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# Asymmetric Doping Effects and Sanctions in Sporting Contests

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**Abstract:** This paper analyses a one-shot game where, prior to a contest, two athletes simultaneously decide whether to engage in doping that is not certain to be detected. Doping is assumed to have at least as great a proportional effect on a naturally weaker athlete's win probability. Given an explicit contest success function, the paper derives an optimal sanction scheme, where sanctions are identically proportional to prizes, sufficient to always induce a no-doping equilibrium. In comparison to previous papers, the winner's optimal sanction, expressed in terms of talent and doping levels and doping costs, may be lower for all detection probabilities.

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# 1. Introduction

The prevalence of cheating in sport is almost as old as sport itself, with some reports emanating from the 3rd century BC. In modern times, the increased rewards of sporting success at the elite level are an obvious temptation in which to seek more favourable outcomes through cheating. As incentives to cheat have increased, the methods employed have becomes ever more sophisticated and can take many forms: the 'professional' foul to prevent a rival from scoring; 'pulling' horses in races to attain a higher handicap or increased betting odds in future races; bribing voters to award a World Cup or Olympic games to a particular country; team 'orders' in motor racing to ensure a particular result. All of these methods, and others, seek to change the outcome of a sporting contest from what would 'naturally' occur based on the best efforts of the participants.

If sporting authorities are serious about eradicating cheating from their sport, then two questions follow: Firstly, how extensive should any efforts to monitor cheating be? Secondly, how severe should be the penalties imposed on those found guilty of cheating? In line with increased incentives to cheat, sporting organisations have increased out-of-competition monitoring and testing of athletes, as well as the punishments for those found guilty of cheating. Also, multi-camera TV coverage of major sporting events increases the likelihood of detecting within-contest cheating. Against this, however, when cheating takes the form of consuming performance-enhancing drugs (PEDs), or 'doping', such policing may be 'behind the curve' and requires the authorities to play 'catch-up' with the athletes, and possibly the medical professionals and pharmaceutical companies that supply athletes with such drugs.

This paper focuses on the latter aspect of cheating. Doping can involve the consumption of synthetic versions of natural hormones such as erythropoietin (EPO), testosterone, nandrolone and human growth hormone (HGH) or artificial substances like tetrahydrogestrinone (THG).[1] Irrespective of the substance employed by an athlete, the objective is to increase the probability of success above that which would prevail in the absence of doping. For sporting authorities and anti-doping bodies, use of synthetic substances by athletes can make it difficult to definitively prove that doping has occurred.

[1] Steroid abuse was widely suspected in baseball for a number of years, particularly in the 1990s when long-standing records were regularly broken. Several years later, Mark McGwire admitted to steroid abuse when setting the single-season home-run record in 1998. Similarly, Alex Rodriguez admitted to using steroids when he was the American League MVP in 2003.

In recent years, the process of 'blood doping' has been suspected, and admitted to, in many endurance sports, particularly cycling, but also in sports like long-distance athletics and cross-country skiing. The basic role of blood doping is to increase an athlete's red blood cell (RBC) count.[2] RBCs transport oxygen to muscles so that during intensive effort over a period of time, higher RBC levels can increase endurance and reduce susceptibility to fatigue, or 'cramp', to enable athletes to 'go faster for longer'. At the elite level, a miniscule improvement in performance can be the difference between winning and losing, where winning may not only confer an immediate benefit in terms of higher prize money, but also increased future earnings through endorsements and appearance fees. Given this, athletes with a naturally low RBC level, and even those with relative high RBC levels, may have an incentive to 'blood dope' in order to increase performance. [3]

Historically, blood doping was achieved by means of a transfusion from a suitable donor or by extracting a blood sample from an athlete some time prior to an event, storing it, and then transfusing immediately before the event.[4] In recent years, medical advances have seen the development of a synthetic form of erythropoietin (EPO), a natural hormone that stimulates RBC production. Given this development, athletes need only inject themselves with some quantity of this synthetic EPO a short time prior to an event. The problem for sporting authorities is that it may be difficult to definitively determine whether such blood doping has occurred, as RBC values can also be naturally increased by, for example, engaging in altitude training. Given this, athletes may be tempted to engage in this form of doping.[5] The downside of blood doping for athletes is that increased RBC levels increase the viscosity of the blood, thereby increasing the stress on the heart as it attempts to push this thicker substance around the body.[6] In light of this, and the premature deaths of many cyclists

[2] This is often referred to as an athlete's haematocrit level.

[3] Blood doping is less prevalent in more 'skilled' events like soccer and golf, as it does not affect technical ability. Nevertheless, a relatively skilled midfielder in soccer may have an incentive to blood dope in order to improve aerobic performance. While the effect on a team's outcome may be insignificant if a player's team mates do not also engage in doping, it may serve to enhance a player's value in the transfer market.

[4] One possible problem of the former method was from receiving a tainted transfusion. This caused several members of the 1984 US Olympic cycling team to contract Hepatitis. The idea of the latter method was that during storage of the extracted sample, an athlete's RBC count would naturally replenish, though maybe not fully, so that the transfusion of the extracted RBCs, though themselves diminished, would boost the athlete's RBC count immediately prior to an event.

[5] According to doping expert Michel Audran, "Artificial boosting of haematocrit levels a week or more before a race can be maintained by micro-dosing with EPO three times a week – and still go undetected", quoted in Jeremy Whittle (2009), Bad Blood: The Secret Life of the Tour de France, p.30.

[6] According to Matt Rendell, "The gel-like blood is great for high performance, but totally unsuited to rest, and at night, when the heartbeat slows, its sheer density becomes a liability..........The athlete has to set his heart-rate monitor to beep whenever his pulse drops below a certain level,......When it sounds, he has to wake up and

in the 1990's, cycling's governing body, the Union Cycliste Internationale (UCI), introduced a rule in 1997 which stipulated that if an athlete's sample RBC (haematocrit) level exceeded 50%, the cyclist would be prevented from competing for two weeks on 'health' grounds.[7,8]

In response to increased doping, most sporting organisations have adopted the World Anti-Doping Agency (WADA) 'Code' that defines doping as, inter alia, "the presence of a Prohibited Substance,....., in an Athlete's Sample" and the "Use or Attempted Use by an Athlete of a Prohibited Substance or a Prohibited Method".[9] WADA also publish a 'prohibited list' of substances that are proven, or perceived, to be illegally performance-enhancing. A first violation of WADA's code can earn an athlete a four (previously two) year ban from all sporting activity, while a third violation can lead to a lifetime ban. [10,11] In more recent times, many sporting organisations, e.g. UCI, IAAF and FIFA, have implemented WADA's Athlete Biological Passport scheme, the purpose of which is to "…monitor selected biological variables over time that indirectly reveal the effects of doping rather than attempting to detect the doping substance or method itself.", though the scheme is not without its issues. [12,13]

This paper seeks to expand on the existing literature by looking at a one-shot game where, prior to an event, two athletes must simultaneously choose whether or not to engage in a given, symmetric, level of doping. This framework has been previously used by, inter alia, Berentsen (2002), Haugen (2004) and Eber (2008). Athletes are subject to penalties if their doping is detected, which is not certain, while doping also incurs a direct cost irrespective of whether it is detected or not. In contrast to some previous contributions, this paper explicitly

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exercise to coax his straining heart into action. For the cyclist, this means that after riding for a living all day, he rides on rollers at night to stay alive", The Death of Marco Pantani: A Biography, p96.

[7] The most famous example of this was in the 1999 Giro d'Italia when race leader, and probable winner, Marco Pantani was disqualified from the event when his haematocrit level was found to be in excess of 52%.

[8] Some argue that this effectively legitimised doping through EPO use. Cyclists with relatively low haematocrit levels could increase them to the 50% threshold without fear of punishment, though this is arguably 'unfair' on those with naturally higher levels. As Whittle (2009), p31, argued, "....donkeys became thoroughbreds....Unwittingly, a level playing field was created, though perhaps not of the kind that the UCI intended."

[9] WADA (2015), World Anti-Doping Code, p.18-20.

[10] For a complete overview of WADA's definition of doping, list of prohibited substances and sanctions regime, see https://www.wada-ama.org/en/resources/the-code/world-anti-doping-code .

[11] Well-known examples of 2-year bans are those given to Tour De France winning cyclists Alberto Contador and Floyd Landis, whose wins were revoked, and to sprinters Dwain Chambers and Marion Jones. Lifetime bans have been given to, inter alia, Canadian sprinter Ben Johnson and, most recently and famously, US cyclist Lance Armstrong, whose seven Tour De France victories from 1999-2005 were revoked.

[12] WADA (2015), available at https://www.wada-ama.org/en/what-we-do/science-medical/athlete-biological-passport

[13] According to page 12 of the 2015 independent UCI report into anti-doping policy, "The biggest concern today is that following the introduction of the athlete biological passport, dopers have moved on to micro-dosing in a controlled manner that keeps their blood parameters constant and enables them to avoid detection".

states a contest success function, where an athlete's win probability relates to its share of the total effective talent in the contest. The paper also looks at the case where symmetric doping levels can have different effects on the win probabilities of the athletes. Specifically, if both athletes dope, the win probability of a naturally weaker athlete is greater than when no athlete dopes. The sanctions scheme also differs from previous papers in that all penalties are identically proportional to winnings for both athletes. Additionally, a doper's penalty can be transferred to its 'clean' rival. Under this framework, the paper seeks to determine a sanction scheme that induces a no-doping equilibrium for all parameter values.

Section 2 gives an overview of the existing literature, while Section 3 introduces the benchmark model. In Section 4, a perfect mechanism to induce a no-doping equilibrium under any circumstances is determined. Section 5 will conclude.

## 2. Literature Review

In recent times, several papers, among them Preston and Szymanski (2003), have outlined the economic arguments surrounding the decision of sporting agents to engage in cheating during a sporting contest. Applying this issue to doping, a number of papers, e.g. Haugen (2004) and Eber (2008), begin by looking at a symmetric two-athlete case, where each athlete has a 50% probability of winning if neither or both athletes dope, while a single doper is the favourite to win.[14] In some cases, doping is assumed to be sufficiently effective in that a single doper wins for certain. These papers and others, e.g. Berentsen (2002) and Stowe and Gilpatric (2010), also discuss the asymmetric athletes case, where one of the athletes is favourite to win if neither or both athletes dope.[15,16]

Berentsen (2002) derives the minimum rankings-based sanctions that satisfy a 'perfect mechanism' where a no-doping equilibrium occurs under any circumstances. Doped athletes, if detected, pay a fine proportional to winnings, though these proportions are not identical for the winner and loser. If the winning athlete is found to have doped, its prize is 're-awarded' to the loser, but only if the latter has not, or has not been determined to have, doped. Doping

[14] See Dilger, Frick and Tolsdorf (2007) for a more extensive overview of the relevant literature.

[15] Berentsen (2002), in the Appendix, discusses a contest success function that relates win probability to expected performance, and where doping has an absolute effect on expected performance, though each athlete's win probability is identical if neither or both dope.

[16] Krakel (2007) examines the case where athletes can use legal and illegal inputs to complement natural talent. In doing so, he uses a multiplicative performance function and analyses a two-stage game where the athletes' 'investment' decisions are endogenous. His model is not directly comparable or applicable to this one.

is not certain to be detected, so a 'false negative' doping result is possible, while a non-doper may falsely test positive. To induce a no-doping equilibrium, the winner's sanction is negatively related to both the probability of detection and the cost of doping, while the loser's fine is exactly equal to its (zero) winnings.

Haugen (2004) looks at a simpler model where athletes are initially assumed to be naturally equally talented. The winner retains its winnings even if detected, though this could be recouped in the cost of being 'exposed' as a drug cheat. There is no explicit health, monetary or emotional doping cost, as such a cost is only incurred if detected. In Haugen's sanctions mechanism, a 'clean' loser does not receive a doped winner's prize. In this model, a no-doping equilibrium may not exist in pure strategies if doping is so effective that a single doper wins for certain. On the other hand, if the athletes are naturally asymmetrically talented and doping is not fully effective for a naturally weaker athlete, a unique no-doping equilibrium may exist depending on the win probability of the weaker athlete, the value of the winner's prize, the likelihood of detection and the penalty if detected.

Eber (2008) extends the Haugen model to introduce 'fair play' norms. Specifically, if there is a single doper, the doper suffers from 'guilt' due to its unfair advantage, while the non-doper suffers from 'resentment'. The effect of fair play norms is to reduce each athlete's expected utility relative to the 'no fair play' case. Eber finds that a doping or no-doping Nash equilibrium may exist depending on the degree of fair play among the athletes.

The above authors assume an imperfect dope-testing system and do not explicitly state a contest success function that relates win probability to talent, performance or doping levels. The authors also assume that if only one athlete dopes, it is more effective for the weaker athlete in terms of a greater increase in win probability. On the other hand, if both athletes dope, win probabilities are identical to when no athlete dopes, so the implicit contest success function assumes a proportional effect of doping on talent or expected performance.

This paper assumes that doping is at least as effective, in terms of increased win probability, for a naturally weaker athlete, irrespective of whether one or both athletes dope. Intuitively, a given level of doping may have relatively greater effects on a weaker athlete by disproportionately increasing its RBC count. To look at this issue, it is necessary to explicitly state a contest success function that underpins individual win probabilities. It is also assumed that doping is not certain to be detected, while there is also an explicit health and/or monetary cost to doping, irrespective of whether doping is detected or not. As with Berentsen, rankings-based sanctions are derived to satisfy a 'perfect mechanism' that induces a no-doping Nash equilibrium under any circumstances, though in this paper, sanctions will be explicitly expressed in terms of talent and doping levels.[17] The difference in this paper is that sanctions are imposed on an equi-proportionate basis on all athletes. Also, if the doping of only one athlete is detected, that athlete's fine is transferred to its 'clean' rival. Finally, and in contrast to Berentsen, this paper assumes that sufficient safeguards exist in WADA doping processes that a 'false positive' outcome can be ignored.

Given the above context, this paper seeks to answer two particular questions: Firstly, for any given anti-doping policy, will doping ever form part of a Nash equilibrium outcome? Secondly, for any given parameters of the models, what are the minimum sanctions required to induce a no-doping Nash equilibrium outcome under any circumstances?

## 3. The model

This paper analyses a one-shot game that is played by two athletes (A and B) who, prior to a contest, must simultaneously choose whether to dope (D) or not dope (ND). The benefit of doping is to increase the effective talent of an athlete that, all else equal, increases the probability of winning. On the other hand, doping is costly in health and/or monetary terms, and possibly if 'detected' and punished by any anti-doping policy.

Doping is assumed to absolutely augment natural talent levels so that the *effective* talent of athlete i is

$$\gamma_i = t_i + d_i \tag{1}$$

where $t_i$ and $d_i$ represent natural talent and doping levels, respectively (i = A,B).[18] It is assumed that $[t_A, t_B] \in [\underline{t}, \bar{t}]$ and that $[d_A, d_B] \in [\underline{d}, \bar{d}]$.[19] It is assumed that $t_A \geq t_B$, so that athlete A is at least as naturally talented as athlete B. The athletes compete for a total prize fund of $w_1 + w_2$, where a 'clean' winner of the contest receives $w_1$, while a 'clean' loser receives $w_2$, where $w_1 > w_2 \geq 0$.[20] For simplicity, it is assumed that the level of doping is

[17] One issue in the Berentsen paper is that the optimal sanction for all parameter values is derived without taking correlated win probabilities into account. This is discussed later in this paper.

[18] If doping was assumed to have a relative effect, then for symmetric doping levels, the athletes' individual win probabilities would be identical in the cases where no or both athletes dope.

[19] The lower bound of talent may reflect a minimum standard required to qualify for competition, while the upper bound may be a rating based on a subjective measure, existing record or perceived limit of performance.

[20] It is assumed that the probability of a tie is approximately zero, or that a mechanism exists to determine a winner. The latter could be extra (over) time, a replay or some verifiable performance measure during a contest.

fixed for each athlete so that $d_A = d_B = d > 0$. Doping also entails a fixed cost of $c < w_1$, identical for each athlete, that can include any health or monetary costs of doping.[21]

In line with much of the sports literature, particularly the competitive balance literature, this paper assumes a logistic contest success function (CSF) where each athlete's win probability is determined by its share of the total effective talent in the contest.[22] Specifically, athlete i's win probability (i = A,B) in any given circumstance is

$$p_i = \frac{\gamma_i}{\gamma_i + \gamma_j} = \frac{t_i + \theta_i d_i}{\sum_{i=A}^{B}(t_i + \theta_i d_i)} \qquad (i,j = A,B\ ,\ i \neq j) \qquad (2)$$

where $\theta_i$ is an indicator variable equal to unity if athlete i dopes and zero otherwise.[23]

As there are four possible outcomes depending on whether no, one or each athlete dopes, then given (2), it is possible to express each athlete's win probability in each case:[24]

**<u>Table 1</u>**

| $p_i$ / A , B | $p_A$ | $p_B$ |
|---|---|---|
| ND , ND | $\bar{p} = \frac{t_A}{t_A + t_B} \geq \frac{1}{2}$ | $1 - \bar{p} = \frac{t_B}{t_A + t_B}$ |
| D , ND | $p' = \frac{t_A + d}{t_A + t_B + d}$ | $1 - p' = \frac{t_B}{t_A + t_B + d}$ |
| ND , D | $\tilde{p} = \frac{t_A}{t_A + t_B + d}$ | $1 - \tilde{p} = \frac{t_B + d}{t_A + t_B + d}$ |
| D , D | $\hat{p} = \frac{t_A + d}{t_A + t_B + 2d} \geq \frac{1}{2}$ | $1 - \hat{p} = \frac{t_B + d}{t_A + t_B + 2d}$ |

Given our assumptions, $0 < \tilde{p} < \hat{p} \leq \bar{p} < p' < 1$. As doping is assumed to be at least as effective for a weaker athlete, in terms of increasing win probability, irrespective of whether one or both athletes dope, it must be the case that $\hat{p} \leq \frac{p' + \tilde{p}}{2} \leq \bar{p}$, which is satisfied given our assumption that $t_A \geq t_B$.[25]

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[21] If $c > w_1$, no rational athlete would ever dope. If the level of doping was a choice variable, marginal doping costs could be increasing in the level of doping.

[22] Papers that utilise such a CSF include El Hodiri & Quirk (1971), Szymanski (2004) and Vrooman (2009).

[23] The marginal effect of athlete i's effective talent on own win probability is positive but decreasing, while the marginal effect of athlete j's effective talent on i's win probability is negative but increasing (i,j = A,B , $i \neq j$).

[24] In table 1, the first column on the left hand side denotes the actions of the athletes, with athlete A's action first. For example, (D,ND) refers to when athlete A is the sole doper.

[25] If doping is at least as effective for the weaker athlete, it must be the case that $(1 - \tilde{p}) - (1 - \bar{p}) \geq p' - \bar{p}$ and that $(1 - \hat{p}) - (1 - p') \geq \hat{p} - \tilde{p}$. Combining these conditions gives the result in the text.

### 3.1. Benchmark Case: No anti-doping policy

When there is no anti-doping policy by the relevant sporting authority, then given (2) and defining $\tilde{w} = \frac{w_2}{w_1} < 1$ and $\tilde{c} = \frac{c}{w_1} < 1$, the athletes' expected payoffs (athlete A's payoff first), divided by the winner's prize, are:

**Table 2**

Athlete B

| Athlete A | | D | ND |
|---|---|---|---|
| | D | $\hat{p}+(1-\hat{p})\tilde{w}-\tilde{c}\,,(1-\hat{p})+\hat{p}\tilde{w}-\tilde{c}$ | $p'+(1-p')\tilde{w}$-$\tilde{c}\,,(1-p')+p'\tilde{w}$ |
| | ND | $\tilde{p}+(1-\tilde{p})\tilde{w},(1-\tilde{p})+\tilde{p}\tilde{w}-\tilde{c}$ | $\bar{p}+(1-\bar{p})\tilde{w},(1-\bar{p})+\bar{p}\tilde{w}$ |

Given our assumptions, doping forms part of a Nash equilibrium outcome if $\tilde{c} < (1-\tilde{w})[\bar{p}-\tilde{p}] \equiv \lambda_1$.[26] To justify the introduction of an anti-doping policy, it is assumed that doping costs are sufficiently low relative to the winner's prize that such outcomes exist.

In Figure 1, where $t_A = 0.8$, $t_B = 0.6$, $d = 0.5$, the unique Nash equilibrium outcome depends on the relationship between doping costs and relative prizes. The lower the prize disparity ($1-\tilde{w} \to 0$), the lower the incentive to dope for any doping costs. Given any prize disparity, if doping costs are low enough, each athlete dopes if their rival does. For both athletes, doping increases their win probability, which would be lower if only its rival doped, and each is willing to incur the, relatively low, health and/or monetary costs to ensure this. As doping costs increase sufficiently highly, the stronger athlete is more likely to not engage in doping if the weaker athlete does. For the stronger athlete, the increased win probability is not sufficient to risk incurring the higher doping cost. On the other hand, the weaker athlete's win probability increases sufficiently to dominate any doping cost. As doping costs increase further, neither athlete engages in doping if its rival does not.[27]

[26] Both athletes doping (D,D) is a Nash equilibrium if $\tilde{c} \le (1-\tilde{w})[\hat{p}-\tilde{p}] \equiv \lambda_2$, which is inefficient for the athletes ('Prisoner's Dilemma') if $\lambda_3 \equiv (1-\tilde{w})[\bar{p}-\hat{p}] < \tilde{c} < (1-\tilde{w})[\hat{p}-\tilde{p}] \equiv \lambda_2$. Athlete B as the sole doper (ND,D) is a Nash equilibrium if $\lambda_2 \equiv (1-\tilde{w})[\hat{p}-\tilde{p}] \le \tilde{c} \le (1-\tilde{w})[\bar{p}-\tilde{p}] \equiv \lambda_1$. If the athletes are naturally asymmetric ($t_A > t_B$), there is never a Nash equilibrium where only athlete A dopes (D,ND). Such an outcome can only occur if the athletes are equally talented ($t_A = t_B$) so that $\hat{p} = \bar{p}$ and only then if $(1-\tilde{w})[p'-\bar{p}] = (1-\tilde{w})[p'-\hat{p}] = \tilde{c}$. A no-doping Nash equilibrium (ND,ND) requires $\tilde{c} \ge (1-\tilde{w})[\bar{p}-\tilde{p}] \equiv \lambda_1$.

[27] Similar incentives arise as doping and relative talent levels increase.

All else equal, a no-doping Nash equilibrium is more likely to occur the more talented is the weaker athlete, while it is less likely the greater the level of doping and the disparity in prizes. Also, as the stronger athlete's talent increases, a no-doping Nash equilibrium is less likely the higher the level of doping.

The main interest of this paper is in determining the doping incentives of the athletes when subject to an anti-doping policy. It is assumed that the dope-testing system is imperfect in that the probability of a doped athlete being detected is r, where $0 < r < 1$ is exogenous to the athletes.[28] Consequently, the probability of a 'false negative' is 1-r. As in Berentsen (2002) and Curry and Mongrain (2009), both winner and loser are penalised if their doping is detected. In this paper, however, penalties are equally proportionate to both prizes. Also, if one athlete is found to have doped, its rival, if 'clean', not only retains its own prize but is also awarded the doper's 'fine'.[29] It is also assumed that neither athlete suffers from 'fair play' norms such as guilt or resentment when there is a single doper as in Eber (2008). In the presence of such an anti-doping policy, the prize structure is as follows:

| | |
|---|---|
| Winner | $w_1$ if both athletes are (i) not doped *or* (ii) doped but not detected<br>$(1-\phi)w_1$ if doped and detected<br>$w_1 + \phi w_2$ if not doped or doped and not detected *and* loser doped and detected |
| Loser | $w_2$ if both athletes are (i) not doped *or* (ii) doped but not detected<br>$(1-\phi)w_2$ if doped and detected<br>$w_2 + \phi w_1$ if not doped or doped and not detected *and* winner doped and detected |

where $\phi \geq 0$ is a penalty parameter that determines an athlete's net payoff.[30] The prize structure ensures that the net prize fund allocated to the athletes never exceeds $w_1 + w_2$.[31]

Given the anti-doping policy, the expected payoffs of the athletes (athlete A first), divided by the winner's prize, are:

[28] It may be that the athletes are not certain to be tested, or testing is certain but doping cannot be definitively proved due to imperfections in the testing system. For example, the doping authorities cannot test for a particular method of doping, the testing procedure itself may be improperly conducted or test samples become contaminated in some way thereby nullifying the test if a positive doping result is appealed by the athlete.

[29] In Berentsen (2012), a clean loser is 're-awarded' the winning prize if the winner is found to have doped. In Curry & Mongrain (2009), however, a clean loser is not certain to be re-awarded the winning prize.

[30] Curry & Mongrain (2009) assume 'limited liability' for athletes in terms of penalties, so that $\phi \leq 1$. On the other hand, Gilpatric (2011) allows for 'outside' penalties in terms of disqualification from future events or reduced future earnings that may effectively ensure $\phi > 1$.

[31] If neither dopes, the prize allocation is simply $w_1 + w_2$. If only the winner is detected, the net prize allocation is $w_1 - \phi w_1 + w_2 + \phi w_1 = w_1 + w_2$. Similarly, if only the loser is detected, the net prize allocation is $w_1 + \phi w_2 + w_2 - \phi w_2 = w_1 + w_2$. Finally, if both are detected, net prize allocation is $(1-\phi)(w_1+w_2) \leq w_1 + w_2$.

**Table 3**

Athlete B

| | | D | ND |
|---|---|---|---|
| Athlete A | D | $\hat{p}+(1-\hat{p})\tilde{w}-\phi r\{[\hat{p}+(1-\hat{p})\tilde{w}]-(1-r)[(1-\hat{p})+\hat{p}\tilde{w}]\}-\tilde{c}$ , $(1-\hat{p})+\hat{p}\tilde{w}-\phi r\{[(1-\hat{p})+\hat{p}\tilde{w}]-(1-r)[\hat{p}+(1-\hat{p})\tilde{w}]\}-\tilde{c}$ | $p'+(1-p')\tilde{w}-\phi r[p'+(1-p')\tilde{w}]-\tilde{c}$ , $(1-p')+p'\tilde{w}+\phi r[p'+(1-p')\tilde{w}]$ |
| | ND | $\tilde{p}+(1-\tilde{p})\tilde{w}+\phi r[(1-\tilde{p})+\tilde{p}\tilde{w}]$, $(1-\tilde{p})+\tilde{p}\tilde{w}-\phi r[(1-\tilde{p})+\tilde{p}\tilde{w}]-\tilde{c}$ | $\bar{p}+(1-\bar{p})\tilde{w}$ , $(1-\bar{p})+\bar{p}\tilde{w}$ |

(see Appendix for a sample of derivations). For simplicity, the payoff matrix is denoted by

**Table 3a**

Athlete B

| | | D | ND |
|---|---|---|---|
| Athlete A | D | $A_{11}$ , $B_{11}$ | $A_{12}$ , $B_{12}$ |
| | ND | $A_{21}$ , $B_{21}$ | $A_{22}$ , $B_{22}$ |

Given Table 3, the Nash equilibrium conditions can be derived in terms of threshold values of the penalty parameter (ϕ) so that

**Table 4**

| Nash equilibrium | Condition |
|---|---|
| ND, ND | $\phi_2 \equiv \dfrac{(1-\tilde{w})(\bar{p}-\tilde{p})-\tilde{c}}{r[1-\tilde{p}(1-\tilde{w})]} \leq \phi$ |
| D, ND | $\phi_3 \equiv \dfrac{(1-\tilde{w})(p'-\hat{p})-\tilde{c}}{r\{1+r\tilde{w}+(1-\tilde{w})[(p'-\hat{p})-\hat{p}(1-r)]\}} \leq \phi \leq \dfrac{(1-\tilde{w})(p'-\bar{p})-\tilde{c}}{r[\tilde{w}+p'(1-\tilde{w})]} \equiv \phi_4$ |
| ND, D | $\phi_1 \equiv \dfrac{(1-\tilde{w})(\hat{p}-\tilde{p})-\tilde{c}}{r\{r+\tilde{w}+(1-\tilde{w})[(\hat{p}-\tilde{p})+\hat{p}(1-r)]\}} \leq \phi \leq \dfrac{(1-\tilde{w})(\bar{p}-\tilde{p})-\tilde{c}}{r[1-\tilde{p}(1-\tilde{w})]} \equiv \phi_2$ |
| D, D | $\phi \leq \dfrac{(1-\tilde{w})(\hat{p}-\tilde{p})-\tilde{c}}{r\{r+\tilde{w}+(1-\tilde{w})[(\hat{p}-\tilde{p})+\hat{p}(1-r)]\}} \equiv \phi_1$ and $\phi \leq \dfrac{(1-\tilde{w})(p'-\hat{p})-\tilde{c}}{r\{1+r\tilde{w}+(1-\tilde{w})[(p'-\hat{p})-\hat{p}(1-r)]\}} \equiv \phi_3$ |

Given our assumptions, $\phi_2 \geq \phi_4$ for all parameter values. Also, as the various model parameters ($t_A$, $t_B$, d, c, $w_1$, $w_2$) change, there may be a unique or multiple Nash equilibria for any given values of the penalty parameter ($\phi$) and probability of detection (r).[32] To determine how Nash equilibrium outcomes are affected by parameter value changes, it is assumed that $\underline{t} = \underline{d} = 0$ and $\bar{t} = \bar{d} = 1$.

Looking firstly at Figure 2, where $t_A = 0.8$, $t_B = 0.6$, d = 0.5, $\tilde{c} = 0.05$ and $\tilde{w} = 0$, the unique Nash equilibrium for most ($r,\phi$) parameter combinations is where no athlete dopes (ND,ND), as the probability of detection and/or the penalty if detected are sufficiently high to discourage doping.[33] As the values of both r and/or $\phi$ begin to decrease from a no-doping equilibrium, there may be a unique Nash equilibrium where only the weaker athlete dopes (ND,D), with the range of penalty parameters at which this occurs decreasing in the detection probability. Given that the stronger athlete does not dope, the weaker athlete has an incentive to dope as, given the parameter values, the increase in win probability from 42.9% to 57.9% can lead to a relatively large prize increase that dominates the expected cost to doping. For the stronger athlete, who, given the parameter values, has a 42.1% win probability when only the weaker athlete dopes, the combination of detection probability and penalties, as well as the possibility that doping by the weaker athlete will be detected and its fine re-awarded to the stronger athlete, dominates any expected gain from its win probability increasing to 54.2% by also doping and running the risk of being detected.

As the penalty parameter decreases further, there may be multiple Nash equilibria where only one athlete dopes. The intuition for the weaker athlete is given above. For the stronger athlete, however, even at relatively high detection probabilities, the lower penalty makes its worth being the only doper as the increase in win probability from 57.1% to 68.4% dominates any expected cost from doping. Given that the stronger athlete dopes, the weaker athlete will choose not to as the increase in win probability from 31.6% to 45.8% is not sufficient to overcome the expected cost while also having the possibility that the winner's fine, if detected, will be transferred to the loser.

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[32] one of the conditions in Table 4 is defined for r = 0. Intuitively, if there is no probability of doping being detected, the Nash equilibrium is as in the benchmark case of no anti-doping policy.

[33] Given the parameter values, the threshold level of $\tilde{c}$ is 0.1559 to ensure that doping forms part of a Nash equilibrium which requires $\tilde{c} \leq (1-\tilde{w})[\bar{p} - \tilde{p}] = \dfrac{d(1-\tilde{w})t_A}{(t_A + t_B)(t_A + t_B + d)}$.

If the penalty parameter is extremely low, then for a given detection probability, there is a Nash equilibrium where both athletes dope. In this case, given that their rival dopes, each athlete will prefer to dope as the increase in win percentage is now sufficiently high to offset any expected cost of doping.

Looking at Figure 3, where athlete A is now absolutely and relatively more talented and doping levels are lower ($t_A = 1$, $t_B = 0.5$ and $d = 0.5$), similar results apply, except that there is never a Nash equilibrium where only the stronger athlete dopes. If the weaker athlete does not dope, then the stronger athlete can make the probability of winning more likely (from 66.7% to 75%) by doping. Despite this, the direct doping cost and the possibility that its doping will be detected and a penalty imposed is a sufficient dis-incentive to the stronger athlete to attempt to increase its win probability by a relatively small amount through doping.

Finally, keeping $t_A = 1$, $t_B = 0.5$ and $d = 0.5$ but increasing direct doping costs to $\tilde{c} = 0.15$, Figure 4 shows that the incentive to dope is so diminished that only the weaker athlete dopes if the probability of detection and penalties are extremely low.

## 4. Perfect Mechanism

For an anti-doping policy to induce a unique no-doping equilibrium for all possible parameter values, it is necessary to determine the minimum sanctions that satisfy a 'perfect mechanism'. Given Table 3a, (ND,ND) is a unique Nash equilibrium when the following conditions are satisfied:

(i) $A_{22} \geq \tilde{w}$ and $B_{22} \geq \tilde{w}$

(ii) $A_{22} \geq B_{22}$

**either** (iiia) $A_{22} > A_{12}$ , $A_{21} > A_{11}$ and $B_{22} > B_{21}$ **or** (iiib) $B_{22} > B_{21}$ , $B_{12} > B_{11}$ and $A_{22} > A_{12}$.

Condition (i) denotes the participation constraints, where reservation payoffs are assumed to be equal to the loser's prize, that are satisfied given $\bar{p} \geq 0.5$ and $0 \leq \tilde{w} < 1$.[34] Condition (ii) reflects the incentive compatibility constraint. As athlete A is at least as talented and weakly favoured to win when neither athlete dopes, its expected payoff is at least as great as that of athlete B. This condition is also satisfied given $\bar{p} \geq 0.5$ and $0 \leq \tilde{w} < 1$.

[34] Reservation levels could be set at zero if sporting organisations can withhold prizes or disqualify contestants due to a perceived or proven lack of effort by participants. Such mechanisms exist in, among others, boxing and horse racing. The necessary condition required to satisfy (i) is unchanged.

Conditions (iiia) and (iiib) ensure that one athlete's dominant strategy is to not dope and, given this, the other athlete's best response is also to not dope. Condition (iiia) is satisfied for the respective conditions if $\phi \geq \phi_4$, $\phi \geq \phi_1$ and $\phi \geq \phi_2$. Similarly, condition (iiib) is satisfied if $\phi \geq \phi_2$, $\phi \geq \phi_3$ and $\phi \geq \phi_4$.[35] Given $\phi_2 \geq \phi_4$, then to induce a unique no-doping equilibrium for all possible parameter values, the anti-doping authorities must derive a value of $\phi$ that is sufficiently high that either (i) $\phi > \phi_1$ and $\phi > \phi_2$ or (ii) $\phi > \phi_2$ and $\phi > \phi_3$.

Given the contest success function in (1), changes in a given win probability can have effects on, possibly all, other win probabilities, depending on what causes the change in the win probability. If, for instance, an athlete's natural talent level increases, possibly due to a new training regime, then this increases its win probability in all possible cases.[36] Consider the case whereby athlete A deviates from (D,D) if $\phi \geq \dfrac{(1-\tilde{w})(\hat{p}-\tilde{p})-\tilde{c}}{r\{r+\tilde{w}+(1-\tilde{w})[(\hat{p}-\tilde{p})+\hat{p}(1-r)]\}} \equiv \phi_1$.

Given (1) and (2), changes in individual talent and doping levels affect *both* $\hat{p}$ and $\tilde{p}$. Consequently, in deriving the maximum values of the various penalty parameter thresholds, one must consider the *total*, rather than the *partial*, effects of such changes in talent and doping levels on individual win probabilities.[37]

Using this approach, consider how a change in athlete B's talent level affects its rival's incentive to deviate from outcome (D,D). Given (1) and (2),

$$\partial\phi_1/\partial t_B \begin{pmatrix} > \\ < \end{pmatrix} 0 \text{ if } \frac{-\partial\hat{p}/\partial t_B}{-\partial\tilde{p}/\partial t_B} \begin{pmatrix} < \\ > \end{pmatrix} \frac{r+\tilde{w}+(1-\tilde{w})\hat{p}(1-r)+\tilde{c}}{r+\tilde{w}+(1-\tilde{w})\tilde{p}(1-r)+(2-r)\tilde{c}} \equiv \psi \qquad (3)$$

so that, all else equal, athlete A is less (more) likely to deviate from (D,D) if the effect of a change in B's talent level on A's win probability when both dope relative to when only A dopes is sufficiently low (high). In (3), it is not possible to determine a definitive relationship between this effect and the $\psi$ term for all talent levels, probability of detection and $\tilde{c} < (1-\tilde{w})(\hat{p}-\tilde{p})$, though (3) can be reduced to $\partial\phi_1/\partial t_B \begin{pmatrix} > \\ < \end{pmatrix} 0 \text{ if } \partial(\hat{p}-\tilde{p})/\partial t_B \begin{pmatrix} > \\ < \end{pmatrix} 0$, which depends on talent and doping levels.[38]

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[35] In respect of the incentives of the athletes, athlete A will not deviate from (ND,ND) if $\phi \geq \phi_4$ or from (ND,D) if $\phi \geq \phi_1$. Similarly, athlete B will not deviate from (ND,ND) if $\phi \geq \phi_2$ or from (D,ND) if $\phi \geq \phi_3$.

[36] In Berentsen (2002), talent changes affect expected performance, which determines win probability, in all cases.

[37] In deriving the maximum threshold penalty parameter, Berentsen's approach derives $\dfrac{\partial\phi_1}{\partial\hat{p}}$ and $\dfrac{\partial\phi_1}{\partial\tilde{p}}$.

[38] $\partial(\hat{p}-\tilde{p})/\partial t_B \begin{pmatrix} > \\ < \end{pmatrix} 0 \text{ if } (t_A-t_B)(t_A+t_B+d)-d(t_B+d) \begin{pmatrix} > \\ < \end{pmatrix} 0$.

If $\partial\phi_1 / \partial t_B > 0$, then given that athlete B dopes and becomes more talented, athlete A is less likely to deviate from (D,D), all else equal. Given $t_A \geq t_B$, $\phi_1$ is at its maximum level when $t_B = t_A = \bar{t}$. From (1) and (2), this implies that $\hat{p} = \frac{\bar{t}+d}{2\bar{t}+2d} = 1/2$ and $\tilde{p} = \frac{\bar{t}}{2\bar{t}+d}$. Substituting these values into our threshold penalty parameter gives $\phi_1^{max} = \frac{d(1-\tilde{w}) - 2(2\bar{t}+d)\tilde{c}}{r\{d(1-\tilde{w}) + (1+\tilde{w})(1+r)(2\bar{t}+d)\}}$. It is easily shown that the threshold sanction is positively related to doping levels and negatively related to doping costs and detection probability, while the effect of the prize disparity will depend on the level of doping, doping costs and detection probability. If doping costs and detection probability levels are relatively low, then for a given doping level, athlete A is less likely to deviate from (D,D) as the prize disparity increases, so a higher maximum sanction is required to prevent doping.

Conversely, if $\partial\phi_1 / \partial t_B < 0$, then given that B dopes, athlete A is more likely to deviate from (D,D) as B's talent increases. In this case, $\phi_1$ is at its maximum level when $t_B = \underline{t}$. Using (1) and (2), $\hat{p} = \frac{t_A + d}{t_A + \underline{t} + 2d}$ and $\tilde{p} = \frac{t_A}{t_A + \underline{t} + d}$ and substituting these values into our threshold penalty parameter gives $\phi_1^{max} = \frac{d(1-\tilde{w})(\underline{t}+d) - \tilde{c}(t_A + \underline{t} + d)(t_A + \underline{t} + 2d)}{r\{d(1-\tilde{w})(\underline{t}+d) + (t_A + \underline{t} + d)[(1+r\tilde{w})(t_A + d) + (r+\tilde{w})(\underline{t}+d)]\}}$. In contrast to the previous case, if doping levels increase, the threshold sanction may increase or decrease depending on detection probability, doping costs and talent levels.

The above procedure can be undertaken for all threshold penalty parameters for changes in the relevant variables (see Appendix for derivations). Given our assumptions, the direction of the effects of these changes are given in the following table:

**Table 5**

| | $t_A$ | $t_B$ | d |
|---|---|---|---|
| $\phi_1$ | - | +/- | + |
| $\phi_2$ | +/- | - | + |
| $\phi_3$ | - | - | + |

From Table 5, an increase in the doping level has a positive effect on all threshold sanction levels, thereby requiring a higher sanction to eliminate any doping incentive. An increase in an athlete's own talent level makes it more likely that they will deviate from an outcome where both athletes dope, thereby also requiring a lower sanction. An increase in the stronger athlete's talent level raises the likelihood that the weaker athlete deviates from an outcome where both athletes dope, so that a lower sanction is required to induce a no-doping equilibrium. On the other hand, how a change in the weaker athlete's talent level affects its rival's incentive to deviate from an outcome where both athletes dope is ambiguous and will depend on detection levels, doping costs, prize disparity and talent levels themselves. Finally, an increase in the stronger athlete's talent has an ambiguous effect on whether its rival will deviate from a no-doping equilibrium, while an increase in the weaker athlete's talent makes it less likely that it deviates from a no-doping equilibrium, thereby requiring a lower sanction.

For each threshold penalty parameter, we can derive (see Appendix) its maximum value as talent and doping levels change. Comparing across these for all $\tilde{c} \leq (1-\tilde{w})(\bar{p}-\tilde{p})$, it can be shown that all values of $\phi_1^{\max}$ and $\phi_3^{\max}$ are dominated by some $\phi_2^{\max}$. Given the various parameters in each case, it is not possible to determine a definitive relationship between all values of $\phi_2^{\max}$, which, under different assumptions, are as follows:

**Table 6**

| | $\tilde{w} \geq 0, 0 \leq t_A, t_B, d$ | $\tilde{w} = 0, t_A, t_B, d \in [0,1]$ |
|---|---|---|
| $\phi_2^{\max}\Big\vert_{\frac{\partial\phi_2}{\partial t_A}<0}$ | $\dfrac{d\underline{t}(1-\tilde{w})-\tilde{c}(\underline{t}+\underline{t})(\underline{t}+\underline{t}+d)}{r(\underline{t}+\underline{t})[(\underline{t}+d)+\tilde{w}\underline{t}]}$ | undefined |
| $\phi_2^{\max}\Big\vert_{\frac{\partial\phi_2}{\partial t_A}>0}$ | $\dfrac{d\bar{t}(1-\tilde{w})-\tilde{c}(\bar{t}+t_B)(\bar{t}+t_B+d)}{r(\bar{t}+t_B)[(t_B+d)+\tilde{w}\bar{t}]}$ | $\dfrac{d-\tilde{c}(1+t_B)(1+t_B+d)}{r(1+t_B)(t_B+d)}$ |
| $\phi_2^{\max}\Big\vert_{\frac{\partial\phi_2}{\partial t_B}<0}$ | $\dfrac{dt_A(1-\tilde{w})-\tilde{c}(t_A+\underline{t})(t_A+\underline{t}+d)}{r(t_A+\underline{t})[(\underline{t}+d)+\tilde{w}t_A]}$ | $\dfrac{dt_A-\tilde{c}t_A(t_A+d)}{rdt_A}$ |
| $\phi_2^{\max}\Big\vert_{\frac{\partial\phi_2}{\partial d}>0}$ | $\dfrac{\bar{d}t_A(1-\tilde{w})-\tilde{c}(t_A+t_B)(t_A+t_B+\bar{d})}{r(t_A+t_B)[(t_B+\bar{d})+\tilde{w}t_A]}$ | $\dfrac{t_A-\tilde{c}(t_A+t_B)(t_A+t_B+1)}{r(t_A+t_B)(t_B+1)}$ |

In all cases, the maximum sanctions are negatively related to the costs of doping, the probability of detection and the relative prize of the loser.

Applying the assumptions of this paper to the results of Berentsen, where sanctions are not proportionally identical for all athletes and $\tilde{w}=0$, the equivalent sanctions of the Berentsen model are $\phi_\omega^B = \dfrac{1-\tilde{c}}{r}$ and $\phi_\tau^B = 1$, where ω and τ denote the winner and loser, respectively.[39] It is easily shown that each possible value of $\phi_2^{max}$ is lower than $\phi_W^B$ for all parameter levels, so the sanctions required to induce a no-doping equilibrium are lower for the winner than in the Berentsen model. This is due to the 're-awarding' scheme in this paper that acts to reduce the incentive to dope, and also the fact that all penalties are equi-proportionate to winnings for each athlete.

In Figures 5-7, using the same values as Figs 2-4, respectively, the optimal minimum sanction of this paper, which is the outer envelope of the various $\phi_2^{max}$, is compared to the equivalent sanction of Berentsen as parameter values change. When the probability of detection is relatively low ($r < 0.5$), the optimal sanction is a relatively large multiple of prizes to ensure that no athlete ever has an incentive to engage in doping. Finally, in Figure 8 where $\tilde{w} = 0.5$, the optimal sanctions are much lower as a proportion of prizes, relative to when $\tilde{w} = 0$, as the reduced disparity in prizes reduces the incentive to dope.[40]

## 5. Summary and Conclusions

This paper has sought to determine a sanctions scheme that will always induce a no-doping equilibrium in a one-shot game, where two athletes must decide, before competing, whether to dope or not. In contrast to other papers, an explicit contest success function is outlined that relates win probability to talent and doping levels. Also, it is assumed that doping is more effective for the naturally weaker athlete, irrespective of whether one or both athlete dope. The anti-doping authorities implement an imperfect testing system whereby the probability of detection if doped is not certain. Doping has a direct cost, in health and/or monetary terms, but may also impose costs in terms of a fine, possibly greater than winnings, if doping is detected.

Introducing fines that are identically proportional to winnings for all athletes, and taking account of the correlation in the various win probabilities, the optimal sanction that always induces a no-doping equilibrium is decreasing in the probability of detection and, for relatively low detection probabilities, are a multiple of prizes. In comparison to previous

[39] In Berentsen (2002), a detected doped winner's net prize is $-S_1$, while a detected doped loser's net prize is $-S_2$, given that there is no nominal loser's prize. A clean loser receives w if the winner's doping is detected. Berentsen's $S_i$ is equivalent to $(\phi\text{-}1)w_i$ in this paper ($i = 1,2$). Normalising by the winner's prize, then Berentsen's $s_1 \Leftrightarrow \phi_\omega^B - 1$, while $s_2 \Leftrightarrow (\phi_\tau^B - 1)\tilde{w}$. On the other hand, this paper assumes that 'dirty' athlete i's fine ($\phi w_i$) is transferred to 'clean' athlete j ($i,j = 1,2$, $i \neq j$).

[40] These sanctions cannot be directly compared to Berentsen where a loser's prize is always zero.

papers, where sanctions were not equally proportionate to prizes for all athletes, the optimal sanction for the winner is lower for all detection probabilities, which may make athletes more likely to participate. Also, the re-awarding mechanism is, in a sense, fairer to non-doping athletes as, for a given detection probability, the expected cost of being a doper is much higher, which again may increase participation.

Regarding future research paths, one may consider a multi-stage game where future reputation effects are present. Also, increasing the number of athletes and making doping levels endogenous for athletes may lead to richer policy implications.

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## Figures

Figure 1: Nash equilibria ($t_A$ = 0.8, $t_B$ = 0.6, d = 0.5)

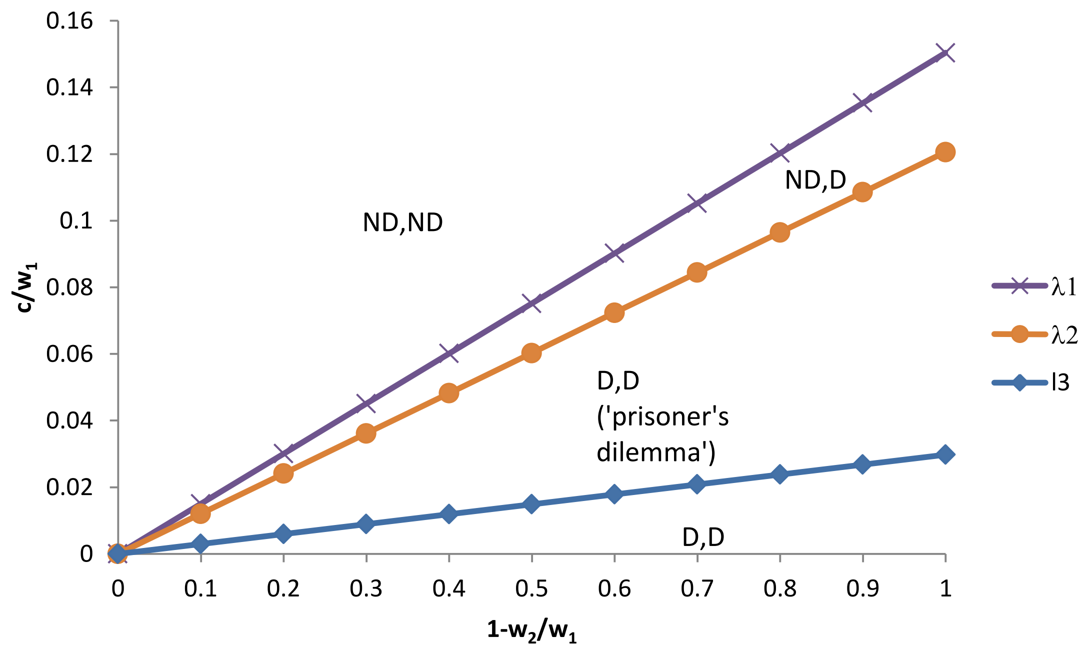


Figure 2: Nash equilibria ($t_A$=0.8, $t_B$=0.6, d=0.5, $c/w_1$ = 0.05, $w_2$=0)

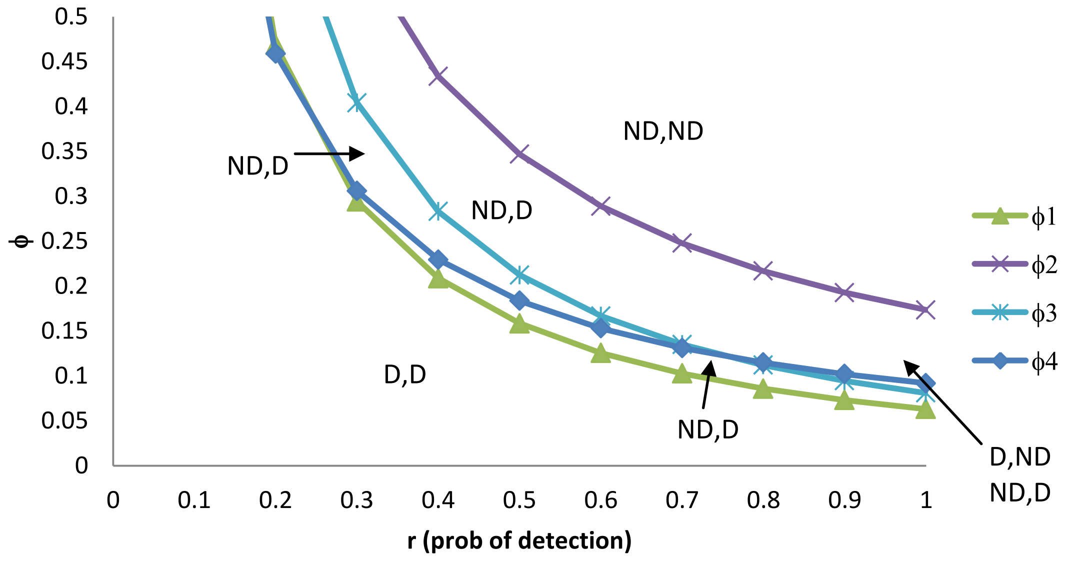
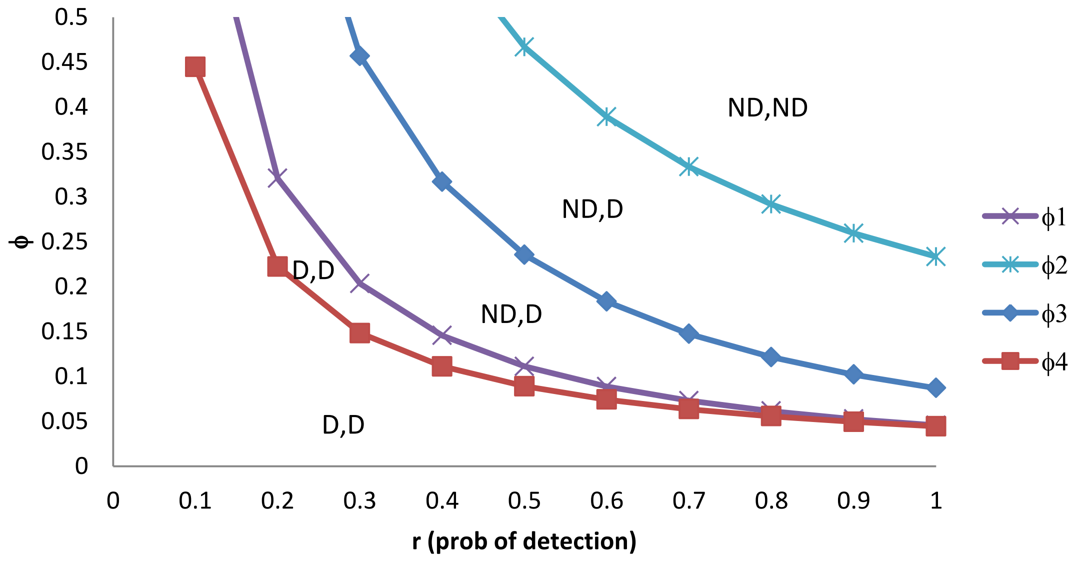



Figure 3: Nash equilibria ($t_A$=1, $t_B$=0.5, d=0.5, $c/w_1$ = 0.05, $w_2$=0)



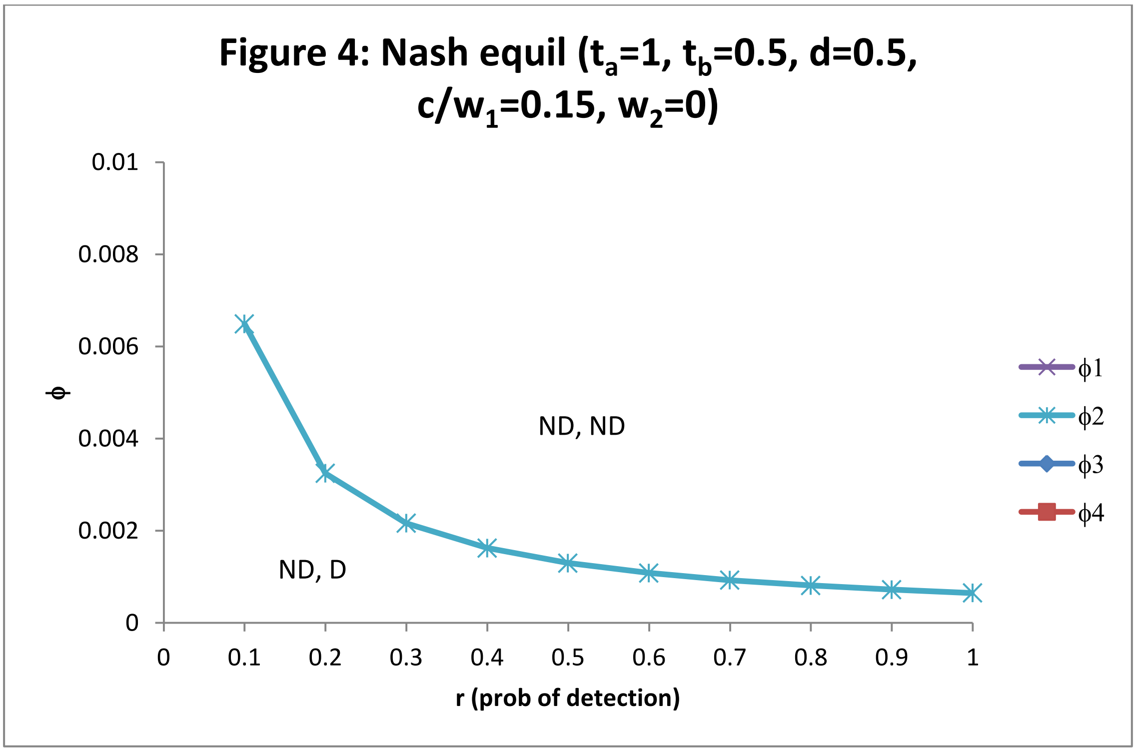


Figure 4: Nash equil ($t_a$=1, $t_b$=0.5, d=0.5, $c/w_1$=0.15, $w_2$=0)
ϕ
0.01
0.008
0.006
0.004
0.002
0
ND, ND
ND, D
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
r (prob of detection)
ϕ1
ϕ2
ϕ3
ϕ4

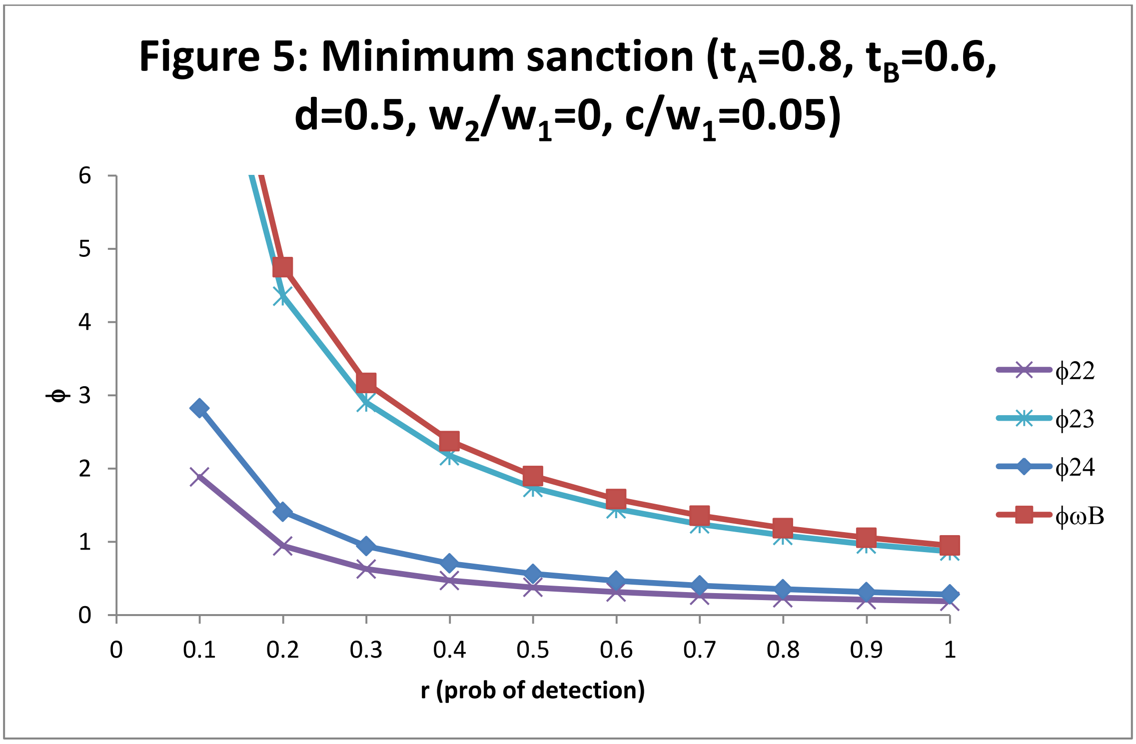


Figure 5: Minimum sanction ($t_A$=0.8, $t_B$=0.6, d=0.5, $w_2/w_1$=0, $c/w_1$=0.05)
ϕ
6
5
4
3
2
1
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
r (prob of detection)
ϕ22
ϕ23
ϕ24
ϕωB

**Figure 6: Minimum sanctions ($t_A$=1, $t_B$=0.5, d=0.5, $w_2/w_1$=0, $c/w_1$=0.05)**

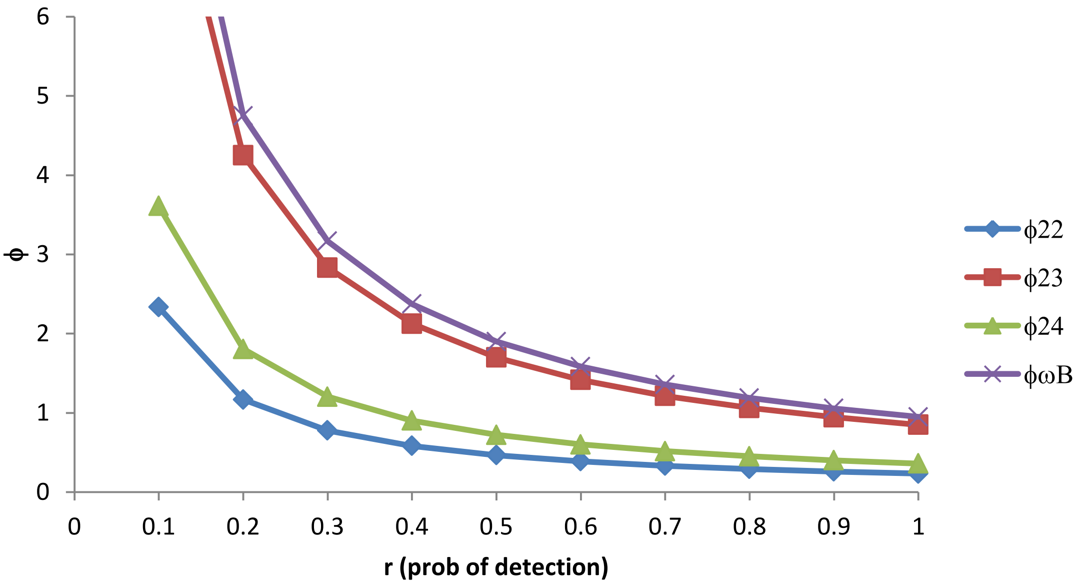
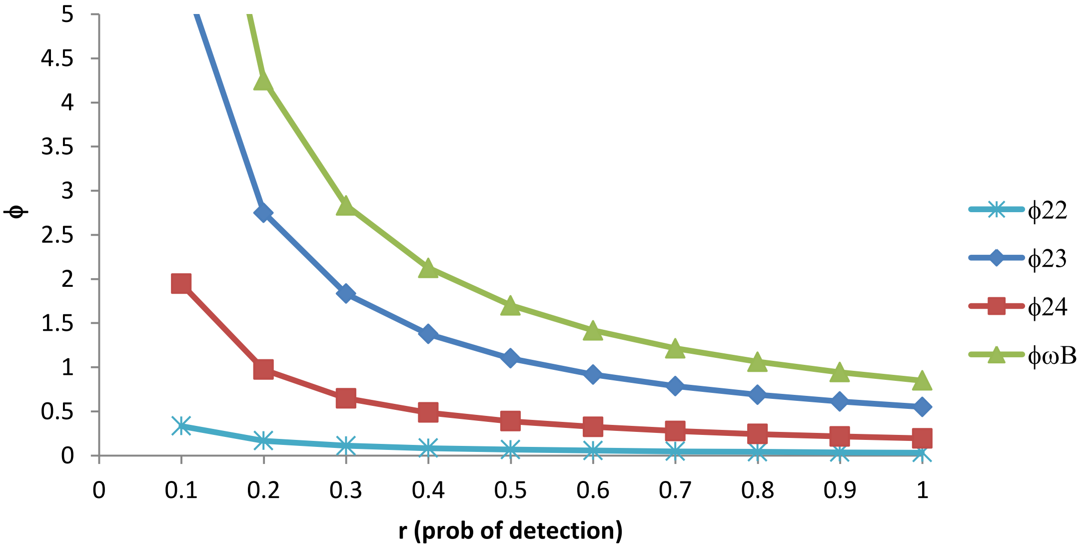



**Figure 7: Minimum sanctions ($t_A$=1, $t_B$=0.5, d=0.5, $w_2/w_1$=0, $c/w_1$=0.15)**



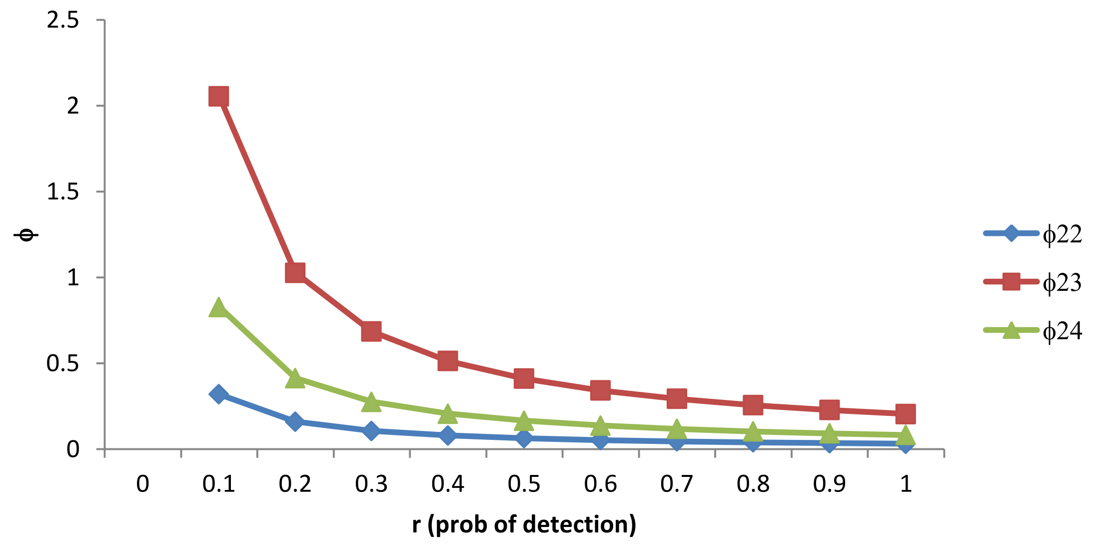



**Figure 8: Minimum sanction ($t_A$=0.8, $t_B$=0.6, d=0.5, $w_2/w_1$=0.5, $c/w_1$=0.05)**

## Appendices

### A.1. Expected payoffs

If athlete A is the sole doper (D, ND), its expected payoff is:

$$p'[(1-r)+r(1-\phi)]w_1+(1-p')[(1-r)+r(1-\phi)]w_2-c \qquad \textbf{(4)}$$

The first term in (4) outlines athlete A's expected payoff if it wins, which occurs with probability $p^{/}$. In this case, the probability of not being 'detected' and keeping prize $w_1$ is (1-r). On the other hand, the probability of detection is r and if detected, the net prize is $(1-\phi)w_1$. The second term in (4) is athlete A's payoff if it loses, which occurs with probability $(1-p^{/})$. The probability of doping not being detected and keeping prize $w_2$ is (1-r). On the other hand, doping is detected will probability r and the net prize is $(1-\phi)w_2$. Finally, the direct doping cost must be incurred irrespective of whether doping is detected or not. Dividing (4) by the winner's prize and simplifying gives the relevant expression in Table 3.

Where both athletes dope (case (D,D)), athlete B's expected payoff is:

$$(1-\hat{p})\{(1-r)[w_1+r\phi w_2]+r(w_1-\phi w_1)\}+\hat{p}\{(1-r)[w_2+r\phi w_1]+r(w_2-\phi w_2)\}-c \qquad \textbf{(5)}$$

The first term in (5) is athlete B's expected payoff from winning, which occurs with probability $1-\hat{p}$. With probability r, B's doping is not detected and he keeps prize $w_1$ but may also be awarded athlete A's 'fine' is the latter's doping is detected, which occurs with probability r. On the other hand, if B's doping is detected, with probability r, the net prize is $(1-\phi)w_1$. The second term outlines B's expected payoff from losing, which occurs with probability $\hat{p}$. Athlete B's doping is not detected with probability 1-r, but his payoff depends on whether athlete A's doping is detected or not. With probability 1-r, athlete A is not detected and B gets $w_2$. On the other hand, athlete A is detected with probability r and the winner's fine is then given to athlete B. Conversely, athlete B's doping is detected with probability r. In this case, its net prize is $(1-\phi)w_2$, irrespective of whether athlete A's doping is detected or not. Again, a direct doping cost must be incurred and is invariant to detection. Dividing (5) by the winner's prize and simplifying gives the relevant payoff $B_{11}$.

Using similar procedures, we can determine each athlete's expected payoff in each possible case to derive the payoff matrix in Table 3.

### A.2 Maximum penalty thresholds

#### A.2.1 Athlete A deviates from (D,D)

Both athletes doping (D,D) is a Nash equilibrium if $\phi < \phi_1$ and $\phi < \phi_3$, and Athlete A will deviate from (D,D) if $\phi > \dfrac{(1-\tilde{w})(\hat{p}-\tilde{p})-\tilde{c}}{r\{r+\tilde{w}+(1-\tilde{w})[(\hat{p}-\tilde{p})+\hat{p}(1-r)]\}} \equiv \phi_1$. Using (1) and (2), a change in athlete A's talent level affects this incentive as

$$\partial\phi_1 \Big/ \partial t_A \begin{pmatrix} > \\ < \end{pmatrix} 0 \text{ if } \psi \equiv \frac{r+\tilde{w}+(1-\tilde{w})\hat{p}(1-r)+\tilde{c}}{r+\tilde{w}+(1-\tilde{w})\tilde{p}(1-r)+(2-r)\tilde{c}} \begin{pmatrix} < \\ > \end{pmatrix} \frac{\partial\hat{p}\big/\partial t_A}{\partial\tilde{p}\big/\partial t_A} < 1 \qquad \textbf{(6)}$$

It is easy to show that $\psi > 1$ if $\tilde{c} < (1-\tilde{w})(\hat{p}-\tilde{p})$, i.e. if (D,D) is a unique Nash equilibrium if no anti-doping policy is in place. Given this, it must be the case that $\partial\phi_1 \big/ \partial t_A < 0$ so that, all else equal, athlete A is more likely to deviate from (D,D) as its talent increases. On the other hand, if $(1-\tilde{w})(\hat{p}-\tilde{p}) < \tilde{c} < (1-\tilde{w})(\bar{p}-\tilde{p})$, then A's incentive is not immediately clear. We can show, however, that $\partial\psi \big/ \partial\tilde{c} < 0$. As an anti-doping policy will only be implemented if

$\tilde{c} \le (1-\tilde{w})(\bar{p}-\tilde{p})$, then substituting this maximum value into (6) above, it can be shown that $\partial\phi_1 / \partial t_A < 0$ for all values of $\tilde{c}$ that induce an anti-doping policy.

As athlete A becomes more talented, a lower sanction is required to ensure a no-doping equilibrium. Intuitively, A's win probability will decrease if only B dopes. However, as A's talent increases, the difference between its win probability when both dope compared to where only B dopes is decreasing.[41] Given this, A is less likely to dope, thereby avoiding any possibility of being detected and paying a fine, while also opening up the possibility of receiving B's fine if the latter's doping is detected.

Given $\partial\phi_1 / \partial t_A < 0$, $\phi_1$ is at its maximum level when $t_A$ is 'low', which occurs when $t_A = t_B = \underline{t}$. From (1) and (2), this implies that $\hat{p} = \frac{\underline{t}+d}{2\underline{t}+2d} = \frac{1}{2}$ and $\tilde{p} = \frac{\underline{t}}{2\underline{t}+d}$. Substituting these values into our threshold penalty parameter gives $\phi_1^{max} = \frac{d(1-\tilde{w})-2(2\underline{t}+d)\tilde{c}}{r\{d(1-\tilde{w})+(1+\tilde{w})(1+r)(2\underline{t}+d)\}}$, sufficient to deter doping for all parameter values.[42]

To determine the effect of an increase in the doping level, it is easy to show that

$$\partial\phi_1 / \partial d \begin{pmatrix} > \\ < \end{pmatrix} 0 \text{ if } \psi \equiv \frac{r+\tilde{w}+(1-\tilde{w})\hat{p}(1-r)+\tilde{c}}{r+\tilde{w}+(1-\tilde{w})\tilde{p}(1-r)+(2-r)\tilde{c}} \begin{pmatrix} > \\ < \end{pmatrix} \frac{-\partial\hat{p}/\partial d}{-\partial\tilde{p}/\partial d} < 1 \qquad \textbf{(7)}$$

It can be shown that $\psi > \frac{-\partial\hat{p}/\partial d}{-\partial\tilde{p}/\partial d}$ for all relevant $\tilde{c}$, so that $\partial\phi_1 / \partial d > 0$ and athlete A is less likely to deviate from (D,D) as doping levels increase. Higher doping levels give a greater incentive to dope so that, all else equal, a higher sanction is required to induce athlete A to deviate from (D,D). Given this, $\phi_1$ is at its maximum value when doping levels are at their highest, i.e. $d = \bar{d}$. In this case, $\hat{p} = \frac{t_A+\bar{d}}{t_A+t_B+2\bar{d}} = \frac{1}{2}$ and $\tilde{p} = \frac{t_A}{t_A+t_B+\bar{d}}$ so that

$$\phi_1^{max} = \frac{\bar{d}(t_B+\bar{d})(1-\tilde{w})-(t_A+t_B+\bar{d})(t_A+t_B+2\bar{d})\tilde{c}}{r\{\bar{d}(t_B+\bar{d})(1-\tilde{w})+(t_A+t_B+\bar{d})[(1+r\tilde{w})(t_A+\bar{d})+(r+\tilde{w})(t_B+\bar{d})]\}}.$$

This case is probably most applicable to elite sports where athletes may have reached the limit of their natural talent and any increased win probability is most easily achieved through doping. Given that effort or natural talent levels may not be easily observed, athletes

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41 $\hat{p} - \tilde{p}$ is the difference in athlete A's win probability when both dope compared to when B is a sole doper.

42 If $\tilde{c} > (1-\tilde{w})(\hat{p}-\tilde{p})$, athlete A will deviate from (D,D) in the absence of an anti-doping policy so that, given $t_A = t_B = \underline{t}$, $\phi_1^{max} < 0$. This idea is consistent across all cases.

have an incentive to attribute any increase in performance or win probability to a new training regime, a more disciplined lifestyle or better tactics rather than admit to doping.

### A.2.2 Athlete B deviates from (D,D)

Athlete B will deviate from (D,D) if $\phi > \dfrac{(1-\tilde{w})(p'-\hat{p})-\tilde{c}}{r\{1+r\tilde{w}+(1-\tilde{w})[(p'-\hat{p})-\hat{p}(1-r)]\}} \equiv \phi_3$. Using (1) and (2) to determine how a change in athlete A's talent level affects this incentive,

$$\partial\phi_3 / \partial t_A \binom{>}{<} 0 \text{ if } \lambda \equiv \frac{1+r\tilde{w}-(1-\tilde{w})\hat{p}(1-r)+\tilde{c}}{1+r\tilde{w}-(1-\tilde{w})p'(1-r)+(2-r)\tilde{c}} \binom{>}{<} \frac{\partial\hat{p}/\partial t_A}{\partial p'/\partial t_A} \qquad \textbf{(8)}$$

From (8), it can be shown that $\partial\lambda/\partial\tilde{c} < 0$. In the absence of an anti-doping policy, athlete B deviates from (D,D) if $\tilde{c} \leq (1-\tilde{w})(p'-\hat{p})$. Substituting this threshold value of $\tilde{c}$ into (8) above, and given $\partial\lambda/\partial\tilde{c} < 0$, it can be shown that $\partial\phi_3/\partial t_A < 0$ for all relevant values of $\tilde{c}$. Given this, athlete B is more likely to deviate from (D,D) as athlete A's talent increases. By deviating, B's win probability decreases but this decrease is diminishing as A becomes more talented. Also, athlete B cannot be penalised for doping and there is also a possibility that even if B loses, it will be awarded the winner's fine if athlete A's doping is detected.

In this case, $\phi_3$ is at its maximum level when $t_A$ is 'low' so that $t_A = t_B = \underline{t}$. From (1) and (2), this implies that $p' = \dfrac{\underline{t}+d}{2\underline{t}+d}$ and $\hat{p} = \dfrac{\underline{t}+d}{2\underline{t}+2d} = \dfrac{1}{2}$. Substituting these values into our threshold penalty parameter gives $\phi_3^{max} = \dfrac{d(1-\tilde{w})-2(2\underline{t}+d)\tilde{c}}{r\{d(1-\tilde{w})+(1+\tilde{w})(1+r)(2\underline{t}+d)\}}$.

Looking at the effect of changes in athlete B's talent level,

$$\partial\phi_3 / \partial t_B \binom{>}{<} 0 \text{ if } \lambda \equiv \frac{1+r\tilde{w}-(1-\tilde{w})\hat{p}(1-r)+\tilde{c}}{1+r\tilde{w}-(1-\tilde{w})p'(1-r)+(2-r)\tilde{c}} \binom{<}{>} \frac{-\partial\hat{p}/\partial t_B}{-\partial p'/\partial t_B} < 1 \qquad \textbf{(9)}$$

Without an anti-doping policy, athlete B deviates from (D,D) if $\tilde{c} > (1-\tilde{w})(p'-\hat{p})$. From (9), $\lambda \geq 1$ when $\tilde{c} \leq (1-\tilde{w})(p'-\hat{p})$. As $\dfrac{\partial\lambda}{\partial\tilde{c}} < 0$, then for all relevant values of $\tilde{c}$, $\lambda$ it must be the case that $\partial\phi_3/\partial t_B < 0$, so that, all else equal, athlete B is more likely to deviate from (D,D) as its talent increases. By not doping when A does, athlete B's win probability is lower but is partially offset by the increase in natural talent. By not doping, B cannot be detected and may possibly attain A's fine if the latter's doping is detected. Given this, as B's talent increases, a lower sanction is required to deter athlete B from doping.

As $\partial\phi_3 / \partial t_B < 0$, $\phi_3$ is at its maximum level when $t_B$ is 'low' so that $t_B = \underline{t}$. From (1) and (2), this implies that $p' = \dfrac{t_A + d}{t_A + \underline{t} + d}$ and $\hat{p} = \dfrac{t_A + d}{t_A + \underline{t} + 2d}$. Substituting these values into our threshold penalty parameter gives $\phi_3^{\max} = \dfrac{d(t_A + d)(1-\tilde{w}) - (t_A + \underline{t} + d)(t_A + \underline{t} + 2d)\tilde{c}}{r\{d(t_A + d)(1-\tilde{w}) + (t_A + \underline{t} + d)[(r+\tilde{w})(t_A + d) + (1 + r\tilde{w})(\underline{t} + d)]\}}$.

To determine the effect of an increase in the doping level, it is easy to show that

$$\partial\phi_3 / \partial d \begin{pmatrix} > \\ < \end{pmatrix} 0 \text{ if } \lambda \equiv \frac{1 + r\tilde{w} - (1-\tilde{w})\hat{p}(1-r) + \tilde{c}}{1 + r\tilde{w} - (1-\tilde{w})p'(1-r) + (2-r)\tilde{c}} \begin{pmatrix} < \\ > \end{pmatrix} \frac{\partial\hat{p} / \partial d}{\partial p' / \partial d} < 0 \qquad \textbf{(10)}$$

As $\lambda > 0$, then $\partial\phi_3 / \partial d > 0$ and athlete B is less likely to deviate from (D,D) as the doping level increases. Given this, then $\phi_3$ is at its maximum value when d is 'high' so that $d = \bar{d}$. Substituting this into the relevant win probabilities, then $p' = \dfrac{t_A + \bar{d}}{t_A + t_B + \bar{d}}$ and $\hat{p} = \dfrac{t_A + \bar{d}}{t_A + t_B + 2\bar{d}}$ and $\phi_3^{\max} = \dfrac{\bar{d}(t_A + \bar{d})(1-\tilde{w}) - \tilde{c}(t_A + t_B + \bar{d})(t_A + t_B + 2\bar{d})}{r\{\bar{d}(t_A + \bar{d})(1-\tilde{w}) + (t_A + t_B + \bar{d})[(1 + r\tilde{w})(t_B + \bar{d}) + (r + \tilde{w})(t_A + \bar{d})]\}}$.

### A.2.3 Athlete B does not deviate from (ND,ND)

Athlete B will not deviate from (ND,ND) if $\phi \geq \dfrac{(1-\tilde{w})(\bar{p} - \tilde{p}) - \tilde{c}}{r[1 - \tilde{p}(1-\tilde{w})]} \equiv \phi_2$. Using (1) and (2) to determine how a change in athlete A's talent level affects this incentive,

$$\partial\phi_2 / \partial t_A \begin{pmatrix} > \\ < \end{pmatrix} 0 \text{ if } \mu \equiv \frac{1 - \bar{p}(1-\tilde{w}) + \tilde{c}}{1 - \tilde{p}(1-\tilde{w})} \begin{pmatrix} < \\ > \end{pmatrix} \frac{\partial\bar{p} / \partial t_A}{\partial\tilde{p} / \partial t_A} \qquad \textbf{(11)}$$

From (11), it is not possible to determine a definitive relationship between the relevant variables. It is easily seen from (11) that $\partial\mu / \partial\tilde{c} > 0$. In the absence of an anti-doping policy, athlete B will not deviate from (ND,ND) if $\tilde{c} > (1-\tilde{w})(\bar{p} - \tilde{p})$. Putting this threshold into (11), it can be shown that $\mu = 1$, so that $\mu \leq 1$ for all relevant $\tilde{c}$. On the other hand, the term on the right hand side of (11) is less (greater) than unity if doping levels are sufficiently low (high).

If $\partial\phi_2 / \partial t_A > 0$, then athlete B is more likely to deviate from (ND,ND) as its rival becomes more talented. In this case, $\phi_2$ is at its maximum level when $t_A$ is 'high' so that $t_A =$

$\bar{t}$. From (1) and (2), this implies that $\bar{p} = \frac{\bar{t}}{\bar{t} + t_B}$ and $\tilde{p} = \frac{\bar{t}}{\bar{t} + t_B + d}$. Substituting these values into our threshold penalty parameter gives $\phi_2^{max} = \frac{d\bar{t}(1-\tilde{w}) - \tilde{c}(\bar{t} + t_B)(\bar{t} + t_B + d)}{r(\bar{t} + t_B)\left[d + (t_B + \tilde{w}\bar{t})\right]}$.

Conversely, if $\partial\phi_2 / \partial t_A < 0$, which requires sufficiently low doping levels, then given that athlete A does not dope, athlete B is less likely to deviate from (ND,ND) as its rival's talent increases due to the reduced effect on win probability from being the sole doper. In this case, $\phi_2$ is at its maximum level when $t_A$ is 'low' so that $t_A = t_B = \underline{t}$. From (1) and (2), this implies that $\bar{p} = \frac{\underline{t}}{2\underline{t}} = \frac{1}{2}$ and $\tilde{p} = \frac{\underline{t}}{2\underline{t} + d}$. Substituting these values into our threshold penalty parameter gives $\phi_2^{max} = \frac{d\underline{t}(1-\tilde{w}) - \tilde{c}(\underline{t} + \underline{t})(\underline{t} + \underline{t} + d)}{r(\underline{t} + \underline{t})[(\underline{t} + d) + \tilde{w}\underline{t}]}$ .

Looking at the effect of changes in athlete B's talent level,

$$\partial\phi_2 / \partial t_B \begin{pmatrix} > \\ < \end{pmatrix} 0 \text{ if } \mu \equiv \frac{1 - \bar{p}(1-\tilde{w}) + \tilde{c}}{1 - \tilde{p}(1-\tilde{w})} \begin{pmatrix} > \\ < \end{pmatrix} \frac{-\partial\bar{p} / \partial t_B}{-\partial\tilde{p} / \partial t_B} > 1 \qquad \textbf{(12)}$$

Given $\mu \le 1$, then it must be the case that $\partial\phi_2 / \partial t_B < 0$, so given that athlete A does not dope, athlete B is more likely to not deviate from (ND,ND) as its talent increases, all else equal. In this case, $\phi_2$ is at its maximum level when $t_B$ is 'low' so that $t_B = \underline{t}$. From (1) and (2), this implies that $\bar{p} = \frac{t_A}{t_A + \underline{t}}$ and $\tilde{p} = \frac{t_A}{t_A + \underline{t} + d}$. Substituting these values into our threshold penalty parameter gives $\phi_2^{max} = \frac{dt_A(1-\tilde{w}) - \tilde{c}(t_A + \underline{t})(t_A + \underline{t} + d)}{r(t_A + \underline{t})\left[d + (\underline{t} + \tilde{w}t_A)\right]}$.

To determine the effect of an increase in the doping level, it is easy to show that

$$\partial\phi_2 / \partial d \begin{pmatrix} > \\ < \end{pmatrix} 0 \text{ if } -\left[1 - \bar{p}(1-\tilde{w}) + \tilde{c}\right]\frac{\partial\tilde{p}}{\partial d} \begin{pmatrix} > \\ < \end{pmatrix} 0 \qquad \textbf{(13)}$$

Given $\partial\tilde{p} / \partial d < 0$, then it must be the case that $\partial\phi_2 / \partial d > 0$ so that athlete B is more likely to deviate from (ND,ND) as the doping level increases, as its win probability increases by more if it is the sole doper. In this case, $\phi_2$ is at its maximum value when d is 'high' so that $d = \bar{d}$. Using (1) and (2), this gives $\bar{p} = \frac{t_A}{t_A + t_B}$ and $\tilde{p} = \frac{t_A}{t_A + t_B + \bar{d}}$ so that $\phi_2^{max} = \frac{\bar{d}t_A(1-\tilde{w}) - \tilde{c}(t_A + t_B)(t_A + t_B + \bar{d})}{r(t_A + t_B)\left[\bar{d} + (\tilde{w}t_A + t_B)\right]}$.