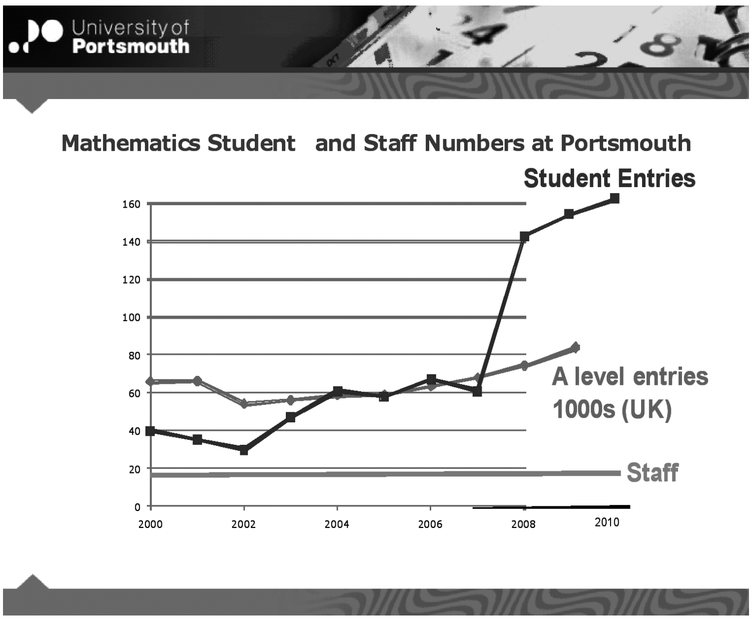

Figure 1: The Growth of Mathematics at the University of Portsmouth

3. Case Study: WileyPLUS

It is common for lecturers to have limited time to prepare printed resources for delivery of a new course unit/ module. It is rare to have enough preparation time for the development of e-learning resources beyond a basic VLE "presence" by uploading notes and presentations. We describe a case study in which there was just two weeks to prepare for the delivery of a first year, second semester, 20-credit unit covering "further calculus and linear algebra" to over 150 students.

Existing resources included two recommended texts from different publishers, written lecture notes, the set of Question Mark Perception e-assessments (see above), in-class question banks for use with "clickers" (PRS handsets) and a minimal presence in the University "Victory" VLE. Weekly delivery was via two 1-hour lectures, one 2-hour "clicker" exercise classes, a 1-hour small group personal tutorial and a 1-hour assessment period during which seven tests were to be completed. Students were allowed to repeat tests to raise their marks, but credit could not be carried over and the same question, answered correctly at the first attempt, might well have to be repeated during subsequent attempts. Random parameters or algorithms were not available and the QM Perception questions developed over ten years earlier for formative and summative assessment were looking outdated and mathematically limited (Figure 2).

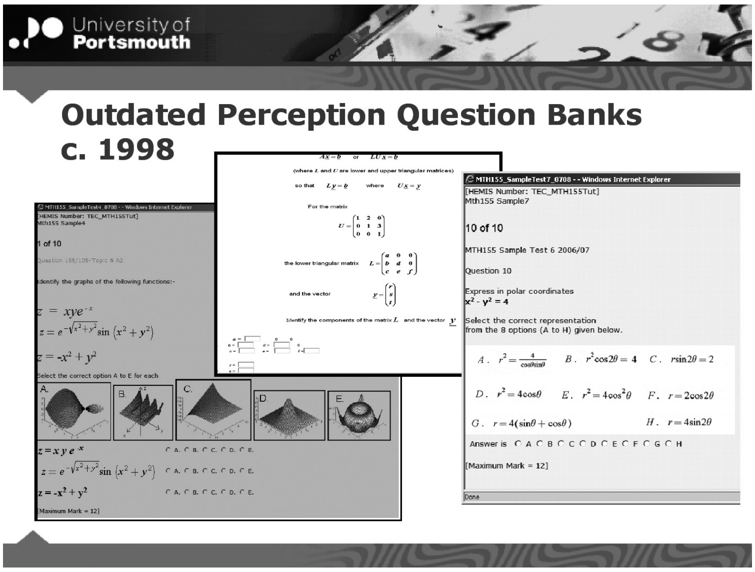

Figure 2: E-assessment Questions in Need of Replacement

With more time the questions could have been updated in QM Perception to eliminate some of the problems or converted to MapleTA to overcome the mathematical limitations. A provisional decision to introduce MyMathLab was switched to WileyPLUS for several reasons:

- it supported one of our adopted calculus textbooks [1]
- its underlying assessment engine was MapleTA (http://www.maplesoft.com/products/mapleta) which we already used
- it provided a comprehensive bank of online assessment questions including feedback and random-algorithms
- there was local Wiley support provided at short notice to set up WileyPLUS access for road testing and live delivery, to deal with technical, especially assessment, issues that arose and to introduce the product to both tutorial staff and students.

The old QM Perception e-assessment provided our "safety net" in the event of problems and the decision was taken to adopt WileyPLUS [2] immediately as a "live pilot". For the trial semester, WileyPLUS was provided free-of-charge, since students had already purchased the associated textbook. There were many advantages of using WileyPLUS:

- a complete electronic copy of the text was available to all student regardless of whether they had purchased a hard copy
- large online question banks with the underlying MapleTA/MapleNet engine
- graded feedback with links from assessments to hints, solutions, tutorials and the book itself.
- learning design underpinning its overall structure
- presentations for lectures including PRS "clicker" questions and summaries
- some applets for interactive activities
- instructor guidance and resources
- extra question banks for use within our "Victory" WebCT VLE (Figure 3)

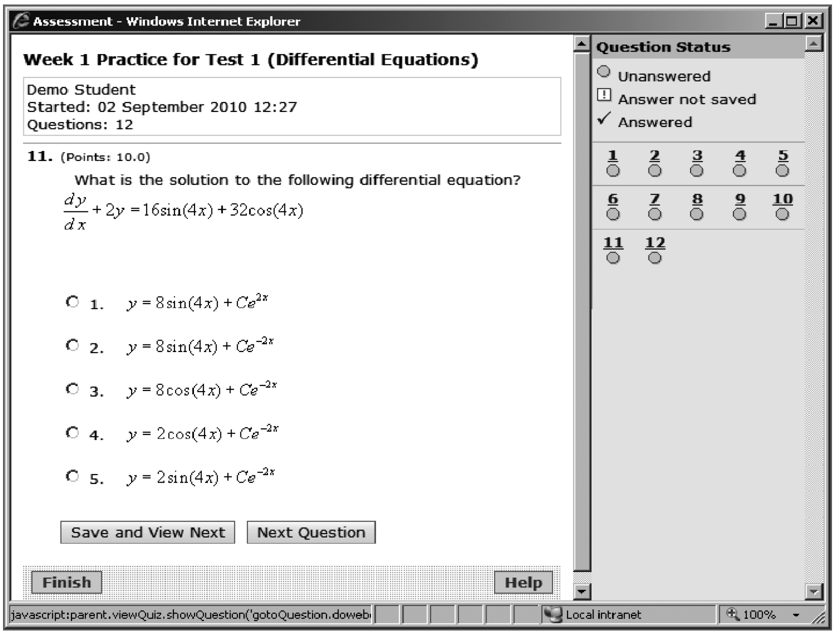

Figure 3: E-Assessment Questions Exported from WileyPLUS into WebCT

The graded feedback (Figure 4) is a particularly strong feature, since it allows progressively more detailed help to be given after each unsuccessful attempt at a question.

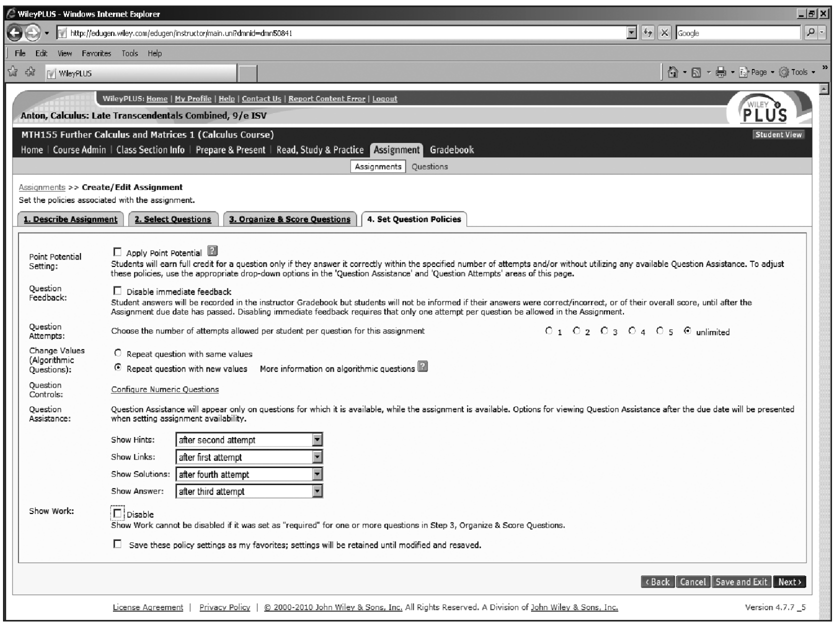

Figure 4: Graded Feedback Options in WileyPLUS

There were also disadvantages in using WileyPLUS (Version 4.7.8) :

- the current product is designed for learning not exams, with an emphasis on formative, not summative assessment
- textbook-based lecture slides which look formulaic and uninteresting
- major limitations on question customisation and in-house authoring
- an inability to import existing MapleTA or QML questions
- an inability to correct errors or modify content
- an inability to hide/show content or perform basic VLE operations to modify the display, e.g. for accessibility

When there were mistakes in e-assessment questions, it was only possible to report the error and avoid using them. The response time for having such corrections made was too long.

Both WileyPLUS and local resources, including MapleTA assessments, were linked through the VLE, which acted as a one-stop shop for the course unit. Formative WileyPLUS assessments (http://wileyplus.com) included:

- standard practice tests for all sub-topics covered in the text
- custom practice tests generated by selecting questions from the bank and setting up appropriate delivery and feedback options
- question selection by difficulty, learning objective or type
- varied answer input, including interactive graphs
- unlimited attempts on algorithmic-randomised questions

Formative MapleTA assessments were authored in-house and are available free of charge for users of MapleTA at http://userweb.port.ac.uk/~mccabeem/mapleta. MapleTA does not include some WileyPLUS features, such as interactive graphical questions (Figure 5) and a smart symbolic input palette (Figures 6 and 7), but this drawback can be weighed against the benefits of greater flexibility and control over question authoring, modification and delivery.

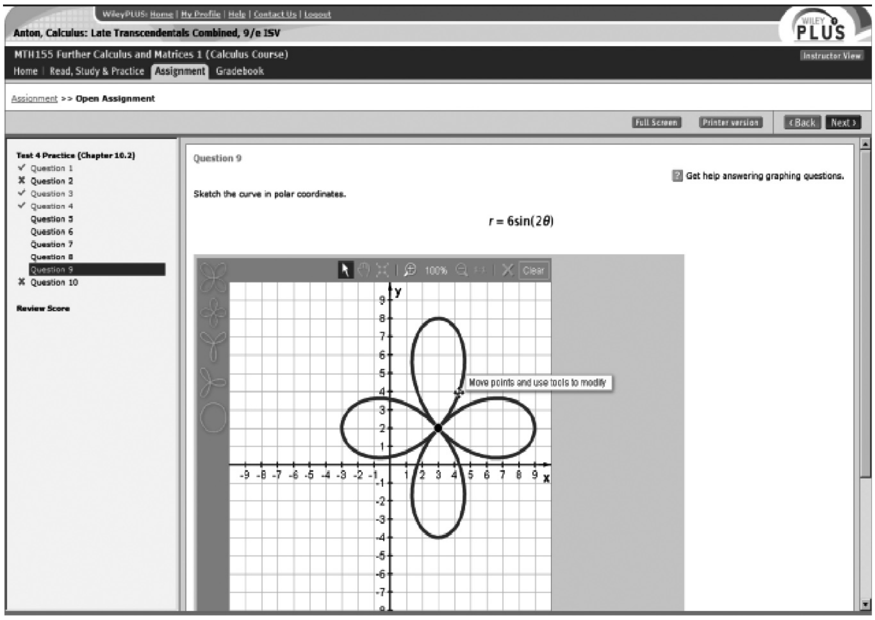

Figure 5: Graphical Input in WileyPLUS

Since a large bank of MCQ and numeric formative assessment questions were available to students within WebCT, they were required to deal with three different interfaces for answering questions. The lack of interoperability

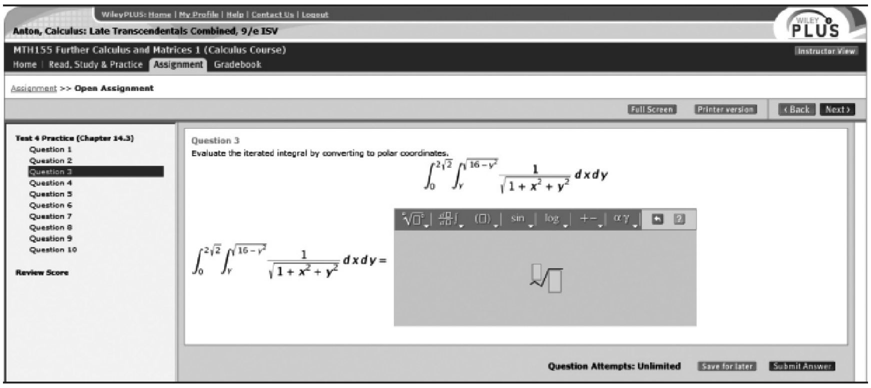

Figure 6: Symbolic Input in WileyPLUS using a Palette

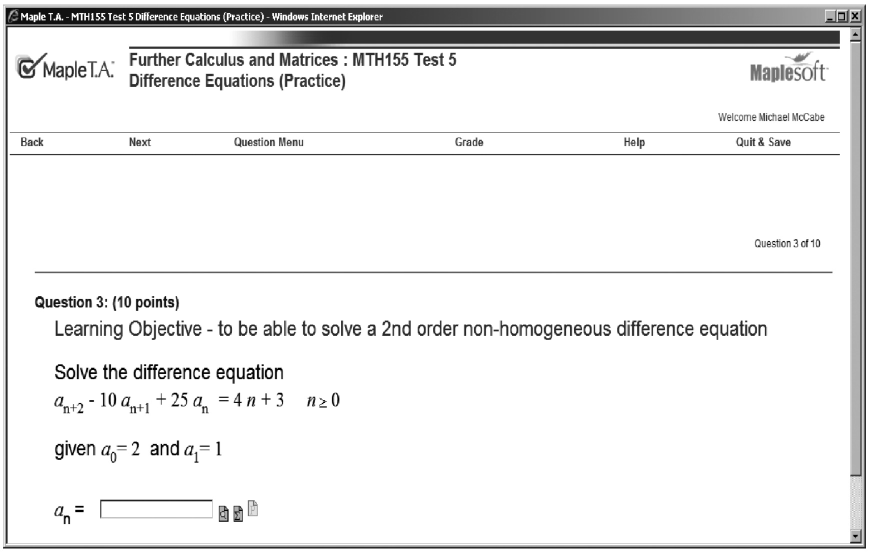

Figure 7: Symbolic Input in MapleTA

between WileyPLUS, MapleTA and WebCT prevented any integration of question banks. Given that MapleTA underpins WileyPLUS, that limitation might be overcome in the future, but remains an issue for the present.

4. Summative Assessment

Of seven exam tests taken by students for summative assessment, four were in WileyPLUS and three in MapleTA. This mix of delivery allowed us to compare their experience in using each of them.

For WileyPLUS:

- custom exam tests were created by selection of suitable questions from the bank
- "baseball" questions were set up so that up to three attempts at each question could be made without any penalty
- each test could be re-entered by students during one or more exam sessions to raise their mark
- questions were selected with a balance of difficulties, learning objectives and types
- administratively time-consuming workarounds had to be developed to account for the lack of test passwords in the WileyPLUS system

The cumulative scoring of "baseball" questions was designed to promote student confidence by allowing them to keep marks from all successful answers without having to repeat similar questions. Traditionally a student would have been expected to repeat a "knockout" test in its entirety with any score below the required threshold being effectively the same as no attempt at all.

Little use was made of the limited question authoring available in WileyPLUS, since it is limited to basic question types (MCQ, text, numeric, essay) using plain text only.

For MapleTA:

- exam security was greatly strengthened through the use of passwords and the administrative overload was reduced
- the setting up of tests with randomised questions was more flexible and easier to control
- all tests were "knockout" in the sense that each one had to be retaken if it was not passed, although it would have been possible to set up similar "baseball" questions to WileyPLUS by allowing up to three attempts

Many issues arise in using the resources in WileyPLUS and, to a lesser extent, MapleTA :

- The products are subject to future development and the release of new versions. There is no control over long-term continuity and stability of existing assessments in the future. Upward compatibility may not be possible and further time may need to be spent in setting up existing tests for annual use.
- The non-uniformity and comparability of questions in terms of their length, time required and scoring
- A need for discrimination between formative questions suitable for learning and summative questions suitable for examination.
- Partial credit for answers which are incomplete or inaccurate

5. Evaluation Method

We were keen to get rapid feedback from students who were learning from on-line resources, introduced on an extremely short timescale. Since it was a "live pilot" we initially relied on verbal comments, which reassured us that students were benefiting from their WileyPLUS experience and that we did not need to revert to existing resources. The "baseball" questions allowed us to track student progress and results were good.

A simple questionnaire was distributed which sought views on all their e-learning resources, with a particular focus on WileyPLUS. For each resource they were asked to rate their features on a Likert scale, e.g. 1 = essential to learning 6 = no use at all. A further set of open-ended question sought positive and negative responses. 68 out of 150 "Further Calculus and Matrices" students at level 1, who had used WileyPLUS throughout, responded. A smaller set of responses were also obtained from 11 out of 120 "Calculus of Several Variables" students at level 2, who had only used WileyPLUS in their final weeks of study in preparation for a final exam.

Evaluation of VLE ('Victory', in WebCT)

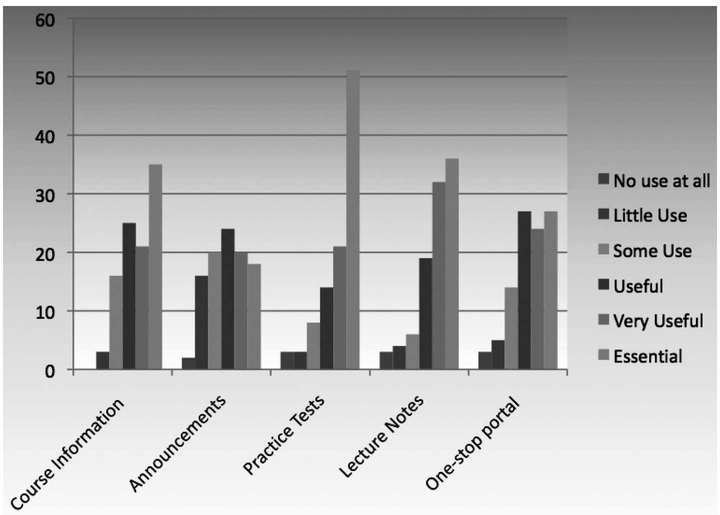

Figure 8: Usefulness of Victory Resources for Learning (%ages)

The Likert-scale questions showed that practice tests were valued most (50% essential for learning), 'just-in-time' lecture notes were highly valued, and the Victory (WebCT) site was also valued for providing a one-stop portal and course information. Victory was valued for providing a one-stop portal (27%) and course information (35%).

The open-ended questions invited student responses on any aspect of the VLE, which is used for most course units. The number of responses on a given point are shown in brackets. There were two overwhelming positives:

1. The VLE provided a necessary one-stop shop linking access to all resources, including WileyPLUS and MapleTA. It was the glue which held all the components together. (12)
2. 'Just-in-time' handwritten lecture notes (Figure 8) provided on a weekly basis as PDF files were greatly appreciated. (16) This came as a surprise given that students already had full access to WileyPLUS and that the rough notes were simply scanned after lectures. Students may have valued them for their focus on process rather than product, for demonstrating mathematical thinking in a digestible summary, for offering a more personalised approach which could be linked to what was said in lectures or for the local course information included. The responses did not go into more details.

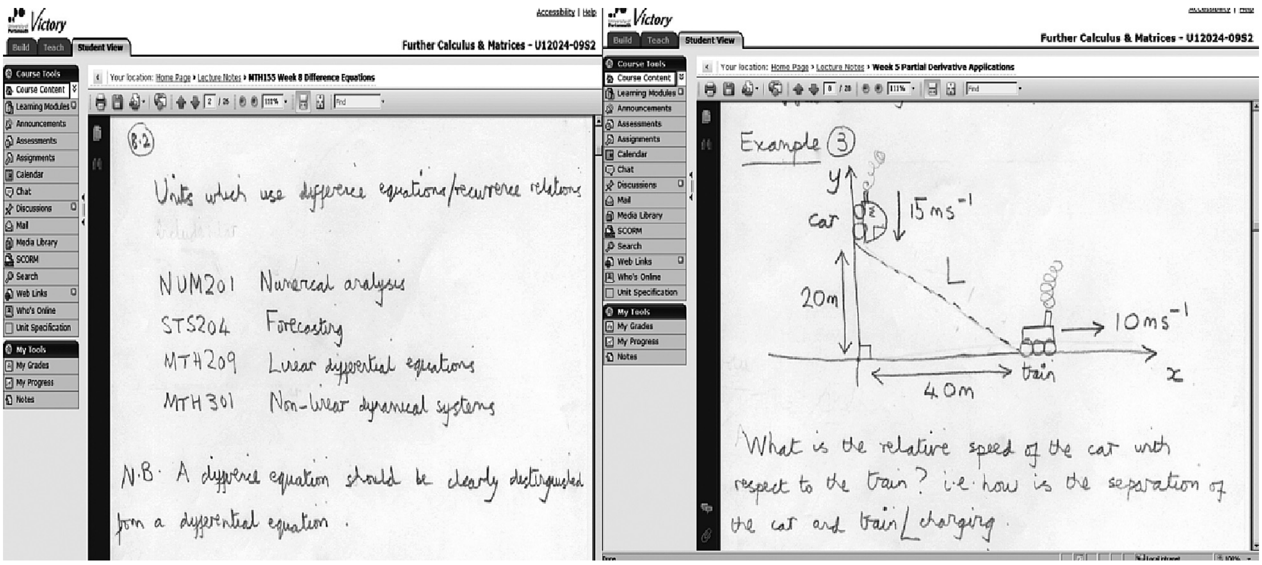

Figure 9: "Just-In-Time" Handwritten Lecture Notes

Amongst the negatives, unreliability (7) , GUI navigation issues (7) and mathematical limitations (5) featured more predictably. None commented on the extra e-assessment resources available within the VLE, suggesting that their lack of integration with WileyPLUS led to them being largely ignored.

6. Evaluation of MapleTA

We were interested in comparing student views on our 'in-house' MapleTA questions as opposed to the "outsourced" WileyPLUS questions. Many commented positively on the value of having a large number of MapleTA practice questions/tests (13) and the ease-of-use (12). The main negative comments were about difficulties with the syntax or format of input, arising from the lack of the palette tool in WileyPLUS (10), unfair or inflexible marking and lack of partial credit (5) and limited feedback such as hints, links and full solutions, which were more regularly available in WileyPLUS (6). A small number also identified the fact that tests were of the "knockout" variety requiring all questions to be repeated rather than having "baseball" questions allowing them to have three attempts (4).

7. Evaluation of WileyPLUS

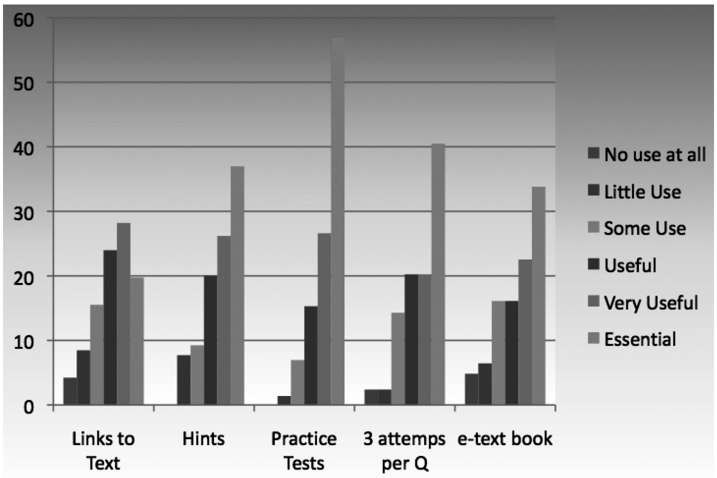

Figure 10: Usefulness of WileyPLUS Resources for Learning (%ages)

The Likert-scale responses indicated very high value for the practice tests (60% essential for learning), followed by the 3 attempts per questions, and the hints and e-textbook. The figures for MapleTA also indicated very high value for the MapleTA practice tests (the other questions did not apply to MapleTA).

Amongst all the open-ended responses, it was the availability of "baseball" questions and cumulative scoring in WileyPLUS that provoked the greatest positive comment (17). The ability to have three attempts at different instances of the same question without penalty seemed to offer greater encouragement to students and a more constructive approach to learning. Other positive comments were on the extensive feedback through book links, hints, solutions and tutorials (18) and the numerous/varied practice set of questions (9). Negative comments related to input difficulties despite the use of symbol palettes (4), GUI issues, e.g. need for multiple windows, and unfair marking (3). One student made the interesting negative comment that it was not always possible to repeat a question when a given instance had been answered correctly. It had incorrectly been assumed that they would not wish to continue practicing on similar questions to reinforce their understanding. Amongst other one-off comments was a reference to "friendly, calm and helpful invigilators" who support the delivery of e-assessments. Such a simple observation is easily overlooked when the focus of an assessment is on its technical delivery and content.

8. Future Directions in a Commercial World

Our experience in using WileyPLUS for calculus has been sufficiently positive that its use has already been extended to linear algebra [3] [4] in 2010/11. It is starting to provide some of the e-learning tools and the content necessary for undergraduate mathematics. At the University of Portsmouth the curriculum for every course, including mathematics, is undergoing a complete revision for 2012/13. Our present expectations are that WileyPLUS will continue to be adopted and integrated into our courses as part of that fresh start. This may well coincide with the next major release of WileyPLUS 5.0, which it is hoped will incorporate such features as exam security with password protection and a greater scope for question authoring, customisation and interoperability. The University also will need to change its VLE from WebCT in 2013, and ongoing change can be expected in the way that e-learning is required to be delivered. The question is whether it is possible to maintain "in-house" development of our own e-learning resources or whether we should become increasingly reliant upon those developed by external providers, either commercial or non-commercial.

The stark choice between open source collaboration and commercial provision often polarises individual academics and even whole institutions, but maybe there can be greater cooperation between commercial and non-commercial developers. It may be that universities themselves become increasingly commercialised, but few other than perhaps the Open University are likely to have the infrastructure necessary to produce the equivalent to WileyPLUS.

We argue that the ideal is a financially sound, hybrid model, which allows for greater customisation, sustainability, extension and interoperability than is currently available from either commercial providers or open source initiatives. There has always been some degree of hybridisation between commercial and non-commercial tools and resources. Commercial software has often been used to develop e-learning and e-assessment resources for non-commercial distribution within HE. Individuals or small groups have developed e-assessment question banks which are freely distributed, while the underlying software remains commercial. Some commercial publishers, e.g. Bedford, Freeman and Worth, develop e-packs for VLEs which can then be freely modified and extended. Conversely, non-commercial software can be used as the basis for commercial products, e.g. Red Hat (http://www.redhat.com/products).

	free resources and add-ons	commercial resources and add-ons
free tool/package	fully open source **STACK, OpenMark, Moodle**	hybrid **Red Hat**
commercial tool/package	hybrid **MapleTA banks (M^3)** **Perception (MG)** **WebCT e-packs** **Toolbook (Mathwise)**	fully outsourced commercial **WileyPLUS** **MyMathLab** **WebAssign** **CALMAT**

Table 1: Commercial vs. Non-Commercial in E-learning and E-assessment

Idealists will always argue for completely free and open software development in the academic world, but this is likely to remain an ideal. Yet WileyPLUS is in danger of becoming as inflexible as a textbook, if it limits customisation, modification and extension of its commercial resources. Furthermore any such changes need to be sustainable when new versions of the product become available. The situation is similar to the release of a new edition of a textbook, requiring a lecturer to update notes and supporting material, but only worse. Written notes may only require a revision to a list of recommended exercises or chapter numbers. Significant effort may be needed to revise e-assessments in WileyPLUS.

At the University of Portsmouth the main driver of change has been a sharp rise in student numbers, the demand for modern e-learning resources and need for both formative and summative e-assessment. It makes sound economic sense to buy in affordable commercial products for large first and second year units. We were able to introduce the calculus course to students and tutorial staff on an extremely short timescale with low overhead. Weekly small group personal tutorials were supported by WileyPLUS both through printed questions and through direct use of the e-assessments.

Ideally the staff time freed up can become available for tailoring the resources to local need or doing other things. A problem with WileyPLUS is the standalone nature of each product in supporting an existing textbook. The printed word remains a constraining factor and it is impossible for a lecturer to mix-and-match e-learning resources or assessments from separate WileyPLUS courses. For standard courses such as first year calculus that may be fine, but elsewhere in the curriculum it is likely to be highly restrictive. We have already noted that there is no distinction in WileyPLUS between those questions which are appropriate for learning, i.e. formative, and those questions which are appropriate for examination, i.e. summative. Our experience suggests that clearer guidance on question use would be appropriate when (and if) the product incorporates greater exam security. Identifying questions by their difficulty, question type or learning objective is insufficient.

To sum up, there are two distinct challenges that face universities – in assessment, and in teaching and learning.

In assessment, there is a need for customisation, which includes the provision of:

- Full authoring tools for academics to create additional assessment materials.

- Security access, and administrative processes and rights, for managing summative assessment, e.g. timed passwords for assessments.
- Interoperability, for instance between MapleTA-based provider materials and user-created materials.

These are essentially technical and systems issues and they need to be addressed rapidly by any provider wishing to maintain market share. They require minimal hybridisation and collaboration between private and public organisations, and would provide a possible modus operandi for collaboration and integration across university and commercial providers.

The challenges for teaching and learning, however, are much broader, and include the need for:

- Resources to be designed primarily for learning, and therefore assessment needs to be designed primarily as formative benchmarking, rather than summative assessment. (This is, as it happens, already the case in WileyPLUS).
- Summative assessment is also required, but it needs to be integrated as far as possible with formative benchmarking. Portsmouth has developed over several years what is in fact a hybrid between summative and formative assessment, and the use of 'three attempts per question' that we have implemented in WileyPLUS is an excellent extension of that principle.
- The tracking data base, which captures information on the use and progress of learners, needs to be designed to provide reports primarily for teaching and for learning, rather than primarily for administration and for management evaluation (necessary as these are).

ASPs such as WileyPLUS provide quite a lot of useful data, which can, already, be exported to administrative reports (which is good), but very little in the way of reports that are of immediate use to learners, tutors and lecturers, even though much of the data that could be used to generate these reports is already being captured.

If ASPs are to provide an integrated 'service', the real challenge in an age of web2.0 and 3.0 and social software is to provide not only the integration of teaching, learning, benchmarking and assessment, but also to provide wider integration with, and links to, the growing wealth of resources and interactive communities available through the Internet.

It is a paradox that people want total flexibility to customise a product such as WileyPLUS in a myriad of different ways, but in practice few have the time or inclination to make those changes. WileyPLUS clearly has far to go in its development, and it is important that commercial developers are aware of academic needs. Some may regard the adoption of ASPs as intellectual suicide on par with Cleopatra, which limits the independence and freedom of academics. In the future, we would like to maintain the benefits which have arisen from our use of WileyPLUS, while eliminating the drawbacks that we have identified from our own experience and student feedback. ASPs are unlikely to become extinct, but they may need to adapt to survive.

References

1. Anton, A., Bivens, I.C. and Davis, S. (2010) *Calculus: Late Transcendentals* 9th Edition Wiley ISBN 9780470398746

2. Anton, A., Bivens, I.C. and Davis, S. (2010) *Calculus: Late Transcendentals* WileyPLUS http://catalog.WileyPLUS.com/Section/Calculus-Late-Transcendentals-Combined-9th-Edition.id-404526.html

3. Anton, A. and Rorres, C. (2011) *Elementary Linear Algebra: Applications Version*, 10th Edition Wiley ISBN 9780470561577

4. Anton, A. and Rorres, C. (2011) *Elementary Linear Algebra: Applications* WileyPLUS http://catalog.WileyPLUS.com/Section/Elementary-Linear-Algebra-Applications-Version-10th-Edition.id-403167.html

DCU Voluntary Maths Tuition Programme

Eabhnat Ní Fhloinn

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Abstract

The Dublin City University (DCU) Voluntary Mathematics Tuition Programme was established in September 2009 as a joint initiative between DCU Mathematics Learning Centre, DCU Access Service and Ballymun Initiative for Third-Level Education. The programme aims to increase the confidence levels and mathematical standards of local secondary school pupils from disadvantaged areas, while raising the profile of mathematics within the schools, through an intervention in which DCU students provide free, one-to-one mathematics tuition on a weekly basis. The requirement of Leaving Certificate Ordinary Level mathematics for entry into third-level in Ireland had been found to be a barrier to these pupils, preventing progression into higher education, and this motivated the development of the programme, which was modelled on similar schemes operating in a number of other universities.

Exceeding initial expectations, a total of ninety DCU students volunteered to be part of the programme, and more than seventy pupils from a local school were involved. The feedback from both tutors and pupils was extremely positive, with both groups identifying significant personal benefits as a result, as well as the mathematics results of school pupils increasing overall. In addition, the school principal singled out the programme as having made third-level education seem like "a reasonable expectation for students from our school".

Here, we discuss the practicalities surrounding the set-up and implementation of such a scheme, including the workload involved, challenges faced and specific comments received from tutors and pupils. In particular, we consider what can be achieved in outreach programmes for which there is little or no funding available. In the coming years, facing increasing cuts in education budgets, the successful implementation of schemes such as this one will become ever more important.

1. Introduction

In the Irish education system, students take two national examinations during the course of their second-level education: the Junior Certificate, at the end of their third year in secondary school, and the Leaving Certificate, at the end of their sixth year [1]. For the latter, students take at least six subjects, with most taking seven or eight, as their best six results are counted for entry into third-level programmes. The Leaving Certificate Mathematics examination can be taken at three different levels: Foundation, Ordinary and Higher. In order to pass a subject, a student must obtain 40% or higher. Often, to qualify for a third-level degree programme, at least a pass in Ordinary Level mathematics is required, with Foundation Level mathematics not normally accepted, although some exceptions are made. Students must also obtain sufficient "points" from their other subjects and satisfy various other criteria, but the specifics of this need not concern us for the purposes of this paper. The requirement of Leaving Certificate Ordinary Level mathematics for entry into third-level had been found to be a barrier to some pupils, preventing progression into higher education [2].

Dublin City University (DCU) is situated to the north of Dublin city, and one of its local neighbourhoods is a socio-economically deprived area known as Ballymun. In 1990, Ballymun Initiative for Third-Level Education (BITE) was established with the aim of encouraging students from the area to fully engage with education, and, to date, BITE has successfully supported over 170 students through third-level education [3]. As a result of detailed discussions between BITE, DCU Access Service (which works with students from socio-economically deprived backgrounds) and DCU Mathematics Learning Centre, along with advice shared by colleagues in the University of Limerick who had previously organised a similar programme [4], DCU Voluntary Mathematics Tuition Programme was formally established in September 2009. The aims of the programme are to increase the confidence levels and mathematical standards of local secondary school pupils, while raising the profile of mathematics within the schools, through an intervention in which DCU students provide free, one-to-one mathematics tuition on a weekly basis.

2. Implementation of the Programme

Clearly, the success of the programme depended entirely on the levels of involvement of both the DCU students and the school pupils. Requests for volunteers from DCU were made in the form of posters, announcements during Orientation Week, and emails sent to class lists by the Mathematics Learning Centre. As a result, a total of 90 DCU students from a range of backgrounds and disciplines volunteered to be tutors in the Voluntary Mathematics Tuition Programme.

After a brief introductory meeting in which some basic training and guidelines regarding working with young people were covered, the tutors attended the school once a week for an hour and helped pupils with homework, revision and exam preparation. The pupils involved were from third, fifth and sixth years and studying a range of Higher, Ordinary and Foundation Level mathematics, with up to seventy pupils participating in the programme. Like many courses in DCU, the comprehensive school involved in the programme does not schedule classes on Wednesday afternoons, so this was chosen as an ideal time for tuition. The voluntary tuition sessions took place from 14th October – 16th December and 10th February – 28th April.

A closing ceremony was held for tutors and pupils on 21st April, in which certificates were distributed and pupils received "goodie packs" provided by DCU Access Service. Martin Conry, Secretary of DCU, who attended the ceremony, remarked that it was "inspiring to see DCU students volunteer so much of their time to help those in the communities surrounding DCU".

3. Challenges Faced

Within any programme of this kind, there are numerous challenges to be faced, and many of these will depend upon the particular school involved. Given that there is a full-time BITE representative working within the comprehensive school involved in this programme, she took responsibility for pairing pupils with suitable tutors, and contacted the tutors via email on a weekly basis to let them know if their pupil would attend tuition. Despite these efforts, absenteeism was a frequent problem, particularly in relation to the school pupils, often due to poor attendance at school in general. In addition, in order to maximise the tuition received by each pupil, it was decided that pupils should come every week, even if it was known in advance that their tutor was unable to attend on occasion. On such days, they were assigned a different tutor. As a result, pupils and tutors did not always work in the same pairings, which was not ideal. Another challenge was that the pupils' timekeeping was often poor, with some arriving up to fifteen minutes late for their hour-long session, and many times they did not come equipped with mathematics textbooks, past examination papers, calculators or sometimes even pens and paper, despite having been at school all morning.

4. Impact of the Programme

During the course of the year, sixth-year pupils took six assessments as part of their mathematics classes. Results were available for 36 pupils who took part in the programme. 15 of the pupils took Ordinary Level for their Leaving Certificate and the average increase in their results was 41%. The nine pupils who were taking Foundation Level from the start of the year saw an average increase of 63% in their results, with one pupil recording an improvement of 103%. 12 students who began the year studying Ordinary Level have seen their results drop by an average of 49% and as a result will take Foundation Level. This decrease can in a large part be explained by poor school attendance in both fifth and sixth year, and emphasises the need for intervention in younger years.

However, in addition to an academic impact, the programme was also successful in promoting third level education within the school. The principal, Pat O'Dowd, commented that

> *"Not only did maths grades increase overall, but meeting a third level student, often for the first time, every week has helped break down the perceived barriers attached to higher education for students from the school. Having DCU on the school's doorstep did not seem to have any great relevance to our students until this initiative. As a result of the programme not only has the profile of maths been raised within the school, but more importantly…DCU is now seen as an accessible resource in Ballymun and third level education has been promoted as a reasonable expectation for students from our school."*

5. Feedback from Pupils and Tutors

In order to improve the programme in coming years, pupils and tutors who took part were asked to complete anonymous surveys to obtain their feedback about the initiative. Twenty-seven responses were received from school pupils, about 80% of which were from sixth-year pupils, with the remainder from third-years. When asked what they found helpful about the programme, pupils most commonly referred to the fact that they received one-to-one help; that the tutors were patient and helpful and "teach at your level"; and that the tuition took place in their school. In addition, the vast majority felt that their maths grades had improved. In response to how the programme had changed how they feel about their maths, most indicated their increased confidence, whereas "before it was very hard and stressful", and the fact that they felt better prepared for examinations, as tutors "showed me how to break questions down." In relation to improvements that could be made to the programme, many stressed the importance of working with the same tutor every week, and several asked for longer or more sessions per week. When asked what they liked best about the programme, the overwhelming majority commented on the tutors, with the importance of the relationship built up over the year again coming to the forefront: "My tutor was there everyday for me." The other factor that pupils identified was simply the opportunity to learn mathematics: "Been able to finally understanding maths work." Overall, their responses are perhaps best summarised in two short comments: "The tutors were very nice and treated us with respect and it was a great experience" and "I learned new things and made a new friend for life."

For the tutor survey, a total of 40 responses were received. All those who responded found taking part in the programme to be a positive experience for themselves, citing benefits such as "interacting with younger people" and "giving back to the community". Some enjoyed sharing their own love of maths, saying they were "happy to show them that maths can be an interesting and manageable subject"; two tutors even said that they now "think becoming a maths teacher might be the way for me to go". Several referred to the fact that "seeing how much they appreciated our effort made the experience all the more worthwhile" and felt that they learned a lot about themselves in the process of the tuition.

When asked to describe the impact they saw the programme having on the pupils they tutored, most referred to the increase they saw in their pupils' self-confidence – "I could see that he was proud of himself and encouraged when he was able to do the maths that I had showed him" – as well as marked improvements in their mathematics – "it was good particularly when your student goes from not being able to add 2 and 2 to being able to do a quadratic". They spoke of pupils "engaging more with actually trying to understand what they were doing when solving problems, rather than just memorising solving procedures" and described how the pupils "had piles of potential" and "wanted to be there to learn maths and they tried there (sic) best." They also referred to the relationships they forged with pupils over the course of the year – "The student I tutored became very attached, she kind of relied on me to help her learn…" – and that they spoke to pupils about the fact that there are "so many options with maths and that they shouldn't just consider it a subject that they need to get through for their exams and forget about it." Several tutors described the process of turning around pupils' poor self-belief in their mathematical abilities: "The first thing every student told me at the start was that they were rubbish at maths, but when we started going through the questions and giving them pointers I was able to show them how much they knew, and changed their attitude towards the subject" and "Well, one girl i helped was very negative towards maths for some time, and i just took my time and listened to her. And then explained the maths in simple form, giving her some useful tips to remember things. And she seemed to understand it far better by the end of the session and seemed delighted with herself that she could do it." In terms of challenges they encountered, the most common was in relation to the teaching of mathematics itself, with tutors observing that "at first it was hard to explain things that I already knew" and "explaining maths in simple English was difficult sometimes". Some tutors mentioned that their pupils did not always turn up for tuition, meaning that they had a wasted journey or worked with a different pupil that day. Others alluded to the difficulty of getting some pupils to focus on their work, or have the correct materials with them.

In common with the pupils, when asked for improvements that could be made to the programme, tutors emphasised the importance of working with the same pupil every week, in order to effect real improvements in their mathematics, although they noted the difficulty of this due to absenteeism. They also suggested closer liaisons with the teachers and more structured work to do during sessions. Several tutors also requested more specific pointers on teaching mathematics, so this will be incorporated into the training session for next year's tutors. Overall, tutors were very positive about the programme – "I am proud to say that I was a part of it" – and keen to take part in the coming year.

6. Conclusion

Overall, the Voluntary Mathematics Tuition Programme has exceeded our expectations, due in no small part to the excellent work of both the DCU students and the school pupils involved in the programme. Both groups seem to have benefited greatly from the experience. However, despite the intervention, no pupil from the comprehensive school took Higher Level mathematics for their Leaving Certificate in June. Therefore, while continuing to run the programme within the school, we hope to extend it to provide additional support to pupils from transition, fifth or sixth year who are considering taking Higher Level mathematics, with these pupils attending one-to-one tuition within DCU twice a week. In addition, subject to volunteer tutor numbers, we hope to encourage more pupils from younger years to take part in the initiative.

Acknowledgements

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References

1. State Examinations Commission (2010), Accessed via http://www.examinations.ie (15 November 2010).

2. Dublin City University Access Service (2009) Internal communication.

3. Ballymun Initiative for Third-Level Education (BITE) (2010), Accessed via http://biteballymun.com (15 November 2010).

4. Ní Ríordáin, M. (2009) Private communication.

Stimulating Techniques in Entry-level Mathematics

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Abstract

This paper reports the outcomes of a project which aimed to build on existing best practice in diagnostic testing of mathematics. The project took existing diagnostic resources and developed tests for the STACK computer aided assessment system. This paper reports on the test design, implementation and how the results of the test were analysed and reported to students and staff.

1. Introduction

In April 2010 the National HE STEM Programme funded a mini-project with the title *Stimulating Techniques in Entry-level Mathematics (STEM) with the STACK computer aided assessment (CAA) system*. The aim of this project was to take existing "diagnostic tests" in core mathematics and develop similar automatic tests for the STACK computer aided assessment system. This paper reports on the outcomes of this project.

1.1 The STACK CAA system

STACK, a System for Teaching and Assessment using a Computer algebra Kernel, is a computer aided assessment system for mathematics. A student enters their answer as a mathematical expression, whose mathematical properties are then established with the use of the computer algebra system Maxima. For many questions the teacher will seek to establish that the answer is (i) algebraically *equivalent* to the correct answer and (ii) in the appropriate *form*, (e.g. factored). However, the answer need not be unique. STACK uses the computer algebra system Maxima to

- randomly generate problems in a structured mathematical way;
- establish the mathematical properties of expressions entered by the student;
- generate outcomes in the form of a numerical score; feedback which may include mathematical computations of the student's answer; and a *note* for later statistical analysis.

STACK was designed and developed at the University of Birmingham, United Kingdom, with support from the Higher Education Academy's Maths, Stats and OR Network. More details of STACK can be found in, for example [1], [2], [3], and [4].

1.2 Diagnostic testing and the nature of effective feedback

Diagnostic testing in mathematics is widespread and has been used for many years. For example [5] found

1. At least 60 Departments of Mathematics, Physics and Engineering give diagnostic tests in mathematics to their new undergraduates. [...]

5. Diagnostic tests play an important part in
    - identifying students at risk of failing because of their mathematical deficiencies,
    - targeting remedial help,
    - designing programmes and modules that take account of general levels of mathematical attainments, and
    - removing unrealistic staff expectations.

Indeed, [5] goes on to recommend that "*students embarking on mathematics-based degree courses should have a diagnostic test on entry*" and that "*prompt and effective support should be available to students whose mathematical background is found wanting by the tests*". An extensive series of case studies in diagnostic testing was published as [6]. Furthermore, there have been specific CAA systems for diagnostic testing such as DIAGNOSYS, [7]. However, there is currently no such diagnostic test for the STACK CAA system.

Notice the importance of feedback, both to students and staff. It is almost axiomatic that "feedback promotes learning". However, the research evidence on this matter appears less clear. For example, the meta-analysis of Kluger, see [8], examined about 3000 educational studies and found that over one third of formative feedback interventions *decreased performance*: a counterintuitive and largely ignored outcome. They concluded that it is not feedback itself but the nature of the feedback which determines whether it is effective. In particular, feedback which concentrates on specific *task related* features and on *how to improve* is found to be effective in a formative setting, whereas feedback which focuses on the *self* is detrimental. A low end-of-test summary mark - hardly a specific form of feedback - may be interpreted as a personal and general comment on the ability of the student, whereas detailed feedback on each task points to where improvement can be made. Following on from this work in [9] Shute recommends that we should be *"cautious about providing overall grades"*, and *"use `praise' sparingly, if at all"*. Furthermore, she suggests we *"avoid using progressive hints that always terminate with the correct answer."* and *"minimize the use of extensive error analyses and diagnosis"*. Clearly it is necessary to gain some broad sense of what a student can and cannot do in order to target help so that they may improve their performance on the task. However, for the purposes of this project we fall short of the level of detail used in the DIAGNOSYS system, see [7], and other intelligent tutoring systems, see for example [10].

Hence we decided that numerical scores on individual questions and tests will *not* be available to students. Instead, text based feedback (which includes e.g. "correct answer") will be used. Further, diagnosis will be at broad topic level, e.g. "algebra", or "calculus", not at the minute detail of individual technical skills.

We also decided to implement confidence based testing. Each question contains a "slider" to enable the student to indicate their confidence with each answer, see [11].

2. Core skills in mathematics

The majority of students taking a diagnostic test such as ours will have been educated in the United Kingdom school system. The purpose of these tests is to assess skills at the start of a university course. We therefore have a responsibility to include within the assessment only those topics which students can be expected to have been taught, and to assess them in a manner similar to the examination questions with which they are familiar. If the users of these tests are not happy with the skills of incoming students then it is the responsibility of the university department to educate their students in different ways. Therefore we took the Qualifications and Curriculum Agency (QCA) *General Certificate of Education (GCE) Advanced Subsidiary (AS) and Advanced (A) Level Specifications Subject Criteria for Mathematics* as our *de facto* curriculum. Of course, we acknowledge that these *Subject Criteria for Mathematics* do miss important mathematical topics, notably complex numbers. The European Society for Engineering Education (SEFI) *Working Group on Engineering Mathematics and Education*, Core Curriculum document [12], for example, describes alternative curricula.

Ultimately, in developing our diagnostic tests we identified 103 "core skills". These are grouped into two levels. The top level skills are listed below, together with the codes with which we refer to them.

1. Logic (LOG)
2. Using conventional forms of notation (FOR)
3. Number (NUM)
4. Arithmetic Operations (ART)
5. Algebraic Manipulation (ALG)
6. Estimation (EST)
7. Interpreting Graphs (GRP)
8. Analysis of Diagrams (DIG)
9. Analysis of Word Problems (WRP)
10. Equations (EQN)
11. Functions (FNC)
12. Geometry (GEO)
13. Sequences and Series (SEQ)
14. Logarithms and Exponentials (LGE)
15. Differentiation (DIF)
16. Integration (INT)
17. Vectors (VEC)

These codes are used in the reporting mechanism. We have chosen to subdivide many of these top level skills into subskills. For example, *Equations* (EQN) includes

1. Solve Linear Equations (EQN-LIN)
2. Solve Simultaneous Equations (EQN-SIM)
3. Solve Quadratic Equations (EQN-QUD)
4. Solve Inequations (EQN-INQ)

There are clearly omissions within this list of topics, notably trigonometry and complex numbers. For the purpose of this project we have stopped short of trying to list all topics, or to write questions which assess them. While it is not difficult to list curriculum topics in mathematics, a curriculum is much more than this. In developing an effective diagnostic test we need to understand something of the *style* of questions and the extent to which topics are linked together.

2.1 An example of structuring

Simply listing a topic such as *integration by substitution* gives us little idea of the style of question. Compare the following four versions.

1. Find $\int \sin^3(x)\sin(2x)dx$.
2. Using integration by substitution find $\int \sin^3(x)\sin(2x)dx$.
3. By using the substitution $u = \sin(x)$, or otherwise, find $\int \sin^3(x)\sin(2x)dx$, giving your answer in terms of x.
4. By using the substitution $u = \sin(x)$, or otherwise, show that $\int \sin^3(x)\sin(2x)dx = \frac{2}{5}\sin^5(x) + c$.

On the OCR specimen paper, [13], it is version 3, i.e. giving the explicit substitution but not asking only to verify the answer, which is asked. We believe this is a general trend, with *verification of a given answer* relatively common. At university we would also expect student to be able to recognize which method should be used, and to make choices within the detail of how this method should be employed. Therefore we have a strong preference for questions which instruct the students to *find the answer*. While this may be a change from the conditions under which students are familiar with answering questions, we feel it is justified.

2.2 Proof, reasoning, and problem solving

In addition, the QCA's *Subject Criteria for Mathematics* discuss more general skills such as proving and problem solving, under the heading *Proof*. In particular they claim that "*these requirements should pervade the core content material set out*". We do not discuss the extent to which current UK school examinations do or do not actually implement these specified assessment objectives. However, we do have technical limitations when trying to assess proof with STACK. Therefore, the extent to which we try to assess these specified objectives will be limited, but not completely abandoned. The following example, known as the *Wason selection task*, seeks to test logical reasoning, [14],[15].

> Imagine you have a deck of cards in which every card has one letter on one side and one number on the other. You can see the following four cards
>
> **D** **3** **K** **7**
>
> Turn over the fewest cards to establish the truth of the following statement. "*Every card which has a D on one side has a 7 on the other.*"

2.3 Conditions of the tests

The project sought to develop tests which are suitable to be taken during a single sitting of one hour; with a pencil and paper to hand, i.e. these are not mental tests; with the help of our own formula sheet but without calculators, computer algebra, Wolfram Alpha or similar.

Of course, teachers can use the questions and tests in a variety of ways. Furthermore, if a teacher needs to ensure students take a test under certain conditions, these will need to be invigilated. This said, each question we developed had random versions, formative feedback and full worked solutions reflecting any randomization. Hence, they can be used in a formative or practice setting. If running a diagnostic test these features can be "switched off" by using the appropriate options within STACK.

3 The User Profile

In STACK, each attempt at a question results in three outcomes:

1. Text based feedback;
2. A numerical score/mark;
3. An internal "answer note".

These broadly equate to the formative, summative and evaluative purposes of assessment. Our questions generate a numerical score and full formative feedback. All but minimal feedback is suppressed to enable them to be used in a summative setting. The answer note records the logical outcome regardless of the random version taken. For example, assume we have a question asking a student to find the derivative of an exponential (DIF-

EXP). If a student gets a question correct which we deem needs this skill, they pick up a tag DIF-EXP-TRUE. If a student gets a question incorrect which we deem needs this skill, they pick up a tag DIF-EXP-FALSE.

The reporting software examines these tags to build the profile and match up appropriate follow-up materials. Further work needs to be done to enlarge the range of materials available.

4. Results

The questions were identified by collaborating with colleagues from the University of Birmingham, University of Aston, Loughborough University and the University of Manchester. Existing paper-based tests were used as an initial starting point for many of these questions. For each STACK question we wrote a full worked solution, and the majority of questions were randomly generated from a template. Ultimately the project wrote 87 questions which sought to assess 70 of the 103 sub-skills identified. In addition to this, resources from the mathcentre website were also linked to questions. The questions have been grouped into three categories, depending on the anticipated educational level of students.

A selection of 16 of these questions were taken by 211 first year students in September 2010 at the University of Birmingham. Interestingly, final score correlated most strongly with the *number of questions attempted* indicating that for most students *time taken* for the questions was the most significant factor. However, at least 45 students appeared to have attempted all questions indicating that the test was not unrealistic. This makes analysis of responses to questions later in the test difficult because many students appear not to have attempted them.

Question 3 was taken from [16]

> A university has 6 times as many students as professors. If S represents the number of students and P represents the number of professors, write an equation expressing the relationship between S and P .

When Clement *et al.,* [16], gave the Students-and-Professors Problem to 150 calculus level students, 37% answered incorrectly with 6S=P accounting for two thirds of all errors. For our students the results were better, only 22% of students were incorrect. Interestingly students were *very confident* in their incorrect and correct answers, with 83% of responses indicating a confidence level of more than 90% in the correctness of their answer.

Results from the *Wason selection task*, discussed above, are also interesting. Only 32% of all attempts were correct, i.e. [D,3]. Popular incorrect choices were D only, or [D,7] at 22% each. The only other significant response was [D,3,7] which would confirm the statement but is not minimal. All other combinations amounted to 15% of responses with no individual response attaining more than 5%. With these incorrect answers the full range of confidence was expressed and more than half of responses were more than 75% confident, with 24% of responses being 100% confident. This misplaced confidence warrants further investigation.

Partial fractions, question 7, was a topic which few students could do: of the 211 students there were only 107 *attempts*. 185 students failed to complete this question, only 24 got this correct.

It will be interesting to compare results on this test with marks at the end of the first year to look for any correlations. In future, more detailed analysis of answers could help us to decide which questions are useful in identifying individuals in need of better initial support.

5. Conclusion

Writing a useful diagnostic test for incoming students in the STEM subjects is considerably more involved than identifying a list of curriculum topics and writing question to assess them. This project has succeeded in developing a range of questions which we assembled into quizzes and made available. Following from our use of the diagnostic test we will further evaluate the outcomes, and decide whether the structures put in place for building student profiles is sufficiently valuable to warrant the effort expended in developing a skill classification scheme and tagging all questions individually. In particular we shall evaluate the results of the diagnostic tests in the light of end of year 1 examinations to decide if the results are in any way predictive.

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References

1. M. Badger and C.J. Sangwin, (2011). My equations are the same as yours!: computer aided assessment using a Gröbner basis approach. In A. A. Juan, M. A. Huertas, and C. Steegmann, editors, *Teaching Mathematics Online: Emergent Technologies and Methodologies*. IGI Global.

2. C. J. Sangwin (2008). Assessing Elementary Algebra with STACK. *International Journal of Mathematical Education in Science and Technology*, Vol. 38(No. 8):987-1002.

3. C. J. Sangwin, (2010). *Who uses STACK? A report on the use of the STACK CAA system*. The Maths, Stats and OR Network, School of Mathematics, The University of Birmingham.

4. I. Wild, (2009). Moodle 1.9 Math. Packt Publishing.

5. T. Hawkes and M. Savage, (2000). *Measuring the mathematics problem*. Technical report, The Engineering Council.

6. D. Lawson. *Diagnostic Testing for Mathematics*, (2003). LTSN MathsTEAM Project.

7. J. Appleby, P. C. Samuels, and T. T. Jones, (1997). DIAGNOSYS - a knowledge-based diagnostic test of basic mathematical skills. *Computers in Education*, Vol. 28:113-131.

8. A. N. Kluger and A. DeNisi, (1886). Effects of feedback intervention on performance: A historical review, a meta-analysis, and a preliminary feedback intervention theory. *Psychological Bulletin*, Vol. 119(No. 2):254-284.

9. V. J. Shute, (2007). *The future of assessment: Shaping teaching and learning*., chapter Tensions, trends, tools, and technologies: Time for an educational sea change. Taylor and Francis Group, pp 139-187.

10. D. Sleeman and J. S. Brown, (1982). *Intelligent Tutoring Systems*. Academic Press.

11. P. Hassmén and D. P. Hunt, (1994). Human self-assessment in multiple choice. *Journal of Educational Measurement*, Vol. 31(No. 2):149-160.

12. L. Mustoe and D. Lawson, (2002). *Mathematics for the European Engineer: A curriculum for the twenty-first century*. Technical report, SEFI Mathematics Working Group.

13. http://www.mathsrevision.net/alevel/ocrspecimen.pdf

14. M. Inglis and A. Simpson, (2004). Mathematicians and the selection task. In *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, volume Vol 3, 89-96.

15. P. C. Wason (1968). Reasoning about a rule. *Quarterly Journal of Experimental Psychology*, Vol 20:273-281.

[16] J. Clement, J. Lochhead, and G. S. Monk, (1981). Translation difficulties in learning mathematics. *American Mathematical Monthly*, Vol. 88(No. 4):286-290.

Dyscalculia in Further and Higher Education

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Abstract

Dyscalculia is one of the newer challenges that face practitioners and researchers, particularly in the post 16 sectors. The focus of this paper is therefore be Further and Higher Education. Dyscalculia is a specific learning difference, which affects the ability to acquire arithmetical skills and an intuitive grasp of numbers. Consideration is given to this and other current definitions, together with a theoretical perspective. The paper also considers the prevalence of dyscalculia, as well as the difficulties dyscalculic students' experience, both in academic life and more generally. The paper highlights DysCalculiUM, a new first-line screening tool for dyscalculia focusing on the understanding of mathematics. The system provides an on-line delivery of the screening tool to identify students at risk with minimal staff input. A Profiler identifies students requiring further investigation. This may take the form of an in-depth interview and referral for further testing. The final section of the paper considers subsequent one-to-one support for students. A case study of a dyscalculic student in Higher Education working with tables of information, percentages and graphs, serves to illustrate some of the ways in which dyscalculic students can be supported on a one-to-one basis.

1. Background

1.1 Towards a Definition

The DSM-IV [1], defines Mathematics Disorder in a person in the following terms: "as measured by a standardised test that is given individually, the person's mathematical ability is substantially less than would be expected from the person's age, intelligence and education. This deficiency materially impedes academic achievement or daily living". One of the key features of this definition is that the mathematical level is significantly lower than expectation. Indeed, Butterworth [2], says: "Most dyscalculic learners will have cognitive and language abilities in the normal range, and may excel in non-mathematical subjects". Another key feature is the impeding of academic achievement and daily living. The National Center for Learning Disabilities [3], says: "Dyscalculia is a term referring to a wide range of life-long learning disabilities involving math… the difficulties vary from person to person and affect people differently in school and throughout life". However, the DSM-IV definition does not expand on what is meant by mathematical ability and this is crucial to our understanding of dyscalculia. The definition is centred on "Mathematics Disorder" and this implies a stable cognitive root, which should not be based on achievement or mastery that is subject to influences of education and environment. Therefore it would seem inappropriate to make assessments through achievement tests, as is often current practice.

The National Numeracy Strategy [4], defines dyscalculia as that which "affects the ability to acquire arithmetical skills. Dyscalculic learners may have difficulty understanding simple number concepts, lack an intuitive grasp of numbers, and have problems learning number facts and procedures. Even if they produce a correct answer or use a correct method, they may do so mechanically and without confidence." The "ability to acquire" is important in that it emphasises acquisition rather than the carrying out of arithmetic procedures. The definition also goes further than the previous one, in that it is more specific about the nature of the mathematical ability: i.e. "difficulty understanding simple number concepts, lack an intuitive grasp of numbers", thus placing understanding at the core of dyscalculia. "A lack of a true comprehension or understanding of maths will be a key characteristic of dyscalculic people" Chinn [5], However, the National Numeracy Strategy [4], continues with the inclusion of "learning number facts and procedures". This would be more indicative of the issues experienced through dyslexia. The dyslexic student is likely to struggle to recall number facts such as times tables and would rely on understanding the mathematics required, rather than well rehearsed procedures learnt by rote or over-learning.

1.2 Prevalence

There is currently no adult data available that gives an estimate of the number identified with dyscalculia. The available data from previous research studies relates to populations of children. Original estimates by Kosc [6] placed this at 6.4% and this is broadly in line with more recent estimates by Butterworth [7], of between 5% and 6%. However, Geary [8], and Desoete [9], both estimate the prevalence of dyscalculia in child populations to be as high as 8%.

1.3 Theoretical Perspectives

Recent advances in neuroscience have greatly increased our understanding of how we conceive number. "An elementary number system is present very early in life in both humans and animals, and constitutes the start-up-tool for the development of symbolic numerical thinking that permeates our western technological societies" [10], Dehaene et al. [11] postulate a triple code theory based on three related neural regions of the brain. The first region is the horizontal segment of the intraperietal sulcus (HIPS) in which numerical quantity is represented and manipulated and which is activated independently of input modality, that is as a digit, word or a collection of items. The second region is the left hemisphere angular gyrus (LAG) in which the verbal processing of numbers takes place and is activated by linguistic rather than quantity processing. It is this area that is responsible for digit naming and learned number facts. The third region is the posterior superior parietal lobule (PSPL) in which visual-spatial processing occurs. The PSPL is activated usually in conjunction with HIPS, during number processing, but is associated with space and time. The PSPL is predominantly within the right hemisphere and the LAG is entirely in the left hemisphere. Thereby, a 3-way system guides and constrains the acquisition of symbolic number skills and, in particular, the HIPS and PSPL are activated during number processing. These are mediated by LAG.

2. A first-line screening tool for dyscalculia

2.1 Development

DysCalculiUM [12], is a first-line screening tool for dyscalculia, developed by Trott and Beacham at Loughborough (due to be published by *Iansyst* in conjunction with *Tribal* in November 2010). The screener focuses on mathematical understanding and has an on-line delivery to identify students at risk of dyscalculia. The tool is designed so as to minimise staff input. A profiler report, given after each student has taken the screening test, identifies those students requiring further investigation (which can be an in-depth interview or referral for further testing). The model for the screener is based on 11 categories, 6 of which are about understanding number and the other 5 relate to common applications. Conceptual understanding of both number and operations relate to activities associated with the HIPS region, while comparisons of number on a visual-spatial plane and with symbolic notation are associated with the PSPL region. Making inferences from given operations requires both manipulation within the HIPS region and visual-spatial processing, thus requiring activations in both HIPS and PSPL. Furthermore, comparisons of number quantity through language would employ the LAG region together with the HIPS region. The model for the DysCalculiUM screening tool is shown in Figure 1.

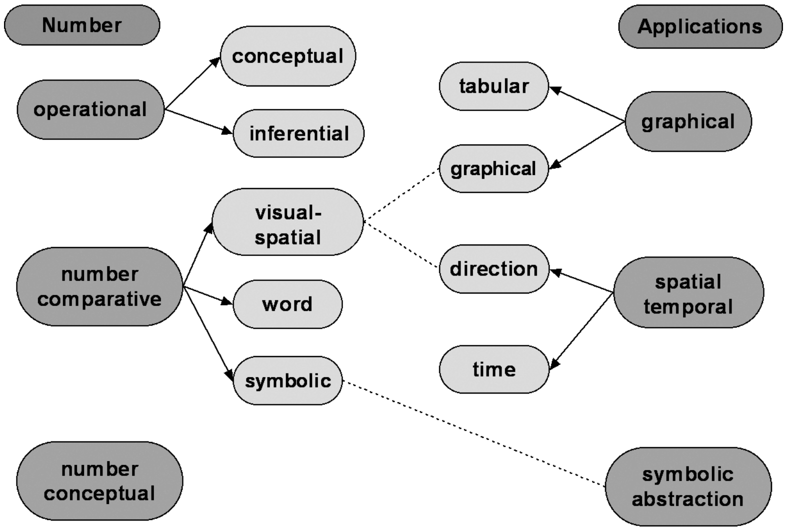

Figure 1: the model for DysCalculiUM, showing the 3 key areas of number, together with their subcategories and the 3 key areas of applications together with their subcategories.

2.2 Trials

The screener has undergone several trials and modifications during its development and the final trials were divided into two parts. In the first, the screener was given to large lecture groups in both Colleges of Further Education and Institutions of Higher Education. The size of each group varied, and the total sample was 504. This provided data for the establishment of "at risk" and "severely at risk" thresholds for the population. The second part involved trials on a one-to-one basis with individuals known to be dyscalculic. This allowed the tool to be verified against a known population. Results from these final trials showed the accuracy of the screener and established reference points for using with future participants. The 8th percentile rank was established as the threshold for "at risk" with the 2nd percentile rank as the threshold for "severely at risk". Of the 51 one-to-one trials with those participants who were known to be dyscalculic, 47 were shown to be "at risk" or "severely at risk" by the DysCalculiUM screening tool. Further information was sought on the four individuals who were not identified by the screener; one assessment report said that the individual was "probably dyscalculic" and the three others were students who were all following science degree courses (biochemistry, physics and computer science). It is likely that these students had developed good coping strategies in order to be able to follow the demands of their courses. These well-developed strategies, that would be frequently repeated during their course, would therefore mediate the score obtained in the screener. This accepted, the screening tool provides a powerful first-line for screening for dyscalculia.

2.3 The Profile

The 11 categories, together with the overall score, are used to create the profile. For each of these, there is an indication of "not at risk", "at risk" and "severely at risk". Guidance is provided on how to interpret the profile and on the subsequent course of action. The procedure is as follows: the learner accesses the DysCalculiUM portal, completes the screener and the results are automatically analysed. The tutor then accesses the portal and reviews the results and profiles, identifying those who are at risk. There can then be a further investigation of the difficulties through an initial tutor-led interview and then by an Educational Psychologist or qualified assessor. A formal identification of dyscalculia can lead to one-to-one support for the learner.

Figure 2 shows an exemplar profile. This profile, for 'Thomas', shows that he is "severely at risk of dyscalculia". This is given, primarily by the overall score indicator, but this is also backed up by the seven categories highlighted as "at risk" or "severely at risk of dyscalculia". The profile further suggested that Thomas had difficulty understanding number concepts and struggled to make numerical comparisons between numbers. It follows that he also had problems with understanding the concept of number operations and in making inferences from them. However, Thomas did not have difficulty with graphical and tabular information, time and spatial directions, which are more visual areas and suggest that Thomas is a visual learner. During the in-depth interview, Thomas revealed that he had

	Severely at risk	At risk	Not at risk
Overall Score:			
1 Number Conceptual			
2 Number Comparative: Word			
3 Number Comparative: Symbol			
4 Number Comparative: Visual Spatial			
5 Graphical			
6 Tabular			
7 Symbolic Abstraction			
8 Spatial Direction			
9 Time			
10 Operational: Conceptual			
11 Operational: Inferential			

Figure 2: DysCalculiUM profile for "Thomas", showing areas of strength, "at risk of dyscalculia" and "severely at risk at dyscalculia"

always had particular difficulties with mathematics, and in school had been placed in the lowest mathematics group. He had achieved outstanding results in many other areas, but had failed to overcome his mathematical difficulties. Consequently, he was very low in confidence and was concerned about the numerical aspects of his course.

3. Dyscalculia: The Social Affects

Although there is still a lack of awareness of dyscalculia as a specific learning difference, number and numerical understanding underpin many of our daily routines, including household budgeting, checking change or telling the time. Thus, dyscalculics face challenges each day. Anxiety, frustration and low self-esteem often result. One dyscalculic student recently said that she always paid with "a purple", meaning a £20 note, thereby ensuring that she had given enough money to cover the cost of her purchases. She could not tender the correct amount nor check her change. She also found it difficult to remember numerous orders when she and her peers went to the cafe. Her embarrassment in this situation led her to stop socialising with her peers and she became increasingly socially isolated. It is therefore important that dyscalculia can be identified effectively so that support can be put in place to enable dyscalculics to reach their full potential and feel more confident in everyday numerical situations. The DysCalculiUM screening tool is a first step in this process.

4. One-to-one Support for Dyscalculia

4.1 Case Study

Following identification, it is important to support the learner in an appropriate way. The following case study will serve to illustrate the ways in which a student with dyscalculia can be supported on a one-to-one basis with the mathematical elements of a course.

'Liam' was a first year student studying transport management and was identified as dyscalculic following initial screening with the DysCalculiUM tool, during his first year at university. Liam had always struggled with understanding basic mathematical concepts and had been placed in the lowest set for mathematics in school. However, he had excelled at other subjects, particularly languages. The initial screening and assessment revealed that he had strengths in several areas, especially verbal reasoning, expressive writing and reading comprehension. At the same time, his dyscalculia resulted in difficulties with conceptual understanding of