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Using bidimensional regression to explore map lineage

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Abstract

This paper is concerned with the exploration of lineage in pre-twentieth century mapping taking Suffolk as an example. A non-parametric bidimensional regression (Tobler, 1994, *Geographical Analysis, 26* 186–212) is used to investigate patterns of distortion in seven maps of the county (Saxton: 1575, Jansson: 1646, Blome: 1673, Overton: 1713, Harrison: 1790, Rowe: 1831, and Wyld: 1891). For each map the locations of 50 towns and villages were digitised and are used to predict the position of the same 50 locations on 20C OS mapping. The residuals are used to interpolate a vector field for each map from which the OS grid may be warped to give a visual indication of distortion. Finally, a statistical comparison of the distortion is made using Akaike's Information Criterion (Akaike, 1973, In B. Petrov, & F. Csaki, *2nd Symposium on Information Theory* (pp. 267–281), Budapest: Akademiai Kiado). The results suggest that pre-nineteenth century map makers were heavily dependent on the work of their predecessors.

Keywords: Bidmensional regression; Map lineage; Akaike Information Criterion; Suffolk

1. Introduction and context

The casual visitor to many a historic property will often notice prints of old maps hung on the walls. They also adorn the walls of country public houses, not a few domestic premises and the reception areas of many departments of geography. There is a thriving antiquarian business in old maps as a visit to a second-hand bookseller will often reveal. Whilst undoubtedly of aesthetic value, do these cartographic items have value as reliable tools in cartometry? This paper arose out of some ideas for using old maps to investigate changes in the location of the coastal

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area around Orford Ness. However, some initial investigation suggested that not only was one of the maps used subject to some local distortion, but others also exhibited a similar pattern of distortion. There was some borrowing between early map makers, indeed John Speed is quite open concerning his use of the earlier work of Christopher Saxton. However, the question remains as to whether we can quantify, model, and visualise distortions on such maps, and also how we might decide whether there is evidence for what might be regarded at best as lineage and at worst as plagiarism.

This problem is by no means new. Tobler (1966) was considering whether it is possible to estimate the projection of an ancient map, using the Hereford map and a Portolan chart as examples. More recently he refined his ideas (Tobler, 1977) into a technique which he terms bidimensional regression and most recently into a non-linear form (Tobler, 1994). The technique has not been widely used in geography, although Kitchin (1993) and Jacobson (1999) are notable, and Miller (1999) discusses its potential in the field of transport modelling. Distortion indices for cognitive maps have been investigated using bidimensional regression (Kitchin, 1996; Lloyd & Heivly, 1987; Lloyd, 1989) and Murphy has investigated the problem of map lineage in a study of early maps of Ulster (Murphy, 1978). Nakaya (1997) considers statistical inference with bidimensional regression. In this paper we illustrate an extension of Nakaya's approach.

However, we cannot begin to examine variation in cartographic accuracy without considering the context of the creation of such maps.

> The context of maps has received little attention . . .apart from biographies of map-makers, but it seems essential to study any map not just as a piece of historical geography or as an example of printing methods, but also as a product of the society and times in which it was produced (Hindle, 1998, vii–viii).

It seems important therefore that the study of map lineage should not only incorporate consideration of the geodetic precision of maps but include an appreciation of the framework in which they were produced. In this way, any quantitative evidence of linkages between maps can be corroborated through the use of secondary evidence.

Finally we consider the weight of evidence for such lineage. We move the informal visualisation of distortion into a formal statistical framework. For any pair of maps we may consider whether the bidimensional functions are the same or different using the Akaike Information Criterion (Akaike, 1973). Using a Bayesian approach we consider the possibilities of similarity between every pair of maps in the study. These may also be visualised using multidimensional scaling (Sammon, 1969).

2. Maps and the study area

The area chosen was originally intended for a study of coastal change at Orford Ness in Suffolk. After some initial experiment it appeared that the problem of map accuracy and lineage needed investigation before this could continue. The county map was decided on as an adequate scale as the continuity of this form of mapping through time in England is unparalleled. From Elizabethan to mid-Victorian times in fact, the county was the basic unit of regional mapping throughout Britain and the county atlas the commonest form of publication. As Tooley (1978) states:

> the county maps of England and Wales have an ancient history commencing in the 16th century, the first English atlas in this field appearing before its counterpart in France, and in succeeding centuries a constant and increasing stream of publications was issued on our national topography (p. 65).

Maps of a size capable of being bound into, if not published in atlas form, are much more likely to have remained intact than large sheet maps, therefore in many cases atlas maps are the only surviving original cartography of a given era (Hindle, 1998). In order to understand the chronology of the maps used in this paper it is necessary to briefly outline the context in which they were produced.

The time period selected for investigation begins in the sixteenth century, at which time the first county maps were produced, and ends in the late nineteeth century, when the OS began to dominate map production. The individual maps within the collection have been chosen so that their dates of production give a relatively even coverage of the time frame in question. In this way conclusions may be reached as to the nature of cartography throughout the period of study, even though only a sample of maps are being analysed. The specific cartographers that have been selected are:

Saxton	1575:	An Atlas of England and Wales
Jansson	1646:	Magna Britannia
Blome	1673:	Britannia
Overton	1713:	A Set of Mape of several Counties
Harrison	1790:	Maps of the English Counties
Rowe	1816:	An English Atlas
Wyld	1891:	An Atlas of English Counties

It should be noted that these form an accessible subset of mapping over this period but are not exhaustive.

3. Methodology

The methodology used to explore the lineage within this selection of maps was similar to that used by Tobler (2001) in which he analyses the Benincasa Portolan chart of 1482. In this analysis Tobler examines the distortions in the representation of the fifteenth century Mediterranean coastline using bidimensional regression. He does this by recording the positions of corresponding 332 observation points on both the Portolan and a modern map. Bidimensional regression is then used to quantify the transformation required to effectively change the fifteenth century co-ordinates into the modern co-ordinates (Tobler, 2001).

3.1. Bidimensional regression

In bidimensional regression there are two variables, an independent variable (Z) and a dependent variable (W), each consisting of n pairs of numbers:

Z	W
$x_1\ y_1$	$u_1\ v_1$
$x_2\ y_2$	$u_2\ v_2$
$x_n\ y_n$	$u_n\ v_n$

The objective is now to relate the variable W to Z through a function $\hat{W}=f(Z)$ in such a way that the mapping $Z \rightarrow \hat{W}$ is, as nearly as possible, the same as the observed association $Z \rightarrow W$. It is therefore possible to give a value for $\hat{W}$ when given a value for Z when there was no observation at that location, or where (u, v) contains an error. In the case of bidimensional regression the function $Z \rightarrow W$ is most commonly written as the two separate functions:

$$u = f(x, y)$$
$$v = g(x, y) \quad \text{(Tobler, 1994)} \quad (1)$$

For the purposes of this paper the transformation carried out can be thought of as any function that transforms the co-ordinate pair (x, y), in this case, a point on the initial map, to another co-ordinate pair (u, v) representing a point on the transformed map. The transformed map is therefore a 'warped' version of the initial map (Gatrell, 1983).

Initially our focus will be on bidimensional regression as a visualisation, rather than statistical tool. We consider the statistical framework of bidimensional regression in Section 6.

3.2. Data capture

The maps were sourced from Suffolk Records Office, Ipswich, where they were photocopied onto A3 paper. Although photocopying introduces a minor degree of inaccuracy through reproduction, there was no viable alternative and the photocopies were deemed of acceptable quality given the age and scale of the original maps.

The positions of 50 towns and villages located throughout Suffolk were identified (Table 1). The 50 towns that were chosen appear on all of the maps from Saxton through to the modern OS base map. The locations of the 50 towns were recorded on each of the old maps. This procedure was also carried out on a modern Ordnance Survey county map of Suffolk.

The symbols denoting the towns fell into two categories, firstly emblematic representations on early maps and secondly planform depictions on later maps. Matters were further complicated as within these categories the cities and towns were often shown by different symbols (Fig. 1).

Table 1
Old and modern names

ID	Modern name[a]	Ancient name[b]	ID	Modern name	Ancient name
1	Mildenhall	Mildnall	26	Needham Market	Nedham
2	Worlington	Worlington	27	Ipswich	Ipswiche
3	Freckenham	Frecknham	28	Aspall	Aspall
4	Newmarket	Newmerket	29	Woodbridge	Woodbridge
5	Haverhill	Hauerhill	30	Orford	Orforde
6	Eriswell	Esewell	31	Wantisden	Wanisden
7	Great Bradley	Magna Bradley	32	Nacton	Nacton
8	Lidgate	Lidgate	33	Wickham Market	Wickham
9	Wickham Brook	Wychhambrooke	34	Dennington	Dinnyngton
10	Clare	Clare	35	Holton	Holton
11	Risby	Risbye	36	Butley	Butley
12	Stansfield	Stanfelde	37	Debach	Debache
13	Elveden	Elden	38	Bungay	Bungey
14	Brockley	Brockley	39	Belton	Bilton
15	Bury St. Edmunds	Burye	40	Hopton	Hopton
16	Sudbury	Sudburye	41	Fritton	Fritton
17	Shimpling	Shimplinge	42	Gunton	Gunton
18	Lawshall	Lawshill	43	Beccles	Beckels
19	Assington	Asington	44	Walberswick	Walderswick
20	Newton	Newton	45	Weston	Weston
21	Preston	Preston	46	Burgate	Burgate
22	Thorpe Morieux	Thorpe	47	Middleton	Midleton
23	Ixworth	Ickesworth	48	Otley	Otley
24	Hadleigh	Hadley	49	Elmswell	Elmeswell
25	Westhorpe	Westhorpe Cotton	50	Thorpeness	Thorpe

[a] Taken from OS 1:50 000 Landranger Sheet, 1993
[b] Taken from Saxton's map, 1575

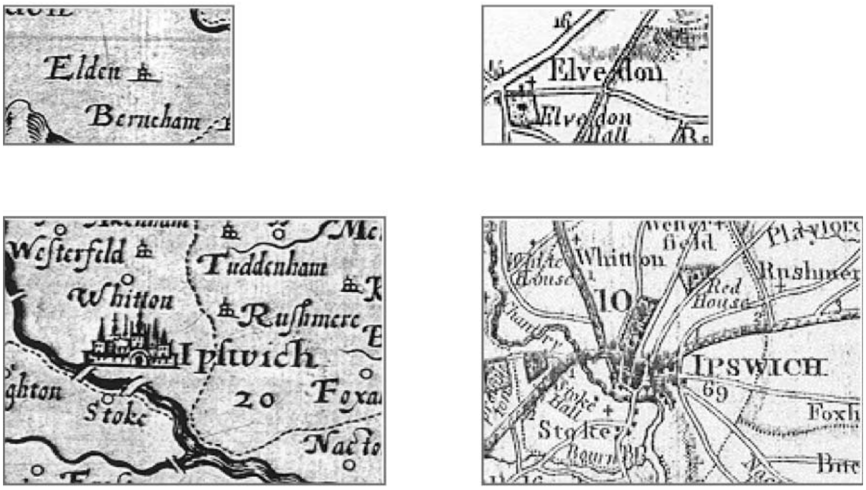
In all cases a location nearest to the perceived centre of the settlement/symbol was digitised, the error introduced by this method was thought to be small enough so as not to affect the study.

It is not necessary to attempt to geo-reference the old maps into OSGR as 'bidimensional regression yields results that are independent of the particular co-ordinate system chosen' (Tobler, 1994, p. 188). The output for every map was a Comma Separated Value file (.csv) comprising of the town ID number, the OSGR (x, y) co-ordinates and 50 town co-ordinates for the digitised map.

3.3. Software issues

The bidimensional regression graphical output was produced using a program written in the statistical computing package R. R was chosen for use in this project because the ease with which it produces well-designed, high-quality graphics (Leisch, 2001) together with a flexible modelling language.

The initial aim of the program was to produce a graphic with arrows (or 'quiver plot') that indicated the degree of transformation required in order to convert the



Saxton Rowe

Fig. 1. The representations of Elvedon and Ipswich on the maps of Saxton (1575) and Rowe (1813).

OS map to the old map. The first plot was constructed through applying the bidimensional regression functions f and g to the co-ordinates digitised from the maps. The first plot constructed was therefore simply a quiver plot with arrows extending from the data points, i.e. the OS position of the digitised towns. The programme was then refined to use a kernel regression technique to interpolate the regression figures between the data points in a regular grid pattern. Kernel regression is a method of local polynomial regression fitting, that is for the point x, the fit is made using points in a neighbourhood of x, weighted by their distance from x (Kim, 2000). This is discussed in Section 6. In this way coverage for most of the county was developed. The interpolations are carried out in a rectangular area which encloses the boundary of Suffolk. This gives rise to some interpolations which lie outside the boundary.

In order to reduce the visual impact of the external interpolations the plots were subjected to a clipping operation. This operation was performed in R using a simple Suffolk boundary derived from Bartholomew's digitised data. Only those interpolated results lying inside the boundary are retained in the finished graphic. The final stage was to add markers indicating the locations of the digitised towns, in order to provide some reference points to allow easier analysis.

A second visualisation technique was also developed in R to aid interpretation. This method employed the same statistical procedure as the quiver plots (Loess local polynomial regression) yet the graphics were presented as a '*warped vector field*'. The warped vector field is essentially an extension of the rubber sheeting idea. The OS map may be thought of as a sheet of rubber onto which the regular OSGR grid has been drawn. The transformation from the OS to the old maps then consists of stretching and contorting the rubber sheet until the old map positions are accurately represented by the modern OSGR town locations. The distortions of the squares that make up the grid therefore give an idea as to the distortions within the old maps.

From this data it is now possible to visualise the transformation from the initial map to the transformed map in two ways. With the distortion grids, the way the squares are distorted, rotated, or sheared gives an idea as to the nature of the distortions in the maps. With the quiver plots the arrows represent the direction and magnitude of any location displacement that is taking place in the map transformation. For each of the maps, both a warped grid and quiver plot were created; an example for Saxton's map is shown in Fig. 2

4. Observations

We will consider the maps in three groups; these have been suggested by our examination of the distortion patterns.

4.1. The maps of Saxton (1575), Jansson (1646), Blome (1673) and Overton (1713)

The four maps in this grouping all show a similar general pattern following that of the Saxton map, yet there are subtle differences between them. The overall pattern will be addressed first with reference to the Saxton map then the differences within the group will be discussed.

It is possible to observe an overall trend pattern of errors from both the quiver plot and distortion grid. The two most major features are the errors in the north-west, where the towns have been placed too far north-west and in the south-east on the coast from Ipswich to Orford Ness, where the towns have been placed too far north. To a lesser extent it can be seen that places in the north-east of the county are too far south and places in the south-west are too far west.

Details depicted on all the maps are sketchy in the north-west section with no features being shown further north-west than Mildenhall, even though the county extends for a further 5 miles in this direction. This fact may account for the inaccuracy in this area. Features on the coast tend to be exaggerated, especially in the southern section, which includes the estuaries of the Rivers Alde, Stour, Orwell and Deben. This may account for the cartographic mistakes in the south and smaller mistakes in the north, both of which indicate that the coastline is elongated on the Saxton map. The most geodetically accurate region of the map is the centre of the county. This pattern of errors may suggest some conjectures:

1. The central region of the map has a high density of settlements, therefore there are many reference points and the distances between the features would have been fairly well known. The geodetic accuracy is therefore relatively high in these areas.
2. The southern coastal section of the map includes many landscape features, i.e. coastal geomorphology, rivers, estuaries and ports. Having such a large number of features in this area, the scale has been exaggerated in order to allow room for them all, therefore the geodetic accuracy is low.

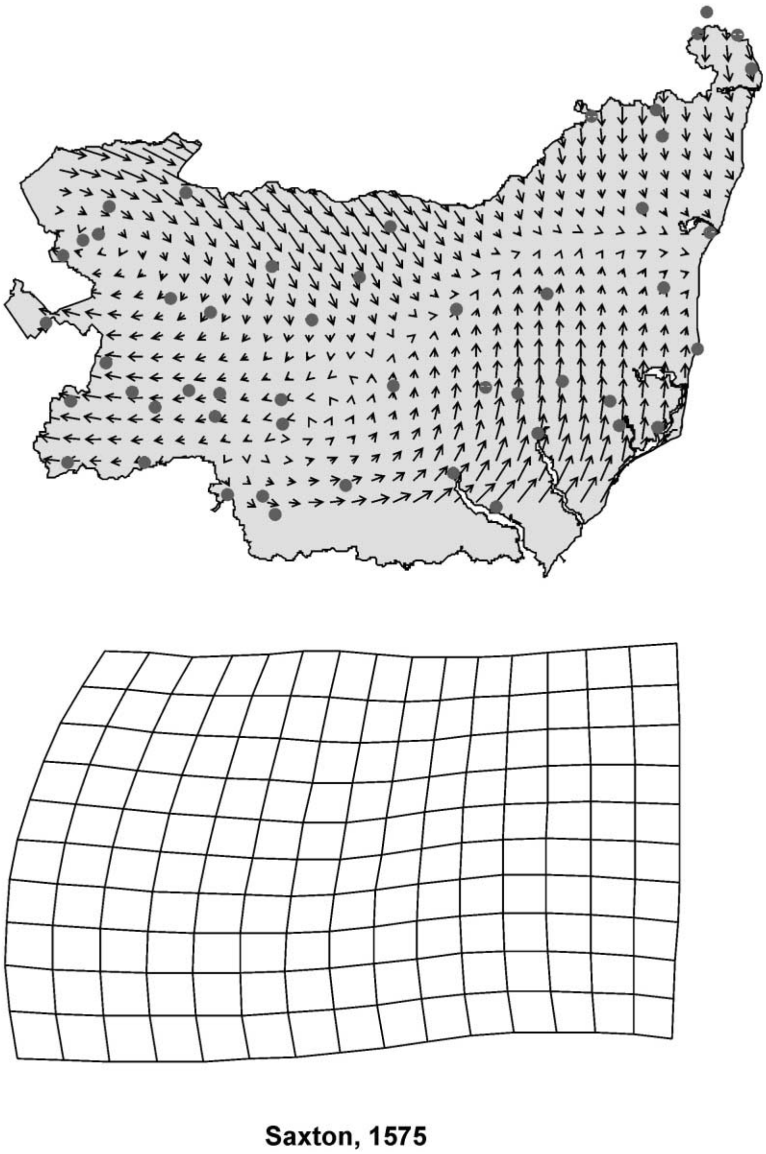


Fig. 2. Quiver plot and distortion grid for Saxton's map.

3. Areas such as the western edge of the county have few settlements or other geographic features, there are consequently only a small number of reference points, therefore lowering the geodetic accuracy.
4. The large distortions seen in the western edge of the grid may have been due in part to the 'edge effect' that was effectively removed from the quiver plots using the 'clip' operation.

Overall, it seems that priority was assigned to certain areas of the county, with lesser attention being paid to others, either intentionally or inadvertently through methodology. It has been said that 'many maps have more of the qualities of a portrait than of a photograph [being] as much perceived as measured' (Harley, 1972, p. 63), given the observed error patterns, this statement may well be true in Saxton's case.

The distortion pattern for the Jansson map is almost exactly the same as that of the Saxton map, yet there are several areas where mistakes have been propagated through reproduction. There appears to be an error divergence in the south-western edge, which was not seen on the Saxton plot. The towns in this area seem to have been placed too far north and west. The deformation grid shows that the coastal distortion is in a similar pattern as on the Saxton map. Jansson's map is in essence the same as Saxton's yet further inaccuracies have been introduced. Reasons for the added error may be hypothesised as:

1. Saxton's map has been stylised to a certain degree by Jansson, with changes being made in the depiction of parkland, rivers, and settlement location. Without an original survey these changes in design cannot be as accurately placed as drawings made directly from a survey; and
2. changes in labelling text or style may also lead into minor added error with difficulties arising as to where to place the revised names and symbols.

The Blome distortion patterns are effectively identical to those seen on the Saxton graphics. It may be said that few errors have in fact been propagated in the copying from Saxton, and as a result this map may be said to be a more accurate representation of Saxton than Jansson's attempt. The map itself, however, is less clear than Saxton's or Jansson's, the text being of no uniform style or orientation, other features have also been extensively stylised including the coast and rivers, although this change in design does not seem to have affected the quality of Blome's copy.

Overton's map is most similar stylistically to that of Blome, the text, symbols and landscape features are all virtually indistinguishable. It is unclear if Overton's map was in fact copied from Blome or directly from Saxton, with its error pattern being a duplicate of both of these earlier works. It would seem likely that Overton has copied Blome's work to use as a base-map for his 'new' road plan.

Our examination suggests that, of these four maps, only Saxton's work is based on an original survey, and that in the process of copying, the three other 'map-makers' had introduced inaccuracies to a greater or less extent.

4.2. Harrison's map (1790)

Harrison's map was the first to show any major change from the distortion pattern of Saxton's map, drawn 215 years earlier. There are still inaccuracies between the Harrison map and the OS base-map used in this paper yet they are much less than those seen in the first group of maps.

Many of the trends on the preceding maps are reversed on this map. This is true of the north-east, north-west, and the south-west. The distortion grid appears less distorted than might be expected from the quiver plot, indicating that the cumulative error in the map is much less than that seen in the first group of maps.

This pattern may be due to several factors:

1. The areas with the highest geodetic inaccuracy are areas with few reference points (i.e. settlements) in the south-east and north-west. There appears to be an optimum reference point-density for mapping above which, and below which, accuracy suffers.
2. Areas in the centre of the county seem to be less accurately located than in previous maps yet there are 'centres of precision' around Bury St. Edmunds in the west and Saxmundham in the east. This may have been due to inaccuracies in the triangulation methodology Harrison is known to have used.

4.3. Rowe (1816) and Wyld (1891)

Although the Rowe and Wyld maps are similar in style, Wyld's map appears to contain fewer errors when both are compared to the OS base-map. Examination of the distortion grid highlights the fact that the errors in the maps are minor yet there are some slight differences in the pattern of distortions that are present. For the Rowe map, the largest errors occur on the western edge and in the central-southern coastal section. Whereas in the Wyld map the errors are seen in the north-west and central areas. Some hypotheses may be formed as to the reasons why some regions exhibit relatively large amounts of distortion:

1. In a similar way to the previous maps in the collection, the areas with distortions seem to be the ones at the edge of the county, with few geographical features, i.e. the north-west of the county and the southern section of the coast. The errors may therefore be linked to a lack of reference points, especially when examining the county in isolation from the neighbouring counties.
2. The vertical edges of the Wyld distortion grid are slightly bowed suggesting there may have been an error introduced through photocopying, or a minor edge effect, the same is true of the horizontal edges of the Rowe distortion grid.

5. Discussion

Saxton's map of 1575, being the first attempt at county mapping, had a large element of inaccuracy yet was an original work, drawn from an original survey. The distortion pattern was concentrated around the county border at the edges of the map, suggesting that Saxton's accuracy was based on reference points and feature density. This would seem a likely conclusion given Saxton's surveying methodology of 'viewing' the countryside and using local knowledge rather than employing extensive ground surveying methods.

Jansson's map of 1646 is without doubt a copy of Saxton's map, which was produced 77 years earlier. This fact was indicated by all of the secondary data sources, the haste of production alluded to by Skelton (1972) and referred to in Section 2 is evident in the analysis. This map is the least accurate copy of Saxton in this map collection with a significant degree of error being added by Jansson.

Blome's map of 1673, produced 98 years after Saxton's original survey is also a copy. In this case the duplication process has not added to the inaccuracies of the original work, yet in many ways this work could be said to be inferior to Saxton's. The stylised representation of places and landscape are for the most part unnecessary and render the chart less clear and harder to use than those it was based upon.

Overton's 1713 map may, like all of the maps preceding it, be said to have been copied from directly from Saxton, even though it was produced 138 years after the original. The distortion patterns are identical to those seen for both Saxton and Blome. The Overton map does have the advantage of showing a rudimentary road layout for the county yet even accurate roads are of little practical use if presented on an outdated base-map.

Harrison's map (1790) is the first to present a departure from the groundwork of Saxton. The distortion pattern is so different for this map that a radical change in methodology must have taken place in both the surveying and representation Harrison employed. This would seem to corroborate the hypothesis that Harrison could have been the first surveyor of Suffolk to utilise triangulation.

Rowe's map of 1816 is a follow-on from the work of Harrison in many ways, yet the distortions, amount of detail and presentation style differ so much between these maps that they must have been drawn from different primary data sets. Both Harrison and Rowe must therefore have carried out separate surveys of Suffolk.

The most modern map in the set is Wyld's map of 1891, solely because of its date this map would be expected to be the most accurate due to the development of cartographic techniques over time. This is indeed the case, although similar in style to the Rowe map, Wyld's is of superior geodetic precision when compared to the OS base-map. As a result it is suggested that this map is likely to have been based on a new survey.

Three separate groups have been identified to classify the seven maps into a coherent lineage. The lineage was determined through the secondary data sources outlined in Section 2 and tested using the methodology described in Sections 3 and 4. For easy comparison, Fig. 3 gives a timeline of distortion grids, with shading denoting the map lineage. The groups are:

"After Saxton" (1575–1790): the maps belonging to this category have been identified as Saxton, Jansson, Blome and Overton;

"Intermediate accuracy mapping" (1790-1816): Harrison's map is the only map in this class; and

"Accurate 19th century mapping" (1816–1891): the maps of Rowe and Wyld fall into this grouping.

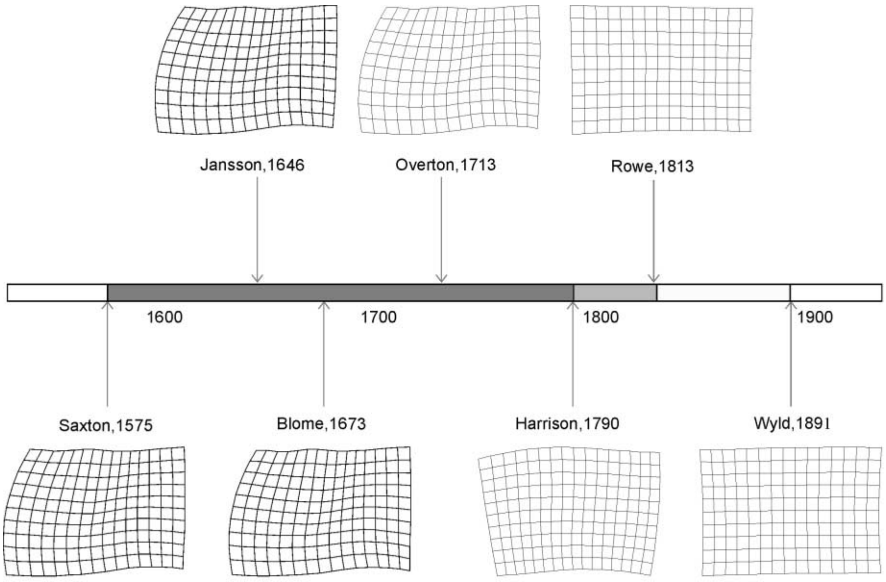


Fig. 3. Bidimensional regression distortion grids organised by date (1575–1891).

6. Bidimensional regression as a statistical technique

Thus far, bidimensional regression is used entirely as a deterministic, visualisation tool. From the graphics produced, patterns are inferred but cannot be tested in any rigorous manner. A recent paper by Nakaya (1997) is the first to propose the establishment of a number of 'formal procedures of significance tests for bidimensional regression models' (p. 169). The proposed tests are:

1. significance tests of estimated parameters;
2. significance tests of bidimensional correlation coefficient; and
3. significance tests for the comparison of bidimensional regression models

The test results can confirm the findings of bidimensional regression studies such as the one undertaken here, therefore enhancing bidimensional regression as a 'more powerful tool for spatial analysis' (Nakaya, 1997, p. 185). We consider an inferential approach in the next section.

The stated aim of the statistical analysis here is to model the historical map points as transformations of the OS map points. However, in practice the digitised historical points will contain a degree of random error in addition to the deterministic warping associated with the map transforms. There are a number of sources of this error—the two principal ones are likely to be digitising errors during the electronic data capture process, and copying error by the original map creators. Clearly, we would not expect two copies of the same map to be identical—just quite similar. Bearing this in mind, a realistic bidimensional regression model is:

$$u = f(x,y) + \varepsilon_u$$
$$v = g(x,y) + \varepsilon_v \qquad (2)$$

Where (u,v) are the historical map coordinates, (x,y) are the OS map coordinates, ε_u and ε_v are independent Gaussian error terms with zero mean and variance σ^2. Here f and g are arbitrary *functions*, and the regression method used is a nonparametric one. This probabilistic model provides a useful tool, since it allows hypotheses about the authorship and lineage of maps to be evaluated on a formal statistical basis.

The methodology used to calibrate the model can be thought of as an extension to the technique of Nakaya (1997) who models f and g as linear functions:

$$u = a + bx + cy + \varepsilon_u$$
$$v = d + ex + fy + \varepsilon_v \qquad (3)$$

We extend this by using the method of kernel regression (Bowman & Azzalini, 1997; Wand & Jones, 1995), enabling us to remove the linearity assumption, and estimate the arbitrary functions f and g directly.

We may use this technique to investigate whether two maps were copied from the same source. Suppose we have two maps, map A and map B. If the two maps have different projections (and by implication are unlikely to come from the same source) then the functions f and g will be distinct for each map. Denoting the coordinates for the respective maps as (u^A,v^A) and (u^B,v^B), we may write:

$$u^A = f^A(x,y) + \varepsilon_u^A$$
$$v^A = g^A(x,y) + \varepsilon_v^A$$
$$u^B = f^B(x,y) + \varepsilon_u^B$$
$$v^B = g^B(x,y) + \varepsilon_v^B \qquad (4)$$

Here f^A, g^A and f^B, g^B are the respective transformation functions for the two maps, and the superscripted εs are the respective error terms. On the other hand, if one map is a copy of the other, so that both maps have the same f and g we may write

$$u^A = f(x,y) + \varepsilon_u^A$$
$$v^A = g(x,y) + \varepsilon_v^A$$
$$u^B = f(x,y) + \varepsilon_u^B$$
$$v^B = g(x,y) + \varepsilon_v^B \qquad (5)$$

This is identical to (Eq. 4) except f and g are no longer distinct for the two maps. Thus, deciding whether the two maps are related to the same source can be informed by considering the relative plausibility of the models in Eqs. (4) and (5).

The statistical framework we use to compare models is the Akaike Information Criterion (AIC; Akiake, 1973). The underlying idea is to compare a number of competing models and decide which is 'closest to reality' (Burnham & Anderson, 1998). Unlike classical statistical inference, this does not involve a decision as to whether a hypothesis is 'true' or 'false' but instead considers how well each model matches the data. In a strict sense this approach assumes that all models are 'wrong', but that some are closer approximations to reality than others. The AIC is a number associated with each model which helps us to decide which model is the closest to reality. Here we consider just two competing models:

1. f and g different for maps A and B (Eq. 4); and
2. f and g the same for maps A and B (Eq. 5)

The utility of the AIC approach becomes apparent if, for example, we acknowledge that the assumption of independent Gaussian error terms may not match reality, so that neither model may be strictly 'correct'. Using AIC we are still able to assess which of the two models is *closest* to reality.

In general, for nonparametric regression, the AIC can be reasonably estimated by the expression

$$\mathrm{AIC}_m = -2\log(L_m) + n\left(\frac{n+k}{n-2-k}\right) \quad (6)$$

(Hurvich & Simonoff, 1998), where L_m is the likelihood of model m, n is the sample size and k is the *effective degrees of freedom* of the model, a concept outlined in the previous reference.

To choose between two competing models, we compute the AIC for each model—the 'best' model is then the one with the smallest AIC value. Note that, as a rule of thumb, a 'serious' difference between two models is generally regarded as one in which the difference in AIC values between the models is at least 3. A useful method for interpreting AICs is to consider *Akaike Weights*. In the two-model case, the Akaike weight for model one is defined to be

$$w_1 = \frac{e^{-\mathrm{AIC}_1/2}}{e^{-\mathrm{AIC}_1/2} + e^{-\mathrm{AIC}_2/2}} \quad (7)$$

with a similar definition for w_2. The w_i's sum to one, and can be thought of as the 'weight of evidence' in favour of each model. Using this approach, we obtain some idea of the relative evidence that each model is 'best', rather than choosing a single candidate. It can be argued that the w_i 's have a Bayesian interpretation—see for example, Akaike (1981, 1994). If we place equal prior probabilities on each of the models, then the w_i 's are the posterior probabilities that each model is 'best' of the two, in the light of the data.

In our study we firstly considered the map pairs in chronological order, and apply the above technique to see whether time adjacent maps and likely to be linked. The results are shown in Table 2.

From this we can see that there is evidence of a linkage between the Saxton, Janssen, Blome and Overton maps. There is strong evidence *against* a linkage between the Overton and Harrison maps and the Harrison and Rowe maps. However, the inter-

Table 2

Contended map pairs	Akaike weight for different *f* and *g*
Saxton (1575) vs. Janssen (1646)	0.0000
Janssen (1646) vs. Blome (1673)	0.0000
Blome (1673) vs Overton (1713)	0.0000
Overton (1713) vs. Harrison (1790)	0.9963
Harrison (1790) vs. Rowe (1813)	0.9999
Rowe (1813) vs. Wyld (1891)	0.3264

pretation is less clear-cut in terms of linkage between the Rowe and Wyld maps—the balance of evidence is that the maps are linked, but the strength of this evidence is considerably less than that for the earlier maps.

Finally we consider Akaike weights for *every* pair of maps—not just temporally adjacent ones. Since there are seven maps in the study, this implies that 21 different Akaike weight computations must be carried out. Although 21 two-way interactions may present quite a large amount of information in tabular form, a better way to highlight patterns is to represent this graphically. To achieve this, we regard the Akaike weight associated with each model pair as a distance metric. This is reasonable, since a weight of zero implies no difference between a pair of maps, and larger values suggest greater degrees of similarities. Clearly this is not a Euclidean metric, but does provide a plausible measure of dissimilarity. Using this we can create a distance matrix for the seven maps, which may be used by a multidimensional scaling technique to create a set of seven points (one for each map) which may be plotted.

The result of this is shown in Fig. 4. Here, the multidimensional scale technique of *Sammon mapping* (Sammon, 1969) is used. This technique is appropriate here since it is a non-metric approach—that is, it does not require the 'distances' or dissimilarities between points to have all of the properties of a well-defined metric, such as Euclidean distance.

The figure shows the four earliest maps to occupy nearly identical points in the plot—these are the maps that the earlier analysis suggests come from the same source. Standing distinctly from all of the other maps is the Harrison (bottom left on the plot). The Rowe and Wyld maps are then distinct from any of these others, but relatively close to one another. This visualisation bears out the conclusion of the earlier analysis—the four earliest maps form one very tight cluster, the Harrison forms a 'cluster' on its own, and the Rowe and Wyld could perhaps form a single cluster, although they could be interpreted as occupying two distinct clusters.

7. Conclusions

In this paper we have produced a method of visualising errors within maps, assessing the errors empirically, hypothesising a lineage within the map collection,

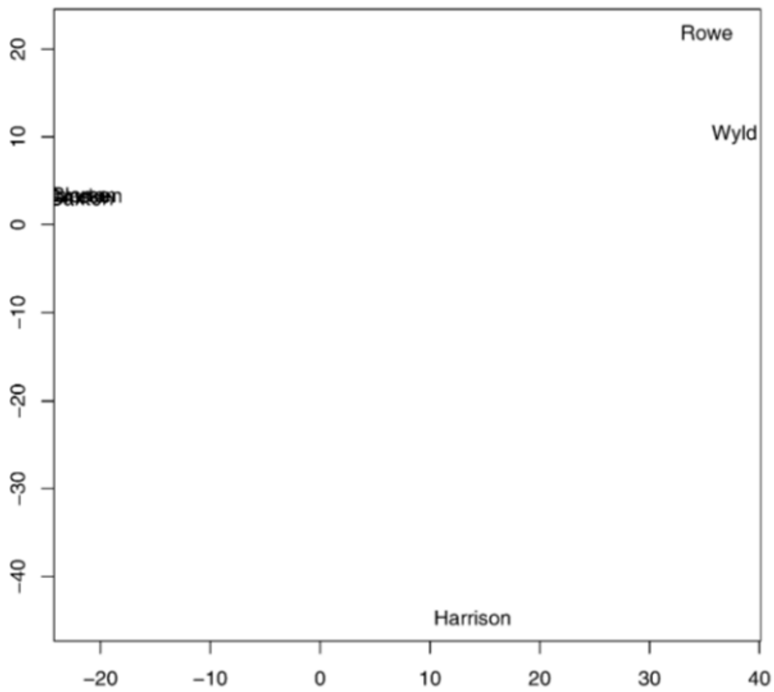


Fig. 4. Sammon mapping of Akaike weights.

and empirically testing the hypothesised lineage. The secondary data indicated that there were three distinct groups within the map lineage:

1. heavily plagiarised early maps;
2. relatively accurate maps forming a transition between ancient and modern cartography; and
3. maps produced contemporarily with early Ordnance Survey maps with modern techniques and good geodetic accuracy.

The main result emerging from this study, however, is the utility of the bidimensional regression technique employed. Given a rudimentary understanding of the methodology and adequate resources, it is possible for any user to apply this technique to any number of maps. This has implications for many areas of study for example, the non-linear bidimensional regression functions (f and g) may be applied to the co-ordinates of graphical objects stored in a GIS, this would essentially remove the mapping error from ancient cartographic sources.

These records may be used in any number of studies to quantify change in any landscape features depicted on maps such as roads, rivers, settlements, or as was originally intended for this study, coastal geomorphological change. It would be possible, using the bidimensional regression technique, to compare the location of a section of coastline on a modern map and on an ancient map source to obtain, for example, some idea of the erosion or accretion in a given area. This element of spatial analysis is termed *space–time process study* or investigation into 'how spatial arrangements are modified by movement or spatial interaction' (Gatrell, 1983 p. 2). From space-time process study, it is a logical progression to think that spatial processes or spatial forecasting may also be examined with the aid of bidimensional regression.

A selection of seven county maps has been analysed. The secondary data collection revealed many more county maps of Suffolk that have been produced between Elizabethan and Victorian times. As only a small sample of maps was analysed, any conclusions reached can only be generalisations of the lineage present within the map collection. However, the methodology developed for this project is applicable to any map of a similar nature to those used in this paper.

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