

# Theoretical and Experimental Investigation of Phase Gratings.

William Lanigan<sup>1</sup>, Ruth Colgan<sup>1</sup>, J. Anthony Murphy<sup>1</sup> & Stafford Withington<sup>2</sup>

<sup>1</sup>Experimental Physics Department, National University of Ireland, Maynooth, Co. Kildare, Ireland

<sup>2</sup>Cavendish Laboratory, Cambridge University, Madingley Road, Cambridge CB3 0HE, England

## Introduction

Phase gratings are now being incorporated into submillimetre-wave array receivers as beam multiplexing devices [1,2]. They have the advantage of being relatively easy to manufacture because of the long wavelength. Transmission gratings can be machined from suitable transparent dielectric materials such as Teflon or quartz. Phase gratings find ready application in submillimetre-wave heterodyne imaging, where optical coupling provides the most efficient way of combining LO power for systems limited to one source.

In this investigation we restrict our study to binary phase gratings in which the optical path length through the grating takes on just two values. A Dammann phase grating [3,4] is an example of a binary grating designed to produce a finite array of diffraction spots of equal intensity. In the paper we summarise our investigations into theoretical modelling of phase gratings using Gaussian beam mode analysis. This can be used as a powerful theoretical tool in predicting grating operation with finite throughput optics, quantifying bandwidth effects and manufacturing tolerances. We also present some experimental measurements for a scale model millimetre-wave phase grating which operates at 100 GHz.

## Beam Mode Analysis of Phase Gratings

The basic operation of a phase grating can be understood in terms of Fourier optics. The grating consists of an array of basic cells with the transmission function of the on-axis cell being given by the aperture function  $t(x,y)$ . The transmission of an ideal grating with rectangular symmetry is given by sum:  $\sum_{m,n} t(x-m\Delta x, y-n\Delta y)$ , where  $\Delta x$  and  $\Delta y$  are the grating periods in the  $x$  and  $y$  directions, respectively. In a typical submillimetre-wave system the grating will be illuminated by some incident quasi-Gaussian field  $\psi(x,y)$  produced by the LO source feed. If one assumes that the extent  $\psi(x,y)$  is less than that of the grating, then at the output plane (the Fourier plane of the plane where the grating is placed), the resultant field obeys the usual Fourier Transform relationship:  $E(u,v) = [\sum_{m,n} T(u,v) \delta(u - m\Delta u) \delta(v - n\Delta v)] * \mathcal{F}\psi(u,v)$ , where capital symbols represent Fourier Transforms. The output field is therefore an array of images of the quasi-Gaussian field  $\mathcal{F}\psi(u,v)$ , with the intensity of the various images determined by the amplitude envelope function  $T(u,v)$  [5].

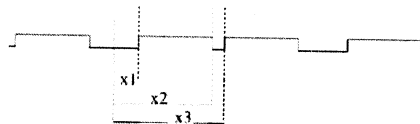
In order to model the effect of the finite throughput of the optics, with the consequent spatial filtering effects on the output array of images, it is convenient to apply a Gaussian beam mode analysis of a typical phase grating set-up. Because of the symmetry of the grating, a phase modulated quasi-Gaussian input field,  $\psi(x,y)\exp(i\phi(x,y))$ , can most conveniently be expressed as a sum of Hermite-Gaussian beam modes,  $\sum_{m,n} A_{mn} \psi_{mn}(x,y)$ . Higher order modes are of much greater spatial extent than the fundamental and may be truncated at optical components between the grating and the output plane. Such stops will act as effective low pass filters for the beam modes, placing a limitation on the maximum number of output beams possible [6]. Clearly, for the system to operate efficiently as a multiplexer, the throughput of the optics between the grating and the output plane must be sufficiently great. A careful modal analysis must therefore be an important aspect of the design of any compact multiplexing quasi-optical system.

Phase gratings only operate correctly over a finite bandwidth, which may pose a limitation for certain applications. Two effects occur if the wavelength of the LO beam does not correspond to the design wavelength of the grating: (i) the actual inter-beam separation in the output plane will change, and (ii) the different phase delays for the grating will not be as designed. The effect is to change  $T(u)$ , and thus the relative intensities of the output array of image beams. These effects can be readily modelled using Gaussian beam modes by recalculating the mode coefficients. It is worth noting that mixer sensitivity to local oscillator power for optimised performance is usually not so critical that such a small variation in the output beam intensities would cause a significant deterioration in performance across the array. Manufacturing tolerances can in a similar fashion be quantified using beam mode analysis by modelling the effects of small resultant variations in the phase delays and cell widths.

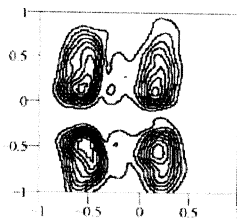
## Experimental Testing of Phase Gratings

We have developed a test facility at Maynooth to evaluate the performance of our grating designs. The facility is built for operation at 100GHz for the scale modelling of submillimetre-wave gratings. This frequency provides a reasonable compromise between the size of the quasi-optical components and their ease of manufacture. The test facility source consists of the Gunn oscillator, isolator and a conical horn antenna. The beam from this source is brought to a waist at the grating plane by an off-axis parabolic mirror and the output from the grating is collected and imaged onto the detector plane by a second matching mirror. The crystal detector is fed by conical horn, and mounted on a XY raster scan mechanism which has a useable scan area of 550mm  $\times$  550mm. This mechanism is computer controlled and has a step resolution of 0.03mm. Data can be collected from any user-defined area within the limits of the scanner and is read from the detector via a variable gain scaling amplifier and a 12 bit A/D converter.

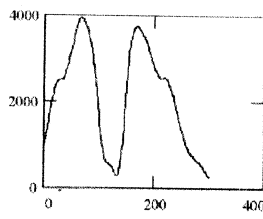
The plots below show the results obtained from our first 2  $\times$  2 Quartz grating. As is clear from the example grating designed and tested, the scheme is particularly suitable for the coupling of LO power in sparse array heterodyne systems.



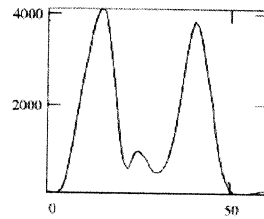
Cross section through two cells.



Contour plot of beam pattern



Cut through bright spots in Y



Cut through bright spots in X

## Conclusions

In this paper we have summarised a theoretical and experimental investigation of phase gratings for LO beam multiplexing at submillimetre wavelengths. Gaussian beam mode analysis was found to be a powerful theoretical tool in analysing the operation and limitation of such gratings in practical systems. The theoretical bandwidth of a typical grating was found to be of the order of 15%, which would be adequate for astronomical array receivers operating in the submillimeter waveband.

## References

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