APPLYING COMPUTER VISION TECHNIQUES TO TOPOGRAPHIC OBJECTS

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ABSTRACT

Automatic structuring (feature coding and object recognition) of topographic data, such as that derived from air survey or raster scanning large-scale paper maps, requires the classification of objects such as buildings, roads, rivers, fields and railways. The recognition of objects is largely based on the matching of descriptions of shapes. Fourier descriptors, moment invariants, boundary chain coding and scalar descriptors are widely used in image processing and computer vision to describe and classify shapes. They have been developed to describe shape irrespective of position, orientation and scale. The applicability of the above four methods to topographic shapes is described and their usefulness evaluated.

1 INTRODUCTION

Automatic structuring (feature coding and object recognition) of topographic data, such as that derived from air survey or raster scanning large-scale paper maps, requires the classification of objects such as buildings, roads, rivers, fields and railways. Shape and context are the main attributes used by humans. Our project combines shape recognition techniques developed for computer vision and contextual models derived from statistical language theory to recognise objects. This paper describes the measurement of shape to characterise features that will then be used as input into a graphical language model.

Much work has been done in computer vision on the identification and classification of objects within images. However, less progress has been made on automating feature extraction and semantic capture in vector graphics. This is partly because the low-level graphical content of maps has often been captured manually (on digitising tables etc.) and the encoding of the semantic content has been seen as an extension of this. However, the successful automation of raster-vector conversion plus the large quantity of new and archived graphical data available on paper makes the automation of feature extraction desirable.

Feature extraction and object recognition are large research areas in the field of image processing and computer vision. Recognition is largely based on the matching of descriptions of shapes. Numerous shape description techniques have been developed in computer vision, such as, boundary chain coding, analysis of scalar features (dimension, area, number of corners etc), Fourier descriptors and moment invariants. These techniques are well understood when applied to images and have been developed to describe shapes irrespective of position, orientation and scale. They can also be easily applied to vector graphical shapes.

A description of the above four methods for shape recognition and their application for classifying objects on large-scale maps is described here. Unlike many applications where the shape categories are very specific (for example identifying a particular aircraft type in a scene), the problem requires the classification of a particular shape into a general class of similar object shapes, for example, building, road or stream. A comparison is made of the effectiveness of these techniques in recognising features on large-scale topographic maps and plans.

2 SHAPE DESCRIPTION TECHNIQUES

The recognition and description of objects plays a central role in automatic shape analysis for computer vision and it is one of the most familiar and fundamental problems in pattern recognition. Common examples are the reading of alphabetic characters in text and the automatic identification of aircraft. Most applications using Fourier descriptors, moment invariants, scalar descriptors and boundary chain coding for shape recognition deal with the classification of such definite shapes. To identify topographic objects each of the techniques need to be extended to deal with general categories of shapes, for example houses, parcels and roads.

Figure 1. Section of a digital map plan

The data used for the experiments described in the following sections was extracted from vector data sets (NTF level 2) representing large-scale (1:1250) plans of the Isle of Man (Kelly and Hilder 1998), an example of which can be seen in figure1. A pre-processing operation was required to transform the vector data from its original form to a new form suitable for further processing. In this case the data was pre-processed to extract closed polygons from lines with the same feature codes. After extracting the required polygonal data from the maps, an interpolation method was applied to sample the shape boundary at a finite number (N) of equi-distant points. These points are then stored in the appropriate format for processing with each shape description technique.

2.1 Fourier Descriptors

Fourier transform theory (Gonzalez and Wintz 1977) has played a major role in image processing for many years. It is a commonly used tool in all types of signal processing and is defined both for one and two-dimensional functions. In the scope of this paper, the Fourier transform technique is used for shape description in the form of Fourier descriptors. The Fourier descriptor is a widely used all-purpose shape description and recognition technique (Granlund 1972, Winstanley 1998). The shape descriptors generated from the Fourier coefficients numerically describe shapes and are normalised to make them independent of translation, scale and rotation. These Fourier descriptor values produced by the Fourier transformation of a given image represent the shape of the object in the frequency domain (Wallace and Wintz 1980). The lower frequency descriptors store the general information of the shape and the higher frequency the smaller details. Therefore, the lower frequency components of the Fourier descriptors define a rough shape of the original object

The Fourier transform theory can be applied in different ways for shape description. One method works on the change in orientation angle as the shape outline is traversed (Zahn and Roskies 1972), but for the purpose of this paper the following procedure was implemented (Wood 1986). The boundary of the image is treated as lying in the complex plane. So the row and column co-ordinates of each point on the boundary can be expressed as a complex number, x + jy where j is sqrt (-1). Tracing once around the boundary in the counter-clockwise direction at a constant speed yields a sequence of complex numbers, that is, a one-dimensional function over time. In order to represent traversal at a constant

speed it is necessary to interpolate equi-distant points around the boundary. Traversing the boundary more than once results in a periodic function. The Fourier transform of a continuous function of a variable *x* is given by the equation:

$$F(u) = \int_{-\infty}^{\infty} f(u) e^{-j2\pi ux} dx$$

(1)

When dealing with discrete images the Discrete Fourier Transform (DFT) is used. So equation (1) transforms to:

$$F(u) = \left(\frac{1}{N}\right) \sum_{x=0}^{N-1} f(u) e^{\frac{-j2\pi x}{N}}$$

(2)

The variable x is complex, so by using the expansion *e[-j A] = cos (A) – j. sin (A)* where *N* is the number of equally spaced samples, equation (2) becomes:

$$F(u) = \left(\frac{1}{N}\right) \sum_{x=0}^{N-1} f(x+jy)).(\cos(Ax) - j.\sin(Ax))$$

(3)

where *A = 2πu/x*.

The DFT of the sequence of complex numbers, obtained by the traversal of the object contour, gives the Fourier descriptor values of that shape.

The Fourier descriptor values can be normalised to make them independent of translation, scale and rotation of the original shape. Simply, translation of the shape by a complex quantity having *x* and *y* components, corresponds to adding a constant *x + jy* to each point representing the boundary. Scaling a shape is achieved by multiplying all co-ordinate values by a constant factor. The DFT results in all members of the corresponding Fourier series being multiplied by the same factor. So by dividing each coefficient by the same member, normalisation for size is achieved. Rotation normalisation is achieved by finding the two coefficients with largest magnitude and setting their phase angle equal to zero (Keyes and Winstanley 1999).

To apply the Fourier descriptor technique to the data set extracted from the Isle of Man map, the points are stored as a series of complex numbers and then processed using the Fourier transform resulting in another complex series also of length N. If the formula for the discrete Fourier transform were directly applied each term would require N iterations to sum. As there are N terms to be calculated, the computation time would be proportional to N^2. So the algorithm chosen to compute the Fourier descriptors was the Fast Fourier Transform (FFT) for which the computation time is proportional to NlogN. The FFT algorithm requires the number of points N defining the shape to be a power of two. In the case of this project it was decided to use 512 sample points.

The FFT algorithm is applied to these 512 coefficients. The list is normalised for translation, rotation and scale. This results in the first two terms always having the values 0 and 1.0 respectively which makes them redundant for classification. Calculation of the Fourier Spectrum builds a new list and disposes of the Fourier transform list. The result is 510 Fourier descriptor terms.

Given two sets of Fourier descriptors, how do we measure their degree of similarity? An appropriate classification is necessary if unknown shapes are to be compared to a library of known shapes. If two shapes, A and B, produce a set of

values represented by a(i) and b(i) then the distance between them can be given as c(i) = a(i) – b(i). If a(i) and b(i) are identical then c(i) will be zero. If they are different then the magnitudes of the coefficients in c(i) will give a reasonable measure of the difference. It proves more convenient to have one value to represent this rather than the set of values that make up c(i). The easiest way is to treat c(i) as a vector in a multi-dimensional space, in which case its length, which represents the distance between the planes, is given by the square root of the sum of the squares of the elements of c(i).

2.2 Moment Invariants

Moment invariants have been frequently used as features for image processing, remote sensing, shape recognition and classification. Moments can provide characteristics of an object that uniquely represent its shape. Invariant shape recognition is performed by classification in the multidimensional moment invariant feature space. Several techniques have been developed that derive invariant features from moments for object recognition and representation. These techniques are distinguished by their moment definition, such as the type of data exploited and the method for deriving invariant values from the image moments. It was Hu (Hu, 1962), that first set out the mathematical foundation for two-dimensional moment invariants and demonstrated their applications to shape recognition. They were first applied to aircraft shapes and were shown to be quick and reliable (Dudani, Breeding and McGhee, 1977). These moment invariant values are invariant with respect to translation, scale and rotation of the shape.

Hu defines seven of these shape descriptor values computed from central moments through order three that are independent to object translation, scale and orientation. Translation invariance is achieved by computing moments that are normalised with respect to the centre of gravity so that the centre of mass of the distribution is at the origin (central moments). Size invariant moments are derived from algebraic invariants but these can be shown to be the result of a simple size normalisation. From the second and third order values of the normalised central moments a set of seven invariant moments can be computed which are independent of rotation.

Traditionally, moment invariants are computed based on the information provided by both the shape boundary and its interior region (Hu 1962). The moments used to construct the moment invariants are defined in the continuous but for practical implementation they are computed in the discrete form. Given a function *f(x,y)*, these regular moments are defined by:

$$M_{pq} = \int\int x^p y^q f(x, y) dx dy$$

(4)

M_{pq} is the two-dimensional moment of the function *f(x,y)*. The order of the moment is (p + q) where p and q are both natural numbers. For implementation in digital from this becomes:

$$M_{pq} = \sum_X \sum_Y x^p y^q f(x, y)$$

(5)

To normalise for translation in the image plane, the image centroids are used to define the central moments. The co-ordinates of the centre of gravity of the image are calculated using equation (5) and are given by:

$$\bar{x} = \frac{M_{10}}{M_{00}} \qquad \bar{y} = \frac{M_{01}}{M_{00}}$$

(6)

The central moments can then be defined in their discrete representation as:

$$\mu_{pq} = \sum_X \sum_Y (x - \bar{x})^p (y - \bar{y})^q$$

(7)

The moments are further normalised for the effects of change of scale using the following formula:

$$\eta_{pq} = {\mu_{pq}} \Big/ {\mu_{00}}^{\gamma}$$

(8)

Where the normalisation factor: $\gamma = (p + q / 2) +1$. From the normalised central moments a set of seven values can be calculated and are defined by:

$\phi_1 = \eta_{20} + \eta_{02}$

$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$

$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (\eta_{03} - 3\eta_{21})^2$

$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{03} + \eta_{21})^2$

$\phi_5 = (3\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2$

$\qquad -3(\eta_{21} + \eta_{03})^2] + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})$

$\qquad \times [3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2]$

$\phi_6 = (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2]$

$\qquad + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03})$

$\phi_7 = (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2$

$\qquad - 3(\eta_{21} + \eta_{03})^2] + (3\eta_{12} - \eta_{30})(\eta_{21} + \eta_{03})$

$\qquad \times [3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{30})^2]$ (9)

These seven invariant moments, ϕ_I, $1 \le I \le 7$, set out by Hu, were additionally shown to be independent or rotation. However they are computed over the shape boundary and its interior region.

2.2.1 New moments

For the purpose of this paper the moment invariants are computed using the shape boundary only and are proven to be invariant under object translation, scale and rotation (Chaur-Chin Chen 1993). Then, using the same notation for convenience, the moment definition in equation (1) can be expressed as:

$$M_{pq} = \int_C x^p y^q ds$$

(10)

For p, q = 0,1,2,3, where $\int_C$ is the line integral along the curve C and $ds = \sqrt{(dx)^2 + (dy)^2}$. The central moments can be similarly defined as:

$$\mu_{pq} = \int_C (x - \bar{x})^p (y - \bar{y})^q ds$$

$$(11)$$

Given that the centroids are as in the regular method:

$$\bar{x} = \frac{M_{10}}{M_{00}} \qquad \bar{y} = \frac{M_{01}}{M_{00}}$$

$$(12)$$

For a digital image, then equation (11) becomes

$$\mu_{pq} = \sum_{(X,Y)\in C}(x - \bar{x})^p(y - \bar{y})^q$$

$$(13)$$

Thus the central moments are invariant to translation. These new central moments can also be normalised such that they are scaling invariant.

$$\eta_{pq} = {\mu_{pq}}\big/{\mu_{00}{}^{\gamma}}$$

$$(14)$$

Where the normalisation factor is: $\gamma = p + q + 1$. The seven moment invariant values can then be calculated as before using the results obtained from the computation of equation's (10) to (14) above.

Using the same data sets as in the Fourier descriptor method described earlier, the moments technique is applied. However, for moments the points extracted from the map are stored not as complex numbers but represent the x and y co-ordinates of the polygonal shape. These points are processed by a moment transformation on the outline of the shape, which produces seven moment invariant values that are normalised with respect to translation, scale and rotation using the formulae above. The resulting set of values can be used to discriminate between the shapes. Classification is carried out using the same method described in section 2.1 for Fourier descriptors.

2.3 Boundary chain coding and scalar descriptors

Boundary chain-coding and scalar descriptors are also used as shape description techniques for this experiment. Chain-code treats the points of a curve and in particular a region boundary, directly and in a strictly local fashion. The basis of the technique is, essentially, to start with a point that is believed to be on the boundary (some local edge point) and to extend the boundary by adding a neighbouring point in the contour direction (that is, the direction which is normal to the gradient direction). This process is reiterated, starting at this new boundary pixel. This boundary chain code technique encodes piecewise linear curves as a sequence of directed straight-line segments called links. There are eight possible directions for a link between a point and its neighbour. These eight directions are numbered '0' through '7' and move counter-clockwise, as shown in figure 3a. Each of these can be considered as an angular direction, in multiples of 45 degrees, which are moved to go from one pixel to the next. The absolute co-ordinates (*x, y*) of the first boundary pixel together with the chain code, represent a complete description of the discrete region boundary.

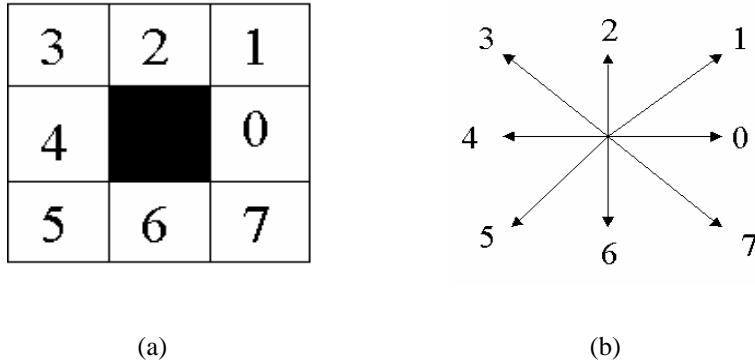(a)   (b)

Figure 3 (a) shows the eight neighbours numbered '0' through '7', counter clockwise. (b), indicates the direction of each link (the direction that can be travelled between a point and its neighbour).
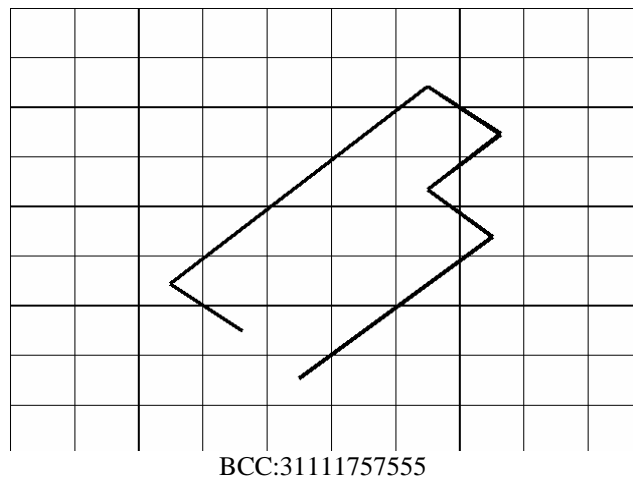

BCC:31111757555

Figure 4 A Boundary Chain Code (BCC) representation of a simple shape.

As tracing around the boundary continues it builds a boundary chain code representation of the contour by recording all the boundary pixels visited. Figure 4 shows a boundary representation of a simple shape, which is based upon the work of Freeman (Freeman 1961). It is assumed that the boundary pixels correspond to the pixels exhibiting the local maximum gradient magnitude and that the gradient direction indicates, approximately at least, the neighbouring boundary pixel. More formally, the boundary direction D of an edge point is given by the edge gradient G as $D = G + 2$ for the forward direction and as $D = G - 2$ for the reverse direction (+90° and -90° respectively). By tracing the boundary, candidates for inclusion in the boundary are given by directions, D, $D + 1$, and $D - 1$. These candidates are, the pixel directly ahead of the current pixel and one pixel either side of it, for example if $D = 1$ (boundary direction is 45°) then pixels 0, 1 and 2 are chosen as candidates. The potential of each of these candidates to be a boundary point is evaluated. If the pixel has been visited previously or if the pixel overlaps the image boundary, then it is assumed to have a negative potential. The candidate with the highest positive potential is selected as the next boundary point from which to continue the trace. This point is implicitly included in the list of boundary points by updating the BCC.

This BCC technique outlined above is dealing specifically with digitised, edge detected pixel images. However, for the purpose of this paper this form of shape description needs to be adapted to deal with the vector map data described in the beginning of section 2, which will be seen in later in the paper. Given the BCC representation of a corpus of shapes in what way can they be classified. While the BCC is a useful method for the representation of shapes, recognition is normally based upon other descriptors derived from the BCC. For example, the moment shape descriptors discussed previously can be generated from the boundary points given by the BCC, which is utilised for this project.

Scalar descriptors are based on scalar features derived from the boundary or an object. They use numerous aspects of the object for performing shape recognition. Simple examples of such features include:

- the perimeter length;

- the area of the shape;

- the ratio of the area of a shape to the square of the length of its perimeter (A/P^2);

- the number of nodes;

- the number of corners;

Recognition and classification of the resulting scalar values, to describe a shape, can be evaluated through the distance between the vectors in the n-dimensional space in the same way as the classification method described in section 2 for the Fourier descriptor and moment invariants techniques.

RESULTS

In this section a sample of the results produced by the application of the Fourier descriptor, moment invariants and scalar descriptor techniques are presented to evaluate and compare their usefulness in shape discrimination of general topographic features. Evaluation of the BCC method is not presented in this work as this is still an on-going experiment. Figure 5 plots the average values, obtained for five categories of objects from the sample maps (using the Fourier descriptor method in this example). This shows that in order to classify shapes with any degree of certainty, the variation within classes must be less than that between classes.
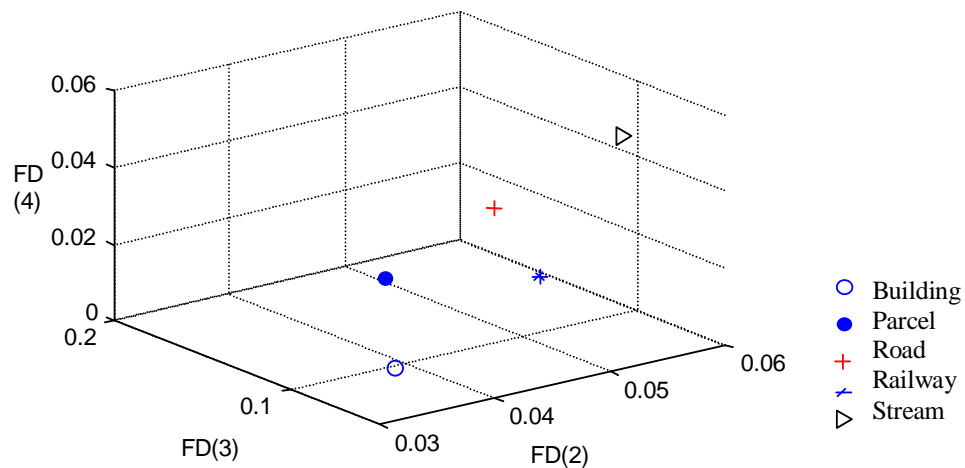

Figure 5. Average values of five sample classes

To compare the three shape recognition techniques used, several sample shapes from the map (buildings and parcels) were used as test images. Figures 6a, 6b, 7 and 8 show plots for the Fourier descriptor, moment invariants and scalar descriptor techniques respectively for a small representative sample of buildings and land parcels. Each plot shows the degree to which the two sets of objects cluster in three-dimensional space. As can be seen in Figure 6 (a), these two sets are not distinct. The evidence therefore indicates that normalised FD's are not very good for use in shape description where the data sets are of a very general shape. Note, that due to normalisation the first two terms, FD(0) = 0 and FD(1)= 1 are redundant in comparison. However, because the polygons are of a known scale the experiment was conducted using a Fourier descriptor technique that is not normalised for scale, that is, FD(1) ≠ 1 and therefore can be used as a comparison. The resulting clusters (figure 6 (b)) are much more significant indicating that using fourier descriptors without scale normalisation gives an improvement in object discrimination.
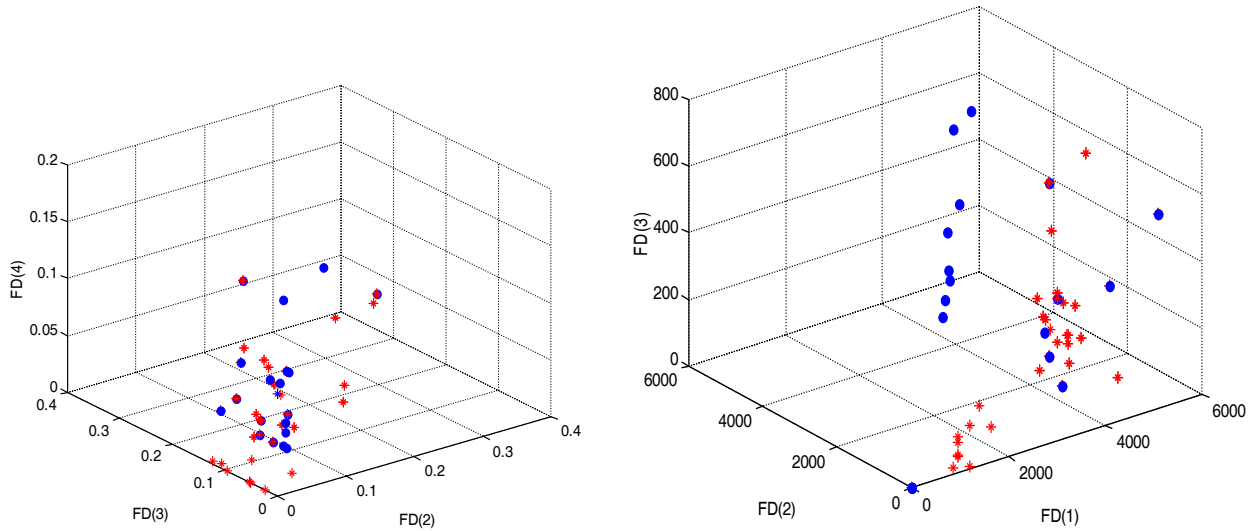
Figure 6, (a): Clustering of the polygon shapes in three-dimensional space of the features FD(2), FD(3) and FD(4), (b): Clustering of the polygon shapes in three-dimensional space of the features FD(1), FD(2) and FD(3), not normalised for scale.

	Buildings	Land Parcels
Number of polygons analysed	537	1095
Mean FD values	FD(2) = 0.0422 FD(3) =0.0795 FD(4) = 0.0416	FD(2) =0.0489 FD(3) = 0.0672 FD(4) = 0.0279
Variance in FD's (σ^2)	FD(2) =0.0073 FD(3) =0.0067 FD(4) = 0.0049	FD(2) =0.0088 FD(3) = 0.0030 FD(4) = 0.0016
Repeatability (3σ)	FD(2) =0.2562 FD(3) = 0.2457 FD(4) = 0.2100	FD(2) =0.2814 FD(3) = 0.1644 FD(4) = 0.1200
Distance between means for buildings and parcels	FD(2) =0.0067 FD(3) = 0.0123 FD(4) = 0.0137	

Table 1: Comparison of repeatability within feature classes and distance between classes for Fourier descriptors.

Table 1 shows the measurements for the sample of building and land parcels. The repeatability of the measurements of the class, represented as 3 times the standard deviation, is sizeably larger than the distance between the mean values for the two classes.
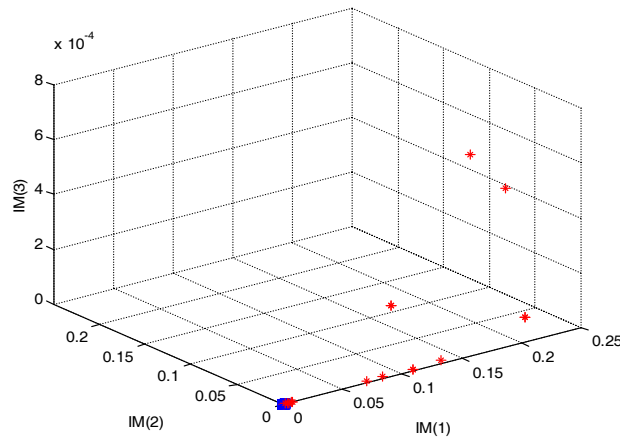

Figure 7. Clustering of the polygon shapes in three dimensional space of the feature IM(1), IM(2) and IM(3).

Producing the same style of graphical and mathematically output for the moment invariants technique, Figure 7 and Table 2,there is a considerable improvement in shape recognition of general shapes on maps over the Fourier descriptor method. Although overlap exits (also seen by the human eye) good classification occurs. In table 2 it can be seen that unlike the Fourier descriptors, the distance between the mean values is smaller than the repeatability for buildings and close to the repeatability for the land parcels.

	Buildings	Land Parcels
Number of polygons analysed	537	1095
Mean MI values	MI(1) = 6.56112e-004 MI(2) =1.8363e-007 MI(3) =6.5318e-012	MI(1) = 0.0750 MI(2) =0.0021 MI(3) =9.7574e-005
Variance in MI's (σ^2)	MI(1) = 2.5518e-007 MI(2) =1.5450e-013 MI(3) =2.2317e-022	MI(1) =0.0059 MI(2) =1.2250e-005 MI(3) =4.1673e-008
Repeatability (3σ)	MI(1) = 0.0015 MI(2) =1.1792e-006 MI(3) =4.4818e-011	MI(1) = 0.2295 MI(2) =0.0106 MI(3) =6.1242e-004
Distance between means for buildings and parcels	MI(1) = 0.0743 MI(2) =0.0743 MI(3) =9.7574e-005	

Table 2: Comparison of repeatability within feature classes and distance between classes for moment invariants.
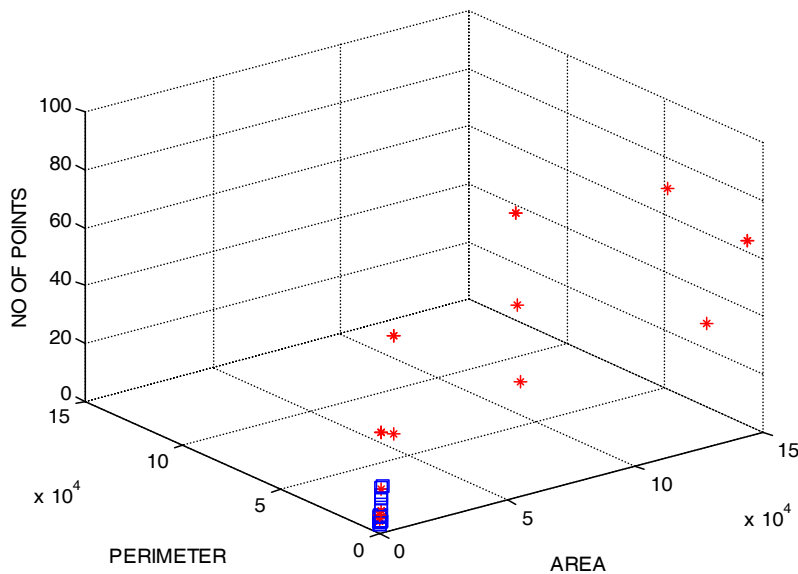

Figure 8. Cluster of the polygon shapes in three-dimensional space of the scalar features area, perimeter and number of points.

Figure 8 shows the cluster graph obtained for the same data sets using the scalar technique. This technique also shows considerable improvement over the Fourier descriptor method but follows closely to the results obtained for the moment method. The clusters are much more distinct with some overlap as would be expected. Again Table 3 shows this mathematically using the repeatability function.

	Buildings	Land Parcels
Number of polygons analysed	537	1095
Mean scalar values	Area = 463.6964 Perim =94.7717 Points =6.3929	Area =3.8695e+004 Perim =682.1834 Points =44.20
Variance in scalar's (σ^2)	Area = 9.1380e+004 Perim =1.6348e+003 Points =15.2103	Area =2.8365e+009 Perim =4.0771e+005 Points =1.0185e+003

Repeatability (3σ)	Area = 906.8734 Perim =121.2972 Points =11.7001	Area =159780.0 Perim =1915.6 Points =95.7411
Distance between means for buildings and parcels	Area = 38231.0 Perim =587.4117 Points =37.8071	

Table 3: Comparison of repeatability within feature 0classes and distance between classes for scalar descriptors.

CONCLUSION

As shape descriptor techniques the evidence to date is that all four techniques, namely Fourier descriptors, moment invariants, boundary chain code and scalar descriptors are very good features to use when dealing with particular types of shapes such as aircraft or alphanumeric characters. The aim of this paper was to investigate and compare their usefulness for shape description of general shapes on maps, for example houses, roads, parcels etc. When tested for the more generalised topographic shapes, Fourier descriptors do not appear to be as conclusive and successful as hoped. However, both the moment invariants and scalar descriptor techniques proved to be significantly more successful in their task. This work is part of on-going research and it is envisaged that all the above methods with their shown capabilities will be combined to produce the optimal results.

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