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SOFT MONEY AND CAMPAIGN FINANCE REFORM*

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We analyze special interest influence on policy when political contributions are capped but the regulation contains soft-money loopholes. The politician chooses between two policy options. We define special interest influence as the probability the politician chooses the policy he would not have chosen in the absence of contributions. Any binding cap reduces special interest influence but the effect may be nonmonotonic. A ban on contributions can result in greater special interest influence than a binding but nonzero cap. The results may also have implications for the policy response to the 2010 Supreme Court ruling on Citizens United *v.* FEC.

1. Introduction

A basic premise of representative democracy is that people elect representatives who either share their policy preferences or are willing to behave as if they do in order to be reelected. However, it is often argued that the need to raise money to run a political campaign may weaken the connection between politicians' policy preferences and their actions, undermining this fundamental premise of representative democracy and leading to greater policy influence of large contributors.

The Bipartisan Campaign Reform Act of 2002 (BCRA) re-regulated campaign contributions in order to achieve a "reduction of special interest influence."[2] An important aspect of the BCRA is that it sets ceilings on each contributor's "hard-money" donation, which is defined as the amount of money donated directly to a candidate, political action committee, or national party. However, the BCRA does not set ceilings on "soft-money" donations to advocacy groups. These entities may engage in political spending giving support to a political candidate but it is illegal for them to coordinate their spending with the candidate, political action committee, or party. The candidate, political action committee, or party has control over the spending of hard-money donations but not over the spending of soft-money donations. Nevertheless, critics fear that these soft-money loopholes may prevent the campaign finance reforms from achieving their goal of reducing special interest influence.

We study the effect of a hard-money contribution cap on special interest influence when there are soft-money loopholes. The incumbent politician chooses between two policy options, such as voting "yea" or "nay" in a congressional committee or on the floor. He has a policy preference, but his choice may be swayed by donations. We define the degree of special interest influence as the probability that the politician chooses the policy option he would not have chosen in the absence of contributions. Two lobbyists compete via simultaneous contributions for a prize that will arise due to the politician's policy choice. To the politician, soft-money contributions are less valuable than hard-money donations since he only has direct control over the spending of hard money. So from a lobbyist's perspective the marginal

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[2] See Title I of Public Law 107–155, 107th Congress. The BCRA is commonly known as the McCain–Feingold Act.

cost of providing value to the politician through soft money is higher than it is through hard-money donations. All contributions are sunk both for the winning and the losing lobbyist. The model is an extension of the all-pay contest of Kaplan and Wettstein (2006) and Gale and Che (2006)—henceforth KW and GC. We incorporate politician preferences into their framework. This extension allows us to capture the concern that policy may be driven by money where the politician makes a different policy decision than he would have made if there were no donations.

If the politician is indifferent between policy positions, KW/GC show that a hard-money contribution cap is completely neutral on the policy outcome and on lobbying costs as long as there are alternate but less efficient ways to contribute in case the lobbyist desires to contribute more than the cap.[3] Circumventing a hard-money contribution cap via soft money is equally onerous for both lobbyists; hence, the cap does not alter the relative strengths of the lobbyists. However, if the politician has a policy preference, we show that a cap with soft-money loopholes is not neutral. The cap affects lobbyists' relative reliance on soft money in an asymmetric fashion. This can result in special interest influence being minimized with a binding but nonzero hard-money contribution cap.

In order to overcome the politician's policy preference, the lobbyist with the policy position that the politician does not favor must exceed the contribution of the favored lobbyist. Hence, if the cap is not too restrictive so that in equilibrium only the unfavored lobbyist resorts to soft-money contributions, a relaxation of the cap only directly affects the unfavored lobbyist. The probability that the politician chooses the policy he does not favor increases; there is an increase in special interest influence. However, if the cap is restrictive enough so that in equilibrium both lobbyists resort to soft-money contributions, the relaxation of the cap leads to a decrease in special interest influence. In this case, although the increase in the cap reduces the cost of contributions for both lobbyists, the decline in the cost is proportionally larger for the favored lobbyist since his rival needs to contribute more in order to overcome the politician's preference. Therefore, the effect of a cap on special interest influence is nonmonotonic whenever a ban on hard-money contributions does not fully suppress all competition. Despite soft-money loopholes, any binding cap reduces special interest influence compared to unregulated contributions. However, a complete ban on hard-money contributions can result in greater special interest influence than a binding but nonzero cap.

In addition, we show that a more restrictive cap may lead to increased aggregate contributions while reducing the degree of special interest influence. This highlights the weakness of using aggregate contributions in the evaluation of the effectiveness of campaign finance legislation.

The results may also have implications for the 2010 Supreme Court ruling on Citizens United *v.* FEC, which allows corporations and unions to buy candidate-specific advertisements from their general funds in the sensitive window of 30 days before a primary and 60 days before a general election. So the ruling enhances the potency of soft-money spending. If prior to the ruling the hard-money contribution cap was set to minimize special interest influence, the model suggests that the cap would now be too low. Hence, in response to the ruling on Citizens United *v.* FEC, increasing—but not eliminating—the hard-money contribution cap may decrease special interest influence.

Section 2 presents the model and its equilibrium. Section 3 gives three equilibrium implications, which provide insight into the effects of a contribution cap that can be circumvented via soft-money contributions. Section 4 provides an illustrative example, and Section 5 concludes with a discussion of potential policy implications in relation to the ruling on Citizens United *v.* FEC.

[3] Since contributing in soft money is less efficient in currying favor than contributing in hard money, Corollary 1 of KW and Corollary 1 to Proposition 1 of GC are directly applicable to our discussion of a hard-money cap with soft-money loopholes.

2. MODEL

Two risk-neutral lobbyists compete for a prize that will be generated by a decision of the incumbent politician. The value of the prize to lobbyist i is denoted by v_i $\forall i \in \{1,2\}$ and $v_1 > v_2 > 0$. The lobbyists make simultaneous contributions that can take the form of hard-money donations directly to the politician (h_i) and/or contributions of soft money (s_i) to groups supportive of the politician. Hard-money donations cannot be higher than the contribution limit, $m \in [0, \infty)$. Although the politician has full discretion over the spending of hard-money contributions, soft-money spending is not under his direct control. Hence, we assume that to the politician one dollar of soft money is worth $1/(1+a)$ dollars of hard money, where a measures the relative inefficiency of soft-money donations, $a > 0$. To the politician the value of a donation package (h_i, s_i) is given by its hard-money equivalent $x_i = h_i + s_i/(1+a)$. Clearly, a lobbyist who attempts to sway the politician's decision with a contribution less than m makes all his contribution in hard money. But if he wants to contribute more than the limit he contributes m in hard money and the rest in soft money. So his cost function is[4]:

$$\text{(1)} \qquad \mathrm{c}(x_i) = h_i + s_i = \begin{cases} x_i & \text{if } x_i \le m, \\ x_i + a(x_i - m) & \text{if } x_i > m. \end{cases}$$

If lobbyist i wins the prize, his payoff is $v_i - \mathrm{c}(x_i)$; otherwise his payoff is $-\mathrm{c}(x_i)$. We can treat the no-cap case as $m \to \infty$.

The politician has a policy preference, either ideologically motivated or induced by the preferences of his constituents. The intensity of the policy preference for the position of lobbyist 2 is put into monetary terms, denoted by $\gamma \in (-\infty, \infty)$.[5] If the politician favors 2's policy position, $\gamma > 0$. If he favors 1's position, $\gamma < 0$. The lobbyist whose policy is favored by the politician is denoted by f. And u denotes the lobbyist with the unfavored policy. If $\gamma = 0$, arbitrarily designate lobbyist 2 as the favored lobbyist, f. The politician awards the prize based on the donations and his preference. He awards the prize to f if $x_f > x_u - |\gamma|$ and to u if $x_f < x_u - |\gamma|$. In case of a tie, each lobbyist has an even chance of winning. Since contributions are sunk, the lobbying game is an all-pay auction.[6] The rules of the game, the valuations of the lobbyists, and the politician's preference are common knowledge.[7]

If the politician's preference is too strong, $\gamma \le -v_2$ or $\gamma \ge v_1$, with or without a binding cap the equilibrium is in pure strategies where neither lobbyist contributes. To win the contest, the unfavored lobbyist needs to contribute at least the hard-money equivalent of $|\gamma|$ even if the favored lobbyist were to contribute nothing. But when $|\gamma|$ is greater than or equal to v_u this is not worth doing irrespective of the level of the cap since no lobbyist would ever be willing to contribute more than his valuation of the prize.

If the preference is not too strong, $-v_2 < \gamma < v_1$, then depending on the inefficiency of soft-money contributions it may be possible to suppress all competition with a sufficiently restrictive cap. Competition is completely suppressed if the cost of a hard-money equivalent

[4] As in KW/GC, the cost function is continuous with a kink at the cap. The composition of donations can be easily recovered from x_i: $h_i = \min(x_i, m)$ and $s_i = \max(0, (x_i - m)(1+a))$.

[5] KW/GC provide results for the case where the politician is indifferent between policy alternatives, $\gamma = 0$.

[6] The all-pay auction without a cap is analyzed by Hillman and Riley (1989) and Baye et al. (1993, 1996). See Konrad (2002), Kaplan et al. (2003), Meirowitz (2008), Siegel (2011), Kirkegaard (2012), and Pastine and Pastine (2012) for all-pay auctions with head starts.

[7] Backward induction in the one-shot game that is analyzed here would have the politician taking his preferred action regardless of contributions since all contributions are sunk. Hence, there would be no contributions. Thus, implicitly we assume that this one-shot game is embedded in a repeated setting so that the politician has an incentive to reward high contributions in order to keep them coming in the future; that is, we assume that the politician can credibly commit to the allocation rule and is motivated to do so.

contribution of $|\gamma|$ meets or exceeds the value of winning the prize for the unfavored lobbyist: $|\gamma| + a(|\gamma| - m) \geq v_u$. So the least restrictive m that can suppress all contributions is

$$m^* = [(1+a)|\gamma| - v_u]/a. \tag{2}$$

If $(1+a)|\gamma| < v_u$ then $m^* < 0$ and competition cannot be completely suppressed even if all hard-money contributions are banned; competition survives via soft-money donations. If soft money is sufficiently inefficient, then $m^* \geq 0$ and for all $m \leq m^*$ competition is completely suppressed; the equilibrium is in pure strategies with no contributions and the prize goes to the lobbyist with the favored policy alternative.

The proposition below describes the equilibrium for all nontrivial cases where there is competition between lobbyists. Lemmata 1–3 in Appendix A, online on the journal's Web site, prove that equilibrium does not exist in pure strategies if $\gamma \in (-v_2, v_1)$ and $m > m^*$. Intuitively, if the favored lobbyist plays the pure strategy of contributing x'_f, the best response of the unfavored lobbyist is either to contribute slightly more than $x'_f + |\gamma|$, guaranteeing victory, or, if that yields a negative payoff, to contribute nothing, incurring no cost. In either case, the favored lobbyist's choice of x'_f would not be optimal and hence he cannot be playing a pure strategy in equilibrium.

The mixed-strategy equilibrium is characterized in two parts depending on which lobbyist has the advantage in the competition. Lobbyist i has the advantage in the contest if he is able to have a positive payoff by contributing enough to win even if j contributes as high as her valuation of the prize. It will prove to be useful to introduce the notation $\bar{x}_i$ as the highest hard-money equivalent contribution that lobbyist i could make and still get a nonnegative payoff from winning. So $\bar{x}_i$ solves $\mathrm{c}(\bar{x}_i) = v_i$ and is given by

$$\bar{x}_i = \begin{cases} v_i & \text{if } v_i \leq m, \\ (v_i + am)/(1+a) & \text{if } v_i > m. \end{cases} \tag{3}$$

If $\bar{x}_f > \bar{x}_u - |\gamma|$, then f has the advantage in the contest. If $\bar{x}_f \leq \bar{x}_u - |\gamma|$, then u has the advantage in the contest. The playing field is level when $\bar{x}_u = \bar{x}_f + |\gamma|$. This is possible only when the low-valuation lobbyist's policy position is mildly favored by the politician $\gamma \in ((v_1 - v_2)/(1+a), v_1 - v_2]$ and the cap is equal to $\tilde{m}$:

$$\tilde{m} = [(1+a)(v_2 + |\gamma|) - v_1]/a. \tag{4}$$

By (3) and (4), for this range of γ, if $m < \tilde{m}$, then $\bar{x}_f > \bar{x}_u - |\gamma|$; that is, the playing field is tilted to the advantage of the favored lobbyist since the restrictive cap makes it expensive for u to overcome the politician's preference. However, if $m \geq \tilde{m}$, then $\bar{x}_f \leq \bar{x}_u - |\gamma|$; that is, the playing field is tilted to the advantage of the unfavored lobbyist. He can offset the disadvantage arising from the preference of the politician since with a less restrictive cap it is less costly to overcome the politician's preference.

PROPOSITION 1. *For all $\gamma \in (-v_2, v_1)$ and $m > m^*$, equilibrium is in mixed strategies and it is characterized by unique cumulative distribution functions $G_f(x_f)$ and $G_u(x_u)$ for the favored lobbyist's and the unfavored lobbyist's hard-money-equivalent political contributions, respectively*.

(i) *If* $\gamma \in (-v_2, 0) \cup (v_1 - v_2, v_1)$ *or if* $\gamma \in ((v_1 - v_2)/(1+a), v_1 - v_2]$ *and* $m < \tilde{m}$, *then* $\bar{x}_f > \bar{x}_u - |\gamma|$ *and*

$$G_u(x_u) = \begin{cases} \dfrac{v_f - \mathrm{c}\,(\bar{x}_u - |\gamma|)}{v_f} & \textit{for } x_u \in [0, |\gamma|] \\ \dfrac{v_f - \mathrm{c}\,(\bar{x}_u - |\gamma|) - |\gamma| + x_u}{v_f} & \textit{for } x_u \in (|\gamma|, \min(m + |\gamma|, \bar{x}_u)] \\ \dfrac{v_f - \mathrm{c}\,(\bar{x}_u - |\gamma|) - am + (1+a)(x_u - |\gamma|)}{v_f} & \textit{for } x_u \in (\min(m + |\gamma|, \bar{x}_u), \bar{x}_u] \\ 1 & \textit{for } x_u \in (\bar{x}_u, \infty) \end{cases}$$

$$G_f(x_f) = \begin{cases} \dfrac{|\gamma| + x_f}{v_u} & \textit{for } x_f \in [0, \min(\max(0, m - |\gamma|), \bar{x}_u - |\gamma|)) \\ \dfrac{(1+a)|\gamma| - am + (1+a)x_f}{v_u} & \textit{for } x_f \in [\min(\max(0, m - |\gamma|), \bar{x}_u - |\gamma|), \\ & \qquad \bar{x}_u - |\gamma|) \\ 1 & \textit{for } x_f \in [\bar{x}_u - |\gamma|, \infty). \end{cases}$$

(ii) *If* $\gamma \in [0, (v_1 - v_2)/(1+a)]$ *or if* $\gamma \in ((v_1 - v_2)/(1+a), v_1 - v_2]$ *and* $m \geq \tilde{m}$, *then* $\bar{x}_f \leq \bar{x}_u - |\gamma|$ *and*

$$G_u(x_u) = \begin{cases} 0 & \textit{for } x_u \in [0, |\gamma|] \\ \dfrac{-|\gamma| + x_u}{v_f} & \textit{for } x_u \in (|\gamma|, \min(m + |\gamma|, \bar{x}_f + |\gamma|)] \\ \dfrac{-(1+a)|\gamma| - am + (1+a)x_u}{v_f} & \textit{for } x_u \in (\min(m + |\gamma|, \bar{x}_f + |\gamma|), \bar{x}_f + |\gamma|] \\ 1 & \textit{for } x_u \in (\bar{x}_f + |\gamma|, \infty) \end{cases}$$

$$G_f(x_f) = \begin{cases} \dfrac{v_u - \mathrm{c}\,(\bar{x}_f + |\gamma|) + |\gamma| + x_f}{v_u} & \textit{for } x_f \in [0, \min(\max(0, m - |\gamma|), \bar{x}_f)) \\ \dfrac{v_u - \mathrm{c}\,(\bar{x}_f + |\gamma|) - am + (1+a)(x_f + |\gamma|)}{v_u} & \textit{for } x_f \in [\min(\max(0, m - |\gamma|), \bar{x}_f), \bar{x}_f) \\ 1 & \textit{for } x_f \in [\bar{x}_f, \infty). \end{cases}$$

PROOF. See Appendix A, which is online on the journal's Web site.[8] $\bar{x}_u$ and $\bar{x}_f$ are given by (3), m^* is given by (2), and $\tilde{m}$ is given by (4). ■

Denote $\bar{m}$ as a barely binding cap where the cap is equal to the maximum of the upper bounds of the supports of the no-cap equilibrium contributions of the two lobbyists. By

[8] In Proposition 1 and in the proof we use the standard definition $[b,c] = \{x \in \mathbb{R} | b \leq x \leq c\}$ so if $b > c$ then $[b,c] = \emptyset$. Open intervals are similarly defined. For all m where $\bar{x}_u \neq \bar{x}_f + |\gamma|$, this game satisfies the conditions for a generic game in Siegel (2011). So the above equilibrium can be found using Siegel's algorithm. However, when $\bar{x}_u = \bar{x}_f + |\gamma|$, the lobbying game is not generic. In this case, Siegel's assumption C3 is violated since the marginal player is not the only player with power equal to 0 (for definitions of the terminology, see Siegel, 2011). In order to show that Proposition 1 holds for all m, including where $\bar{x}_u = \bar{x}_f + |\gamma|$, Appendix A online derives the equilibrium using traditional methods in the spirit of Hillman and Riley (1989) and Baye et al. (1993). This task is simplified using results from Siegel (2009) for the generic cases.

Proposition 1, when $\bar{x}_f > \bar{x}_u - |\gamma|$ the upper bound of the support of u's equilibrium strategy is $\bar{x}_u$. Since f is favored by the politician, f never needs to exceed $\bar{x}_u - |\gamma|$. When $\bar{x}_f \le \bar{x}_u - |\gamma|$, the upper bound of the support of f's equilibrium is $\bar{x}_f$. So u never needs to exceed $\bar{x}_f + |\gamma|$. Using (3), a barely binding cap is given by[9]

$$\bar{m} = \begin{cases} v_u & \text{for } \gamma \in (-v_2, 0] \cup (v_1 - v_2, v_1), \\ v_f + |\gamma| & \text{for } \gamma \in (0, v_1 - v_2]. \end{cases} \tag{5}$$

The cap is binding if $m < \bar{m}$.

Given the equilibrium cumulative distribution function (c.d.f.) of i, lobbyist j is indifferent among all contributions in the support of his equilibrium strategy. The kinks in the equilibrium cumulative distributions arise from the kinks in the rivals' cost functions. When f has nonzero probability of positive contributions both above and below m (where his marginal cost of contributions increases from 1 to $1 + a$), u has a kink in his equilibrium c.d.f. at $m + |\gamma|$. Similarly, when u puts nonzero probability of positive contributions on both sides of m, f has a kink in his equilibrium c.d.f. at $m - |\gamma|$.

Lobbyist u never contributes in $(0, |\gamma|)$, since contributing zero would give the same probability of victory. So if $m < |\gamma|$, u contributes via soft money for all positive contributions in the support of his equilibrium strategy and hence f does not have a kink in his equilibrium c.d.f.[10] However, when the binding cap is greater than $|\gamma|$, lobbyist u has a positive probability of contributing both below and above m. This induces a kink in f's equilibrium c.d.f. at $m - |\gamma|$.[11]

Lobbyist f does not have a positive probability of contributing above the cap if $x_f^{\sup} \le m$. Define m'' as the lowest level of the cap such that f never contributes in soft money when $m \ge m''$. From Proposition 1, the upper bound of f's equilibrium strategy is $x_f^{\sup} = \bar{x}_u - |\gamma|$ when f has the advantage and it is $x_f^{\sup} = \bar{x}_f$ when u has the advantage. Using (3) and Proposition 1, solve for the lowest m where $x_f^{\sup} \le m$:

$$m'' = \begin{cases} v_u - (1+a)|\gamma| & \text{when } \gamma \in (-v_2, 0) \cup ((v_1 - v_2)/(1+a), v_1)\,, \\ v_f & \text{when } \gamma \in [0, (v_1 - v_2)/(1+a)]\,. \end{cases} \tag{6}$$

If the cap is not very restrictive ($m \ge m''$), then f never resorts to soft-money contributions. Hence, u's equilibrium c.d.f. does not have a kink at $m + |\gamma|$.[12]

3. RESULTS

This section presents three corollaries of Proposition 1 that provide insight into the effects of a binding contribution cap that can be circumvented via soft-money contributions. We analyze the degree of special interest influence on policy: the probability that the unfavored policy is enacted, Prob_u. This is equal to the equilibrium probability that the lobbyist with the unfavored policy position contributes at least the hard-money equivalent of $|\gamma|$ more than his rival: $\text{Prob}_u = \int_{|\gamma|}^{\infty} G_f(x - |\gamma|) g_u(x) dx$ where $g_u(\cdot)$ is the equilibrium probability

[9] When $\gamma \in ((v_1 - v_2)/(1+a), v_1 - v_2]$ Part (ii) of Proposition 1 applies when $m \ge \tilde{m}$, so $\bar{m} = v_f + |\gamma| > \tilde{m}$.

[10] If $m < |\gamma|$, in $G_f(x_f)$ the ranges $[0, \min(\max(0, m - |\gamma|), \bar{x}_f)) = \emptyset$ and $[0, \min(\max(0, m - |\gamma|), \bar{x}_u - |\gamma|)) = \emptyset$ in Parts (i) and (ii) of Proposition 1, respectively.

[11] From (3) and (5), when $m \ge \bar{m}$, in Part (i) of Proposition 1 $\min(\max(0, m - |\gamma|), \bar{x}_u - |\gamma|) = m - |\gamma|)$ and in Part (ii) $\min(\max(0, m - |\gamma|), \bar{x}_f) = m - |\gamma|$.

[12] From (3) and (6) when $m \ge m''$, in $G_u(x_u)$ the ranges in Part (i) $(\min(m + |\gamma|, \bar{x}_u), \bar{x}_u] = \emptyset$ and $\min(m + |\gamma|, \bar{x}_u) = \bar{x}_u$ and in Part (ii) $\min(m + |\gamma|, \bar{x}_f + |\gamma|) = \bar{x}_f + |\gamma|$.

density function (p.d.f.) of the unfavored lobbyist's contribution.[13] Using the equilibrium cumulative distributions one can also calculate the expected contributions in hard and soft money:

$$E(h_u + s_u + h_f + s_f) = \int_0^m x g_u(x)dx + \int_m^\infty [x + a(x - m)]g_u(x)dx + \int_0^m x g_f(x)dx + \int_m^\infty [x + a(x - m)]g_f(x)dx.$$

Although straightforward, these calculations are tedious. One needs to keep track of the min/max arguments in Proposition 1 to employ the relevant set of equilibrium distribution functions depending on the level of the cap. Furthermore, if the advantage in the contest switches from one lobbyist to the other in response to changes in the level of the cap, then one needs to keep track of the move from one set of equilibrium distribution functions to the next. Section B of the Appendix gives the changes in $E(h_u + s_u + h_f + s_f)$ and Prob_u with respect to small changes in m for all $\gamma \in (-v_2, 0) \cup (0, v_1)$ and $m^* \le m < \bar{m}$. Naturally, an increase of a nonbinding cap has no effect on the equilibrium of the contest.

COROLLARY 1. $\lim_{\Delta m \to 0^+} \Delta E(h_u + s_u + h_f + s_f)/\Delta m \neq 0 \; \forall \, \gamma \in (-v_2, 0) \cup (0, v_1)$ *and* $m \in [m^*, \bar{m})$; *that is, with strict politician preferences a binding cap on hard-money contributions is never neutral on expected aggregate political contributions.*

PROOF. See Section B of the Appendix. The values of m^* and $\bar{m}$ are given in (2) and (5). ■

In KW/GC, the cap is always neutral on lobbying effort when there is a monetary cost of exceeding the limit. The cap introduces a kink in the cost functions of the lobbyists, but it does not change lobbyists' relative strengths. However, when the politician has a strict policy preference, a contribution cap affects the two lobbyists in an asymmetric fashion since u needs to contribute more than f to win. If the cap is only effectively binding for the unfavored lobbyist ($m'' \le m < \bar{m}$) so that in equilibrium only u has positive probability of contributing in soft money, a relaxation in the cap decreases only u's cost of donation. If the cap is effectively binding for both lobbyists ($m < m''$) so that both have a positive probability of contributing in soft money, a relaxation in the cap reduces both lobbyists' costs. But the reduction is proportionally higher for the favored lobbyist, since in order to win, the unfavored lobbyist must exceed the rival's contribution to overcome the politician's policy preference. So a change in the cap alters the lobbyists' relative strengths and therefore changes the intensity of the competition and expected aggregate contributions.

COROLLARY 2. $\text{Prob}_u|_{m<\bar{m}} \le \text{Prob}_u|_{m \ge \bar{m}} \; \forall \, \gamma \in (-v_2, 0) \cup (0, v_1)$; *that is, with strict politician preferences any binding cap on hard-money contributions reduces the degree of special interest influence on policy outcomes compared to unregulated contributions.*

PROOF. See Section B of the Appendix. The value of $\bar{m}$ is given by (5). ■

In order to win the lobbying contest, u must exceed the contribution of f by at least the hard-money equivalent of the politician's policy preference $|\gamma|$. Hence, with a binding cap, u will be more reliant on soft money than f. So any binding cap reduces Prob_u compared with unregulated contributions.[14]

[13] The p.d.f. $g_u(.)$ is not uniquely determined, but it must be almost everywhere equal to the derivative of $G_u(.)$. Any such p.d.f. will integrate to the same value and hence for any m, Prob_u is uniquely determined and can be calculated using the derivative of $G_u(.)$ for $g_u(.)$.

[14] The inequality in Corollary 2 is strict except at $m = 0$, when $\gamma \in ((v_1 - v_2)/(1 + a), v_1 - v_2]$.

The stated aim of the current legislation (BCRA) is a reduction of special interest influence. This goal may or may not be desirable, but one reasonable interpretation of it is an attempt to reduce the likelihood that the politician enacts policy despite his policy preference. In practice, a politician is likely to have different degrees of preference across policy issues. The value of the prize to the interest groups varies depending on the policy area. However, irrespective of the degree of politician preference and lobbyists' valuations, the corollary above shows that any binding contribution cap leads to weaker influence of money on policy compared to unregulated contributions.

In the absence of soft-money loopholes, a ban on hard-money contributions would result in the politician always enacting his preferred policy, so the influence of special interest money would be eliminated.[15] However, when there are soft-money loopholes, Corollary 3 shows that special interest influence is not minimized with a complete ban on hard-money contributions whenever $m^* < 0$.

COROLLARY 3. *For all* $\gamma \in (-v_2, 0) \cup (0, v_1)$*, the degree of special interest influence on policy,* Prob_u*, is minimized with a binding but strictly positive cap on hard-money contributions of* $m'' \in (0, \bar{m})$ *whenever a ban on hard-money contributions does not fully suppress all competition,* $m^* < 0$.

PROOF. See Section B of the Appendix. The values of m^*, $\bar{m}$, and m'' are given by (2), (5), and (6), respectively. ■

If the cap is not too restrictive ($m'' \leq m < \bar{m}$), then only the unfavored lobbyist has a positive probability of contributing in soft money. Therefore, an increase in the cap reduces only u's expected cost of contributions. This induces u to contribute more aggressively and hence special interest influence goes up. However, if the cap is restrictive enough such that both u and f contribute in soft money with positive probability ($0 \leq m < m''$), an increase in the cap lowers the cost of contributions for both u and f. But the decline in cost is proportionally smaller for u since he relies more heavily on soft money to counteract the politician's policy preference. So special interest influence declines.[16]

4. AN EXAMPLE

This section considers an illustrative example employing particular ranges of parameter values. Figure 1 graphs Prob_u and $E(h_u + s_u + h_f + s_f)$ as functions of m for $\gamma \in ((v_1 - v_2)/(1 + a), v_1 - v_2)$, where the politician's preference for the policy position of the

[15] With politician preferences, caps without soft-money loopholes are analyzed in Pastine and Pastine (2010).

[16] The U-shape of the Prob_u is driven by the politician's policy preference, which causes the lobbyists' relative reliance on soft money to depend on m. Hence the U-shape is likely to hold in a variety of other model specifications. For instance, modify the model such that there are $n > 2$ lobbyists and let the politician's preference for the policy of i be $\gamma_i \geq 0\ \forall i \in \{1,2, \ldots, n\}$ and $\gamma_i \neq \gamma_j\ \forall\ i \neq j$. $x_i + \gamma_j$ is the player's "score." First consider $m = 0$. The cost functions would be linear with marginal cost equal to $1 + a$ and would satisfy the noncrossing condition in Siegel (2011). So his algorithm applies for parameterizations that yield generic games. This implies that in equilibrium there is at least one player k with a positive probability of choosing a score just above his head start (with negligible spending). For each such score, there must be at least one additional player who chooses the same score with positive probability—otherwise k could decrease his contribution without affecting his win probability. But the additional player(s) must be spending a strictly positive amount to achieve that score, and hence their γ's must be lower than γ_k. So when m increases slightly from zero, it does not alter the marginal cost for the less favored lobbyist(s) in this group of players. But it decreases the marginal cost for the most favored lobbyist in the group. Hence it is likely to lead the most favored lobbyist to contribute more aggressively and to increase his win probability. In the absence of a contribution cap, cost functions are also linear, with marginal cost equal to 1. Let z be the lobbyist with the highest contribution in the support of his equilibrium strategy, x_z^{sup}. There must be at least one more lobbyists who puts a positive probability on the same score ($x_z^{\text{sup}} + \gamma_z$), as z would get contributing x_z^{sup}. Otherwise z could lower his spending without decreasing his win probability. But of these lobbyists, z has the lowest γ, since he spends the most to achieve that score. Hence, a barely binding cap only increases the cost for this less preferred lobbyist and so it is likely to decrease his win probability.

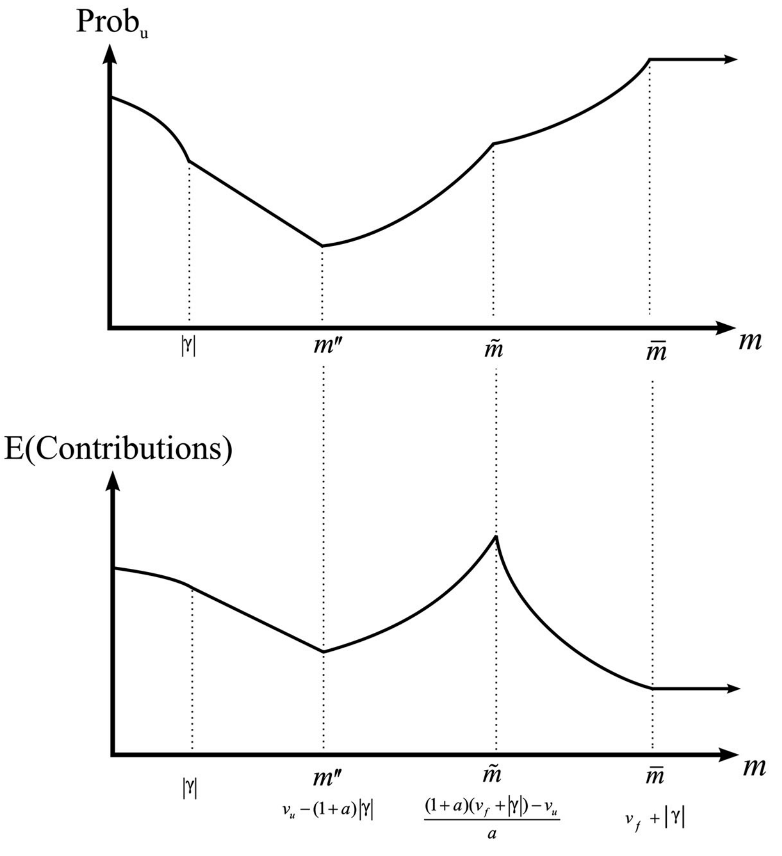

FIGURE 1

THE PROBABILITY THAT THE UNFAVORED POLICY IS ENACTED AND EXPECTED AGGREGATE CONTRIBUTIONS FOR $\gamma \in ((v_1 - v_2)/(1+a), v_1 - v_2)$, $a \in (0, (2v_2 - v_1)/(v_1 - v_2))$, AND $2v_2 > v_1$

low-valuation lobbyist is mild, and $a \in (0, (2v_2 - v_1)/(v_1 - v_2))$, so the inefficiency of soft-money contributions is not too strong and $2v_2 > v_1$.[17] For the parameter values in this example $m^* < 0$, see (2). So even if there is a ban on hard-money contributions, there is competition via soft money and therefore $\text{Prob}_u > 0$ when $m = 0$. The barely binding cap is equal to $\bar{m} = v_f + |\gamma|$; see (5). So $m > v_f + |\gamma|$ has no effect on the equilibrium of the game and hence does not affect Prob_u nor expected contributions. By Equations (4)–(6), $|\gamma| < m'' < \tilde{m} < \bar{m}$.

When $m < \tilde{m}$ Part (i) of Proposition 1 applies so f is advantaged. In the range $m \in [0, m'')$, both f and u have a positive probability of contributing in soft money. Relaxing the cap tilts the already uneven playing field further in favor of f since the reduction in f's cost of contributions is proportionally larger. This reduces the intensity of the competition, and hence the expected contributions and Prob_u decline. In the range $m \in [m'', \tilde{m})$, only u has a positive probability of contributing in soft money. Hence, relaxing the cap helps level the uneven playing field, which induces more intense competition, so aggregate contributions and Prob_u increase. Expected contributions are maximized at $\tilde{m}$, as a level playing field where $\bar{x}_u = \bar{x}_f + |\gamma|$ yields the fiercest competition. When $m \geq \tilde{m}$ Part (ii) of Proposition 1 applies so u is advantaged. Relaxing the

[17] The comparative statics are shown in the Table B1 in Section B of the Appendix. For these parameter values, the relevant ranges of m in Table B1 are $m \in [0, |\gamma|)$ range I, $m \in [|\gamma|, m'')$ range III, $m \in [m'', \tilde{m})$ range IV, and $m \in [\tilde{m}, \bar{m})$ range VIII.

cap from $\tilde{m}$ tilts the playing field toward u, which discourages f, leading to less effort by f and hence by u, so the overall expected contributions decrease.[18] The effect of the cap on special interest influence is nonmonotonic.[19] The probability that the politician enacts his less favored policy (Prob_u) is minimized at m'', see (6).[20]

By explicitly incorporating politician preferences, we are able to capture the sense in which special interest money can overwhelm the politician's natural inclinations, the sense in which it can have an undue influence. Note that the example demonstrates that the choice of measure matters in the evaluation of campaign finance legislation that contains soft-money loopholes. For the parameter values in this example when $\tilde{m} < m < \bar{m}$, a more restrictive cap leads to increased aggregate contributions while at the same time reducing the degree of special interest influence.

5. DISCUSSION

The model restricts attention to contributions motivated by policy favors. In reality, a lobbyist may also be contributing to gain access to the politician or to improve the politician's electoral prospects if he shares the policy preference of the lobbyist. Nevertheless, the model may provide a potentially useful albeit incomplete framework to think about policy issues.

For example, in the case Citizens United *v.* FEC (2010), the Supreme Court struck down the BCRA's "electioneering communications" restrictions on soft-money spending. The ruling allows corporations and unions to buy candidate-specific advertisements in television and radio stations, cable television, or satellite systems within 60 days prior to the general election or 30 days prior to a primary. So the ruling enhances the effectiveness of soft-money spending. This increases the value of soft-money donations to the politician and reduces parameter a.

In the model, whenever a ban on hard money does not completely suppress competition via soft money, the cap that minimizes special interest influence is given by m'' (Corollary 3). Note that m'' is weakly decreasing in a; see (6). If previously the cap was set to minimize special interest influence, with the reduction in a, at the ongoing cap not only the unfavored lobbyist but also the favored lobbyist would have a positive probability of contributing in soft money. Hence, the cap would be too restrictive to minimize special interest influence. Of course, the model's parameter values, v_u and γ, and hence m'', vary over policy issues. But for each issue, the level of cap that minimizes special interest influence is weakly decreasing in a. Hence, the

[18] Che and Gale (1998) show that a more restrictive cap can lead to increased aggregate contributions when the cap is rigid. The introduction of politician preferences retrieves the result of Che and Gale (1998) even though the cap is non-rigid as in KW/CG. There exists a range of m where expected aggregate political contributions strictly increase with a more restrictive cap: $m \in [\max(m'', |\gamma|), \bar{m})$ for $\gamma \in (0, (v_1 - v_2)/((1+a)]$ and $m \in (\max(\tilde{m}, |\gamma|), \bar{m})$ for $\gamma \in ((v_1 - v_2)/((1+a), (v_1 - v_2)]$.

[19] This result is also likely to hold in frameworks that yield pure-strategy equilibria. Consider the incomplete information framework in Kirkegaard (2012) with head-start advantage; the equilibrium is applicable to our discussion when there are no kinks in the cost functions, when $m > \bar{m}$ (marginal cost equal to one) and when $m = 0$ (marginal cost equal to $1 + a$). Valuations are private information and are drawn from continuous distributions on $[0, \bar{v}_i]\, \forall i \in \{u, f\}$ and $\bar{v}_f < \bar{v}_u$. Equilibrium is in pure strategies: A player makes no contribution if his valuation is below an endogenous critical score v_i^c, whereas for valuations above v_i^c his contribution is a continuous increasing function of v_i. Consider an ex ante barely binding cap that is just below the maximum of contributions that either lobbyist would choose with positive probability in the absence of a cap. In equilibrium, the highest valuation type of u (with $\bar{v}_u$) spends $|\gamma|$ more than the highest valuation type of f (with $\bar{v}_f$). A reduction in m only directly affects the player with the highest unconstrained spending. Therefore, restricting the cap at the margin only increases the marginal cost for the highest valuation u type and leaves the marginal cost for all f types unaltered. Hence, a slight decrease in m is likely to decrease Prob_u. With $m = 0$, examine the type of u with a valuation just above v_u^c. He contributes just above $|\gamma|$, although the type of f with a valuation just above v_f^c contributes just above 0. So raising the cap from $m = 0$ leaves u's marginal cost unaltered and reduces f's marginal cost, giving him an additional incentive to contribute aggressively. As the marginal costs of the higher valuation types of u and f are unchanged, the effects of the relaxation of m on their choices are second order. Hence, a slight increase in m is likely to decrease Prob_u.

[20] Note that when soft money is sufficiently inefficient, $a > (v_u - |\gamma|)/|\gamma|$, m'' is negative and m^* is positive. This implies that f never exceeds m and only u is directly affected by a relaxation of m. Prob_u then does not have a U-shape as depicted in Figure 1. It is 0 for all $m \leq m^*$, and it is an increasing function of m for all $m \in (m^*, \bar{m})$.

model suggests that in response to the Citizens United *v.* FEC (2010) ruling, relaxing—but not eliminating—the cap on hard-money contributions may decrease special interest influence.

APPENDIX

A. *Proof of Proposition 1.* Online at the journal's Web site.

B. *Proof of Corollaries.* From Proposition 1 it is straightforward to calculate $\text{Prob}_u = \int_{|\gamma|}^{\infty} G_f(x-|\gamma|)g_u(x)dx$ and $E(h_u+s_u+h_f+s_f) = \int_0^m xg_u(x)dx + \int_m^\infty [x+a(x-m)]\,g_u(x)dx + \int_0^m xg_f(x)dx + \int_m^\infty [x+a(x-m)]\,g_f(x)dx$. To prove the corollaries, we calculate the changes in these values w.r.t. changes in m for all $\gamma \in (-v_2, 0) \cup (0, v_1)$ and $m > \max(0, \text{m*})$.

The equilibrium c.d.f.s are uniquely determined. The equilibrium p.d.f.s are equal to the derivatives of the c.d.f.s almost everywhere. However, since at individual points the p.d.f.s may deviate from the derivatives of the c.d.f.s, Prob_u and $E(h_u + s_u + h_f + s_f)$ need not be differentiable. Hence, to find the comparative statics w.r.t. changes in m, we work with limits instead.

In Table B1, we present $\lim_{\Delta m \to 0^+}(\Delta E(h_f+s_f+h_u+s_u)/\Delta m)$ and $\lim_{\Delta m \to 0^+}(\Delta \text{Prob}_u/\Delta m)$ for all $m \in [\max(0, m^*), \bar{m})$. The c.d.f.s and hence these limits, depend on which part of Proposition 1 applies and on the relationship between m, $|\gamma|$, and m'', given by (6). To simplify the replication of the results, Table B1 first shows the value of the min/max arguments that appear in the c.d.f.s. Finally, Table B1 is employed to prove the three corollaries.

B.1. *Determining the value of min/max arguments for the ranges in Table 1*

B.1.1. *When Part (i) of Proposition 1 holds and $m < \bar{m}$.* *Ranges I and III* ($0 \le m < m''$): For the values of γ where Proposition 1, Part (i) applies, $m'' = v_u - (1+a)|\gamma|$ from (6). Hence, $m'' < v_u$ and $\bar{x}_u = (v_u + am)/(1+a)$ by (2). Since $m < m'' = v_u - (1+a)|\gamma|$, $\min(m'' + |\gamma|, \bar{x}_u) = m + |\gamma|$. In Range I ($m<|\gamma|$ and $m < m''$), $\min(\max(0, m-|\gamma|), \bar{x}_u - |\gamma|) = 0$ as $0 \le m < m = v_u - (1+a)|\gamma|$ implies $\bar{x}_u > |\gamma|$. In Range III ($m \ge |\gamma|$ and $m < m''$), $\min(\max(0, m-|\gamma|), \bar{x}_u - |\gamma|) = m - |\gamma|$ since $m \ge |\gamma|$ and $m < \bar{x}_u$.

Ranges II and IV ($\bar{m} > m \ge m''$): From (5) if $\gamma \in (-v_2, 0) \cup (v_1 - v_2, v_1)$ then $\bar{m} = v_u$. The other possibility where Proposition 1, Part (i) can apply is when $\gamma \in ((v_1-v_2)/(1+a), v_1 - v_2]$ and $m \le \tilde{m} = [(1+a)(v_f + |\gamma|) - v_u]/a$, which implies that $m < v_u$ since $\gamma < v_1 - v_2$. Since in either case $m < v_u$, so by (2) $\bar{x}_u = (v_u + am)/(1+a)$. For the values of γ where Proposition 1. Part (i) applies, from (6) $m'' = v_u - (1+a)|\gamma|$. Therefore, $m \ge m''$ implies $\min(m + |\gamma|, \bar{x}_u) = \bar{x}_u = (v_u + am)/(1+a)$. In Range II ($m < |\gamma|$ and $m \ge m''$), $\min(\max(0, m-|\gamma|), \bar{x}_u - |\gamma|) = 0$ since $m \ge m^*$ implies $\bar{x}_u - |\gamma| \ge 0$. In Range IV ($m \ge |\gamma|$ and $m \ge m''$), $\min(\max(0, m-|\gamma|), \bar{x}_u - |\gamma|) = m - |\gamma|$ since $m \ge |\gamma|$ and $m < v_u$.

B.1.2. *When Part (ii) of Proposition 1 holds and $m < \bar{m}$.* *Ranges V and VII* ($0 \le m < m''$): These ranges are possible only if $\gamma \in (0, (v_1 - v_2)/(1+a)]$. If $\gamma \in ((v_1 - v_2)/(1+a), v_1 - v_2)$ then to be in Proposition 1, Part (ii) it must be that $m > \tilde{m}$; however $m'' = v_u - (1+a)|\gamma|$ by (6), and so $\tilde{m} > m''$, a contradiction of $m < m''$. If $\gamma \in (0, (v_1 - v_2)/(1+a)]$, then from (6) $m'' = v_f$. In both ranges V and VII $m < m'' \Rightarrow m < v_f$ and by (2) $\bar{x}_f = (v_f + am)/(1+a)$. Note that $\bar{m} = v_f + |\gamma|$ by (5). Therefore, $\min(m + |\gamma|, \bar{x}_f + |\gamma|) = m + |\gamma|$. So in Range V ($m < |\gamma|$ and $m < m''$), $\min(\max(0, m-|\gamma|), \bar{x}_f) = 0$ since $m < |\gamma|$. But in Range VII ($m \ge |\gamma|$ and $m < m''$), $\min(\max(0, m-|\gamma|), \bar{x}_f) = m - |\gamma|$ since $m \ge |\gamma|$ and $m < \bar{m}$.

Ranges VI and VIII ($\bar{m} > m \ge m''$): If $\gamma \in (0, (v_1 - v_2)/(1+a)]$, then from (6) $m'' = v_f$. If $\gamma \in ((v_1 - v_2)/(1+a), v_1 - v_2)$, then m must be greater than $\tilde{m}$ for Proposition 1, Part (ii) to apply. From (4) $\tilde{m} > v_f$, so $m > v_f$. In both ranges VI and VIII, $m \ge m'' \Rightarrow m \ge v_f$, and by (2) $\bar{x}_f = v_f$. Therefore, $\min(m + |\gamma|, \bar{x}_f + |\gamma|) = v_f + |\gamma|$. Note that $\bar{m} = v_f + |\gamma|$ by (5). So in Range VI

TABLE B1
COMPARATIVE STATICS $\forall\gamma \in (-v_2, 0) \cup (0, v_1)$ AND $m \in [\max(0, m^*), \bar{m})$

Proposition 1, Part (i) Applies and		$\bar{x}_u$	$\min(m + \lvert\gamma\rvert, \bar{x}_u)$	$\min(\max(0, m - \lvert\gamma\rvert), \bar{x}_u - \lvert\gamma\rvert)$	$\lim_{\Delta m \to 0^+}\left(\frac{\Delta E(h_f + s_f + h_u + s_u)}{\Delta m}\right)$	$\lim_{\Delta m \to 0^+}\left(\frac{\Delta Prob_u}{\Delta m}\right)$
I	$m < \lvert\gamma\rvert$ and $m < m''$	$\frac{v_u + am}{1+a}$	$m + \lvert\gamma\rvert$	0	$am\left(\frac{v_f - v_u}{v_u v_f}\right) = \begin{cases} >0 \text{ if } f = 1 \\ <0 \text{ if } f = 2 \end{cases}$	$\frac{-am}{v_u v_f} < 0$
II	$m < \lvert\gamma\rvert$ and $m \geq m''$	$\frac{v_u + am}{1+a}$	$\bar{x}_u = \frac{v_u + am}{1+a}$	0	$\frac{a(\bar{x}_u - \lvert\gamma\rvert)}{v_u} + \frac{a\lvert\gamma\rvert + a^2(\lvert\gamma\rvert - m)}{(1+a)v_f} > 0$	$\frac{a(\lvert\gamma\rvert + a(\lvert\gamma\rvert - m))}{(1+a)v_u v_f} > 0$
III	$m \geq \lvert\gamma\rvert$ and $m < m''$	$\frac{v_u + am}{1+a}$	$m + \lvert\gamma\rvert$	$m - \lvert\gamma\rvert$	$a\lvert\gamma\rvert\left(\frac{v_f - v_u}{v_u v_f}\right) = \begin{cases} >0 \text{ if } f = 1 \\ <0 \text{ if } f = 2 \end{cases}$	$\frac{-a\lvert\gamma\rvert}{v_u v_f} < 0$
IV	$m \geq \lvert\gamma\rvert$ and $m \geq m''$	$\frac{v_u + am}{1+a}$	$\bar{x}_u = \frac{v_u + am}{1+a}$	$m - \lvert\gamma\rvert$	$\frac{a(\bar{x}_u - m)}{v_u} + \frac{am}{(1+a)v_f} > 0$	$\frac{am}{(1+a)v_u v_f} > 0$
Proposition 1, Part (ii) Applies and		$\bar{x}_f$	$\min\left(m + \lvert\gamma\rvert, \bar{x}_f + \lvert\gamma\rvert\right)$	$\min\left(\max(0, m - \lvert\gamma\rvert), \bar{x}_f\right)$	$\lim_{\Delta m \to 0^+}\left(\frac{\Delta E(h_f + s_f + h_u + s_u)}{\Delta m}\right)$	$\lim_{\Delta m \to 0^+}\left(\frac{\Delta Prob_u}{\Delta m}\right)$
V	$m < \lvert\gamma\rvert$ and $m < m''$	$\frac{v_f + am}{1+a}$	$m + \lvert\gamma\rvert$	0	$am\left(\frac{v_f - v_u}{v_u v_f}\right) = \begin{cases} >0 \text{ if } f = 1 \\ <0 \text{ if } f = 2 \end{cases}$	$\frac{-am}{v_u v_f} < 0$
VI	$m < \lvert\gamma\rvert$ and $m \geq m''$	v_f	$v_f + \lvert\gamma\rvert$	0	$-a < 0$	0
VII	$m \geq \lvert\gamma\rvert$ and $m < m''$	$\frac{v_f + am}{1+a}$	$m + \lvert\gamma\rvert$	$m - \lvert\gamma\rvert$	$a\lvert\gamma\rvert\left(\frac{v_f - v_u}{v_u v_f}\right) = \begin{cases} >0 \text{ if } f = 1 \\ <0 \text{ if } f = 2 \end{cases}$	$\frac{-a\lvert\gamma\rvert}{v_u v_f} < 0$
VIII	$m \geq \lvert\gamma\rvert$ and $m \geq m''$	v_f	$v_f + \lvert\gamma\rvert$	$m - \lvert\gamma\rvert$	$\frac{a(m - (v_f + \lvert\gamma\rvert))}{v_f} - \frac{a(m - \lvert\gamma\rvert)}{v_u} < 0$	$\frac{a(m - \lvert\gamma\rvert)}{v_u v_f} > 0$

($m < |\gamma|$ and $m \geq m''$), $\min(\max(0, m - |\gamma|), \bar{x}_f) = 0$ since $m < |\gamma|$. But in Range VIII ($m \geq |\gamma|$ and $m \geq m''$), $\min(\max(0, m - |\gamma|), \bar{x}_f) = m - |\gamma|$ since $m \geq |\gamma|$ and $m < \bar{m}$.

B.2. *Determining the change in $E(h_f + s_f + h_u + s_u)$ and $Prob_u$ w.r.t. changes in m in Table B1.* It is tedious but straightforward to calculate the limits shown in the final two columns of Table B1. In the interest of space here we only demonstrate the calculation of $\lim_{\Delta m \to 0^+}(\frac{\Delta \text{Prob}_u}{\Delta m})$ in the first row of Table B1. In row I of Table B1 Part (i) of Proposition 1 applies along with the min/max arguments derived above and presented in Table B1. $m < \bar{x}_u - |\gamma|$ since $\min(m + |\gamma|, \bar{x}_u) = m + |\gamma|$, so from (1) $c(\bar{x}_u - |\gamma|) = (1 + a)(\bar{x}_u - |\gamma|) - am$. $\bar{x}_u = (v_u + am)/(1 + a)$, so $c(\bar{x}_u - |\gamma|) = v_u - (1 + a)|\gamma|$. Therefore, Proposition 1, Part (i) yields the equilibrium cumulative distribution functions:

$$G_u(x_u) = \begin{cases} \dfrac{v_f - v_u + (1+a)|\gamma|}{v_f} & \text{for } x_u \in [0, |\gamma|] \\ \dfrac{v_f - v_u + a|\gamma| + x_u}{v_f} & \text{for } x_u \in [|\gamma|, m + |\gamma|] \\ \dfrac{v_f - v_u - am + (1+a)x_u}{v_f} & \text{for } x_u \in \left(m + |\gamma|, \dfrac{v_u + am}{1+a}\right] \\ 1 & \text{for } x_u \in \left(\dfrac{v_u + am}{1+a}, \infty\right) \end{cases}$$

$$G_f(x_f) = \begin{cases} \dfrac{(1+a)|\gamma| - am + (1+a)x_f}{v_u} & \text{for } x_f \in \left[0, \dfrac{v_u + am - (1+a)|\gamma|}{1+a}\right) \\ 1 & \text{for } x_f \in \left[\dfrac{v_u + am - (1+a)|\gamma|}{1+a}, \infty\right) \end{cases}.$$

The probability that the unfavored policy is enacted is given by

$$\text{Prob}_u = \int_{|\gamma|}^{m+|\gamma|} G_f(x - |\gamma|)g_u(x)dx + \int_{m+|\gamma|}^{\bar{x}_u} G_f(x - |\gamma|)g_u(x)dx.$$

Although the p.d.f. $g_u(\cdot)$ is not uniquely determined, it must be almost everywhere equal to the derivative of $G_u(\cdot)$. Any such p.d.f. will integrate to the same value, and hence for any *m*, Prob_u is uniquely determined and can be calculated using the derivative of $G_u(\cdot)$:

$$\begin{aligned} \text{Prob}_u &= \int_{|\gamma|}^{m+|\gamma|} \left(\frac{(1+a)x - am}{v_u}\right) \frac{1}{v_f} dx + \int_{m+|\gamma|}^{\bar{x}_u} \left(\frac{(1+a)x - am}{v_u}\right) \frac{(1+a)}{v_f} dx \\ &= \frac{1}{2v_u v_f}((1+a)(m+|\gamma|)^2 - (1+a)|\gamma|^2 - 2am(m+|\gamma|) + 2am|\gamma|) \\ &\quad + \frac{(1+a)}{2v_u v_f}\left((1+a)\bar{x}_u^2 - 2am\bar{x}_u - (1+a)(m+|\gamma|)^2 + 2am(m+|\gamma|)\right). \end{aligned}$$

Row I of Table B1 applies for $m \in [\max(0, m^*), \min(|\gamma|, m''))$. Take a level of cap m in this range and $\Delta m \in (0, \min(|\gamma|, m'') - m)$ so that both m and $m + \Delta m$ are in the specified range. Let $\Delta \text{Prob}_u = \text{Prob}_u|_{m+\Delta m} - \text{Prob}_u|_m$ and take the limit as $\Delta m \to 0$. This yields $\lim_{\Delta m \to 0^+}(\frac{\Delta \text{Prob}_u}{\Delta m}) = \frac{-am}{v_u v_f} < 0$.

A similar exercise is conducted to find the other limits in Table B1.

B.3. *Proof of Corollary 1.* From Table B1 $\forall \gamma \in (-v_2, 0) \cup (0, v_1)$ and $m \in [\max(0, m^*), \bar{m})$ changes in m are never neutral on expected contributions. ■

B.4. *Proof of Corollaries 2 and 3.* In each part of the Proposition 1, at each x_i and for each player $G_i(x_i)$ is clearly continuous in m. When $\gamma \in (\frac{v_1 - v_2}{1+a}, v_1 - v_2]$ the distribution switches from that of Part (i) of Proposition 1 to that of Part (ii) at $m = \frac{(1+a)(v_2+\gamma)-v_1}{a} = \tilde{m}$. However, for each distribution, at each x_i, $\lim_{m \to \tilde{m}^-} G_i(x_i) = G_i(x_i)\big|_{m=\tilde{m}}$. If $a \leq [v_u - |\gamma|]/|\gamma|$ then all competition is curtailed at $m^* = [(1+a)|\gamma| - v_u]/a$. At m just above that point, the favored bidder is advantaged, so the distribution of Proposition 1, Part (i) applies and $\lim_{m \to m^{*}+} G_i(0) = 1$ results in no competition between the bidders. Hence, in all cases, at each x_i and for each player $G_i(x_i)$ is continuous in m and so Prob_u is also continuous in m.

When $m^* < 0$ from (3) $v_u > (1+a)|\gamma|$ and from (5) and (6) $m'' \in (0, \bar{m})$, so Table B1 shows that Prob_u is U-shaped. It decreases in m for all $m \in (0, m'')$ and is weakly increasing in m for all $m \in (m'', \bar{m})$. Moreover, (5) implies that $\bar{m} > |\gamma|$, so range VI in Table B1 (where there is no effect of changes in m on Prob_u) must be strictly below $\bar{m}$. This completes the proof of Corollary 3. It also shows that whenever $m^* < 0$, then Prob_u will be maximized at one of the endpoints, either $m = 0$ or $m = \bar{m}$.

If $\gamma \in (0, (v_1 - v_2)/(1+a)]$ then $m^* \geq 0$ is not possible. Hence, from Equations (5) and (6), whenever $m^* \geq 0, m'' = v_u - (1+a)|\gamma| \leq 0$. So $m > m'' \, \forall m \in (m^*, \bar{m})$, and thus Table B1 shows that Prob_u is weakly increasing in m over this entire range. Since range VI in Table B1 (where there is no effect of changes in m on Prob_u) must be strictly below $\bar{m}$, it follows that if $m^* \geq 0$, any level of cap $m < \bar{m}$ will reduce Prob_u compared to no contribution cap.

It only remains to show that when $m^* < 0$ the level of Prob_u is weakly lower at $m = 0$ than at $m = \bar{m}$. Without a cap, when $\gamma \in (-v_2, 0) \cup (v_1 - v_2, v_1)$ then Part (i) of Proposition 1 applies and

$$\text{(B.1)} \qquad \text{Prob}_u = \int_{|\gamma|}^{\infty} G_f(x - |\gamma|) g_u(x) dx = \left(v_u^2 - \gamma^2\right) / (2 v_u v_f).$$

When $m = 0$ and $\gamma \in (-v_2, 0) \cup ((v_1 - v_2)/(1+a), v_1)$, then Part (i) of Proposition 1 applies and

$$\text{(B.2)} \qquad \text{Prob}_u = \int_{|\gamma|}^{\infty} G_f(x - |\gamma|) g_u(x) dx = \left[v_u^2 - (1+a)^2 \gamma^2\right] / (2 v_u v_f),$$

which is strictly less than (7) and so Part (i) holds when $\gamma \in (-v_2, 0) \cup (v_1 - v_2, v_1)$. When $\gamma \in (0, v_1 - v_2]$ and there is no cap then Part (ii) of Proposition 1 applies and

$$\text{(B.3)} \qquad \text{Prob}_u = \int_{|\gamma|}^{\infty} G_f(x - |\gamma|) g_u(x) dx = 1 - v_f / (2 v_u).$$

When $\gamma \in ((v_1 - v_2)/(1+a), v_1 - v_2]$ Prob_u is given by (8) when $m = 0$, which is strictly less than (9) for $\gamma > (v_1 - v_2)/(1+a)$, so Corollary 2 holds for this range of γ. Finally, when $m = 0$ and $\gamma \in (0, (v_1 - v_2)/(1+a)]$, then Part (ii) of Proposition 1 applies, so $\text{Prob}_u = \int_{|\gamma|}^{\infty} G_f(x - |\gamma|) g_u(x) dx = 1 - v_f/(2v_u)$, which is equal to (9), so for this final range of γ Corollary 2 holds weakly for a complete ban on contributions and holds strictly for any $\gamma \in (0, \bar{m})$.

An Addendum online on the journal's Web site expands Table B1 based on parameter values, which allows the reader to easily find the comparative statics results for any $\gamma \in (-v_2, 0) \cup (0, v_1)$ directly without having to work through the concise summary in Table B1.

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