Equality and Curriculum in Education

A Collection Of Invited Essays

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Chapter 3

Accessing the maths curriculum; applying psychological theory to help students with specific learning difficulties and with benefits for all.

Maeve Daly

It is important to take an ecological view of the problem of failure in mathematics, not only recognising the difficulties that are embedded at an individual level (cognitive and emotional factors), but also contextual factors (inappropriate instruction, the subject itself, curriculum and assessment) that impact on the problem. With the aim of promoting deeper conceptual understanding of mathematics, the contextual issues surrounding teaching approach in mathematics will be discussed. The extent to which appropriate instruction, which is grounded in developmental methods, alleviates difficulties at the individual level and the contextual level will be examined. This will be done through a discussion of the theory that has informed the practice of developmental methods and examples of this practice in the classroom will also be highlighted. The nature of specific learning difficulty (SLD) in the area of mathematics and of mathematics itself will be analysed first to determine why students underachieve in mathematics.

Failure in mathematics can be a consequence of dyscalculia but it can also happen as a consequence of other factors associated with the learning of mathematics. Dyscalculia is recognised as a deficit in number-sense with Butterworth (2003) describing it as a deficit in the most basic capacity for number upon which everything in number work seems to be built. SLD in mathematics, however, seems to account for children with dyscalculia but also for children with difficulties in mathematics for other reasons related to the demands of the subject and the teaching approach. Chinn (2008) notes that students may present with what looks like dyscalculia where they show symptoms (low motivation, anxiety towards mathematics, poor attitude) and claims that whilst this is not a direct result of dyscalculia, it arises as a consequence of mismatch between the learner and the instruction given. Westwood (2000) claims that maths difficulty is more of a reflection of the teaching approach used and the curriculum being taught, than the pre-disposition of the learner and this statement prioritises the need for better teaching approaches to mathematics. For students with SLD in the area of mathematics, there is speed of processing difficulties at a practical level where time limits expect students with SLD to read and process questions rapidly where copying difficulties lead to chaos and untidiness on the page. There are difficulties with the language of mathematics which 'is high in density and low in redundancy' proving difficult for a poor reader who may normally rely on contextual cues for decoding language, (Coventry, Pringles, Rifkin & Weldon, 2001, p. 128). At a conceptual level, there are difficulties with working memory (WM) in manipulating and processing incoming material. Coventry et al (2001) describe WM as a large workbench where something can be done to manipulate the new information and to store it more meaningfully. Research shows that WM deficit is very characteristic of SLD where rote retention of arbitrary information is especially hard, (Chinn, 2008). This is why a shift away from rote learning towards discovery learning is crucial for success in the subject for students with SLD. Adopting a multi-sensory approach to learning is important and experiences of success at mathematics are crucial to address self-esteem as an inhibitor to mathematics performance. The important role of developmental methods, in addressing this aspect of intelligent learning in mathematics, will become clear.

Developmental methods appeal to the very nature of mathematics as mathematics is a subject that is cumulative in style. Mathematics has been described as a subject where one learns the parts, with the parts build on each other to make a whole and that, knowing the whole enables one to reflect with more understanding upon the parts (Chinn & Ashcroft, 1998). Mathematical thinking is akin to intelligent thinking in that it too evolves through a series of stages. Development of mathematics seems almost synonymous with the development of schemas and intelligent learning. The latter has been described as "the formation of conceptual structures communicated and manipulated by means of symbols" (Skemp, 1971, p.16). The content and structure of mathematics as a contributing factor to student's difficulties cannot be ignored because it is a subject that makes demands on numerous abilities, for example, the ability to order and sequence information. It also demands exactitude and precision and is dependent upon arbitrary symbols. It requires effective storage, access, retrieval and a flexible working memory for information. It combines creativity with rigorous mental organisation and the ability to learn and apply rules, the ability to process high density and low redundancy text effectively and the ability to draw simultaneously upon a range of learning styles and preferences. Coventry et al (2001) state that these are not qualities that is typical of the SLD learner and must be explicitly taught. Langer (1957) recognised that mathematical data are arbitrary sounds or marks called symbols and that difficulty in symbol interpretation may be a major cause of failure at mathematics. In addition to this, mathematical lessons all too often involve learning series of facts in the absence of relational understanding which is merely rote learning of mathematics. Whilst the nature and demands of the subject cannot be eliminated, however, it is possible that adjustments in instructional approach go far in alleviating the difficulties associated with mathematics for learners with SLD. With this goal in mind, developmental methods, in place of rote learning, as an effective teaching approach for this purpose will be examined.

To teach conceptual understanding, the use of the concrete- representational- abstract sequence is an effective developmental method. This sequence for mathematical teaching is derived from developmental theories of cognition such as those proposed by Piaget and Bruner. Piaget makes two important claims about the sequence of stages in cognitive development; each stage derives from the previous stage, incorporate and transforms that stage and prepares for the next stage. That is, the previous stage paves the way for the new stage. Secondly, "the stages must follow an invariant sequence" (Miller, 2002, p.35). Since the first stage does not have all the building materials needed for the third stage, the second stage is required for learning to occur. Most importantly then, no stage in this learning sequence can be skipped when using developmental methods for teaching mathematics. Learners must progress through a series of stages beginning with the most concrete in order to lay the foundation for conceptual understanding. Where the foundation is not there, a gap in understanding arises and pupils miss a vital link. Over reliance on meaningless memorisation or rote-learning of rules is a consequence of missing links and lead to mathematical failure, particularly for pupils with SLD where impairment in working memory is a common problem (Chinn, 2008). According to Piaget's theory, all children discover the world through the manipulation of objects. Children should discover information themselves and learn through experience by the construction of schematic models to form an internal representation of the world around them (Miller, 2002). Schema about events, objects and situations in the world are changing constantly as the child discovers new information. Children experience temporary disequilibrium when they encounter new properties of objects that do not fit into their present cognitive structures or set of schema. A process of assimilation (interpreting the new experience) - accommodation (reorganisation of thought) happens to restructure cognitive schema when new information is processed and the discrepancy between them is resolved to restore equilibrium. In learning mathematics, therefore, pupils must be allowed to play with objects to learn concepts at the concrete level. In teaching fractions, for example, the teacher introduces the topic by allowing students to manipulate fraction bars and card bars. At this concrete level, they should make their own bars, cut them into fractions and discover that 4 quarters makes a unit, 3 thirds makes a unit, 5 fifths makes a unit and so on. Once they physically compare their cut-out pieces, they should discover that 2 quarters makes a half, 2 sixths makes 1 third and so on (Kinsella, personal communication, February 2011). In the representational stage of the lesson, they should compile all their own newly discovered information into tables. Pupils then identify fractions that are bigger and smaller and discover that all fractions with the same top and bottom are equal to 1, while those in which the top number is smaller are less than 1 and those with the top number bigger are greater than 1. This leads onto identifying proper fractions, improper fractions and mixed fractions. In this approach the pupils have discovered many basic principles about fractions before any formal work begins and it is only once this is well consolidated, that work on operations such as addition and subtraction ensues. An example of a programme that has empirical support for increasing student achievement in mathematics following the concrete-symbolic-abstract learning sequence is *'The Strategic Math Series'* by Mercer & Miller (as cited in Mercer & Miller, 1997, pp. 103- 110). Researchers found use of this sequence, in combination with systematic and explicit instruction, the provision of feedback to pupils in addition to teaching mnemonic strategies and fluency practice, resulted in achievement in mathematics for their sample of over three hundred students, (Mercer & Miller, 1997). The sequence of learning involves solving problems through the manipulation of concrete devices only. It is only once the pupil understands the maths skills at this level that representational instruction may begin. At this level, the concrete materials are replaced by pictures and tallies that represent these objects. As before, only once the student understands the skill at the representational level, may abstract level instruction begin where problems are now solved using only number symbols. Memorisation and fluency building only enters instruction at the level of abstraction. Hutchinson (as cited in Mercer & Miller, 1997, p.102) promotes teaching students specific strategies to help in problem solving. Mnemonic devices help students with memory deficits in remembering the steps to take and promote independent problem solving. For example, the RIDD strategy, Read, Imagine, Decide and Do. Many similar strategies fit in well into a learning sequence based on developmental methods, particularly at the abstract level.

Similarly, Bruner's theory states that children's learning is an active process and they construct knowledge actively through the linking of new information to previously acquired information. An internal representation of the world that makes sense to the child is built up through the representation of an event, object or situation in three forms through three media, that of action (the enactive mode), through imagery (the iconic mode) and through language (the symbolic mode). Bruner claims that intellectual development occurs through these modes and in this order whilst each mode has a set of skills that depends on the accomplishment of skills from the previous mode (Bruner, 1966). In the enactive mode the child learns the how to do something, learning procedural knowledge while in the symbolic mode the child learns declarative knowledge where they can manipulate symbols without concrete props. This can't come about, however, without passing through the middle stage of learning which is the iconic mode where a child gets used to using sets of images that stand for a concept. Imagine then a classroom where mathematics is taught purely through language in the symbolic mode, the mode which the teacher is most likely to have representations of the concept. A clear mismatch between the teacher's mode of representation and the pupil's mode ensues and so the child does not develop conceptual understanding but rather learns off a collection of unintelligible rules (Skemp, 1971). Knowledge over procedure is important to emphasise with the aim of building conceptual understanding in mathematics. The outcome of learning based on 'relational mathematics' versus someone who has an 'instrumental' understanding of a concept is illustrated through an analogy of a cognitive map of a town (Skemp as cited in Frederickson, Miller & Cline, 2008). In this analogy, instrumental understanding shows how pupils can find their way only from fixed points with a very limited range of application and this corresponds to the outcome of rote learning in mathematics. Again, Skemp (1989) shows concern over the mismatch between teacher's style of teaching and the content of modern mathematics curricula where he states that there is evidence to suggest that strands of material that is relational in nature is actually being taught in an instrumental way. A music lesson is described to illustrate this and this analogy serves to illustrate the point that despite recent moves in curricula towards discovery learning, mathematics still has the potential to be taught in an instrumental way leaving it boring, heavy on memory work and rote learning. With Bruner's theory informing developmental methods, however, it is the responsibility of the teacher to create opportunities for children to experience concepts through action, image and language followed by enabling pupils to link up existing representations through new insights in each mode. To succeed with this, teachers must follow the developmental stages in teaching mathematics, starting with manipulating concrete examples (enactive). For example, concepts of factors, divisors, multiples and prime numbers should be introduced by making square and triangular numbers from coins or counters (Connolly, personal communication, March 2005). These patterns should be recorded in picture form, word form and symbol form. This activity is followed up with tabulating newly discovered information (iconic) such as colouring in every third, sixth, ninth square in a 100 square (Connolly, personal communication, April 2005). Finally material from the first two modes should be used to scaffold work in the symbolic mode using laminated tables of factors and multiples to support calculations until the pupils can do without them. Eventually as automaticity develops in the abstract level of learning topics, scaffolds can gradually be removed. The idea of scaffolding is for the teacher to take control only when needed and to hand over the responsibility to the students whenever they are ready. Drawing from developmental methods to inform practice, the teacher takes the children through the full journey from enactive to iconic to symbolic modes of representing concepts from topics in mathematics (Bruner, 1964). Most crucially, each stage is linked to coincide with linking internal representations and no stage is skipped.

Successful use of developmental methods and the concrete-symbolic-abstract sequence is dependent on the identification of the Zone of Proximal Development (ZPD) and on the notion of optimal discrepancy from Piagetian theory. The ZPD is "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers" (Vygotsky, 1978, p. 86). The ZPD therefore denotes the space between actual knowledge and potential knowledge. The first step in using a concrete-symbolic-abstract sequence is establishing where child is at in relation to mathematical thinking. This allows informed teaching and is a crucial ingredient in pitching material at the right level. Monitoring progress and providing feedback, which becomes an integral part of every lesson in the use of developmental methods, helps to establish a pupil's ZPD. Miller and Mercer (1997) recommend the use of informal assessment procedures to verify student understanding of content and grouped learning embedded in developmental methods can help this. In addition to qualitative assessments, norm-referenced and criterion-referenced tests can also be used. When pupils are set to work in groups to help each other, learning is facilitated for students with SLD. What children can do with the assistance of others is "even more indicative of their mental development than what they can do alone" (Vygotsky, 1978, p. 85). Also, two distinct learning styles recognised in the current literature and this has implications for teachers in their approach. Bath, Chinn, and Knox (1986) identified two styles, the inchworm approach (looks for relevant formulae and works in serially ordered steps but is unlikely to check and evaluate answers) and the grasshopper approach (who has flexible methods and a holistic approach but rarely documents methods). A mismatch in teacher and pupil's cognitive style contributes to pupil's underachievement in mathematics. Group learning may be an opportunity for learners to influence one another and become flexible in the use of both styles. Krutetskii (as cited in Bath et al, 1986) favours use of both styles in becoming a good mathematician. There are two key elements at the first stage of learning, the experience is child-centred where the child is making their own discoveries and optimal discrepancy between old information and new information is also crucial. The notion of optimal discrepancy should address maths anxiety as an inhibitor to learning and also arouse motivation for learning in the student. If the discrepancy between old information and new information is too small, there will be no intrinsic motivation for the child to become interested and no learning will take place. The teacher must organise learning appropriately to achieve optimal discrepancy. Sousa (2008) emphasises the importance of meaningful learning and how teachers must become the link between mechanical calculations and the meaning behind them. On the other hand if the discrepancy is too large, it may invoke a state of anxiety in the child, thus creating a barrier to learning. Skemp (1986) noted that the reflective activity of intelligence is inhibited by anxiety. Teachers strive to create new learning for pupils which in the context of developmental psychology means throwing a child's existing schema into a state of disequilibrium thus creating a feeling of dissatisfaction with current information so that they will learn. The challenge for an effective facilitator lies in creating a dilemma for pupils from problems that are structured to provide insights into different mathematical concepts. The teacher must ensure not to be presenting information that is neither too easy nor hard because in either case learning will not occur. The teacher plays a crucial role in organising the learning environment to facilitate pupils in forming their own internal models of mathematical concepts they meet. It is unlikely that standard textbooks are still useful in lessons with this approach as problems that are presented at optimally discrepant levels are the key to learning. Through the use of developmental methods the teacher is enabling the child to learn with their own intelligence. This marks a complete shift away for traditional transmission models of learning towards a student-centred developmental approach of learning.

The application of theories of developmental psychology to the mathematics classroom means teachers work to elaborate pupil's schemata for mathematical concepts and to link up the enactive, iconic and symbolic representations pupils hold about the world to link into meaningful chunks for pupils. Upon examination of the theory that informs developmental methods, development of conceptual understanding is the hallmark of intelligent learning and can only be achieved once certain conditions are met in the classroom. Identifying the

ZPD in pupils, optimal discrepancy of information, scaffolding pupil's learning, setting up group learning to create a child-centred classroom, addressing the teacher's role as facilitator and following a concrete- symbolic- abstract sequence are all crucial elements in the use of developmental methods. Through these methods, a complete shift from rote-learning to a student-centred environment that promotes intelligent learning can be achieved. In this way, barriers to the learning of mathematics such as pacing of the curriculum, lack of differentiation, mismatch between teacher's style of instruction and a learner's cognitive style, underdevelopment of automaticity, badly designed tasks, introduction of material that is outside the student's range, reading difficulties, beginning of formal work too early in classrooms, can all be alleviated to some extent for students with SLD in the area of mathematics. Developmental methods, as outlined above, go a long way towards meeting the diverse and often competing needs of learners with SLD in the mathematics classroom. It is fair to conclude that developmental methods play a pivotal role in the quest to replace rote-learning of mathematics with intelligent learning for learners of all ages in mathematics.

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