# Fourier Descriptors as A General Classification Tool for Topographic Shapes

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**Abstract**. Automatic structuring (feature coding and object recognition) of topographic data, such as that derived from air survey or raster scanning large-scale paper maps, requires the classification of objects such as buildings, roads, rivers, fields and railways based on their shape. There is a considerable body of published work on the identification and classification of objects within images. Recognition is based on the matching of descriptions of shape. Several techniques have proved useful such as boundary chain encoding and moment invariants. The technique used here uses Fourier Descriptors. Based on a Fourier analysis technique applied to the boundary co-ordinates of an object expressed as complex numbers, Fourier descriptors are widely used in image processing to describe and classify shapes. The shape descriptors generated from the Fourier coefficients numerically describe shapes and can be normalised to make them independent of translation, scale and rotation. Classification is performed by comparing descriptors of the unknown object with those of a set of standard shapes, finding the closest match. Most applications using Fourier Descriptors deal with the classification of definite shapes, for example identifying a particular type of aircraft. To identify topographic objects the technique needs to be extended to deal with general classes of shape. Fourier descriptors are evaluated as general classifiers applied to broad classes of topographic shape (buildings, fields, roads etc.). To analyse their effectiveness, a corpus of shapes of classified objects was extracted from topographic large-scale digital maps. The descriptors of each shape were calculated and the results analysed. These indicate that normalised Fourier descriptors alone are unsuitable for such general classification. However, when applying the same Fourier method combined with other techniques it was found that they could help to discriminate between some classes of objects.

**Keywords**: shape analysis, shape description, object recognition, Fourier descriptors

## 1. Introduction

The technology to capture paper-based cartographic data through scanning is well founded and the production of raster data relatively easy [1]. The vectorisation of raster data, although not perfect, also is widespread in mapping organisations although it usually requires user intervention to ensure the quality of data [2].

Vectorisation produces vector graphical data but most applications require the data to be structured so it models not only the geometry and topology but also logical contents often stored as a set of attributes attached to the geometry. These are usually captured manually by a human operator but this process of classifying and entering attributes can be a severe bottleneck in the production flow. This can result in both a scarcity of suitable searchable data and/or a sparseness in its accuracy and detail. Automation of the recognition of objects is the obvious solution but this is a complex problem [3].

Much work has been done in computer vision on the identification and classification of objects within images [4]. However, less progress has been made on automating feature extraction and semantic capture for digital cartography and geographic information systems. This is partly because the low-level graphical content of maps has often been captured manually (on digitising tables etc.) and the encoding of the semantic content has been seen as an extension of this. However, the successful automation of raster-vector conversion plus the large quantity of new and archived graphical data available on paper makes the automation of feature extraction desirable.

Feature extraction and object recognition are large research areas in the field of image processing and computer vision [5]. Recognition is largely based on the matching of descriptions of shapes. Numerous shape description techniques have been developed, such as analysis of scalar features (dimensions, area, number of corners etc.), Fourier descriptors, moment invariants and boundary chain coding. These techniques are well understood when applied to images and have been developed to describe shapes irrespective of position, orientation and scale. They can be easily applied to topographical shapes.

The application of one of these techniques, Fourier descriptors, to classify objects on large-scale maps is described here. Unlike many applications, where the shape categories are very specific (for example, identifying a particular aircraft type in a scene), the problem requires the classification of a particular shape into a general class of similar object shapes, for example building, road or stream.

## 2. Fourier Descriptors

### 2.1 Background

Fourier transform theory [6] has played a major role in image processing for many years. It is a commonly used tool in all types of signal processing and is defined both for one and two-dimensional functions. It has a wide range of applications in image processing and continues to be a topic of interest in theoretical as well as applied work in this field. Fourier transforms can be used in image enhancement, restoration, encoding and description.

In this paper the Fourier transform technique is used for shape description in the form of Fourier Descriptors. The Fourier Descriptor is a widely used all-purpose shape description and recognition technique [7-13]. These Fourier descriptor values produced by the Fourier transformation of a given image represent the shape of the object in the frequency domain [14]. The lower frequency descriptors store the information about the general shape and the higher frequency descriptors store the information about the smaller details of the image. Therefore, the lower frequency components of the Fourier descriptor define the rough shape of the original object.

## 2.2 Theory

The Fourier transform theory can be applied in different ways for shape description. One method works on the change in orientation angle as the shape outline is traversed [15] but for the purpose of this paper it was decided to implement the following procedure [16]. The outline (boundary) in the image is treated as lying in the complex plane, the vertical being the imaginary axis and the horizontal the real axis. So the row and column co-ordinates of each point on the boundary can be expressed as a complex number, x + jy where j is sqrt(-1). Tracing once around the boundary in the counter-clockwise direction at a constant speed yields a sequence of complex numbers, that is, a one-dimensional complex function over time (figure 1).

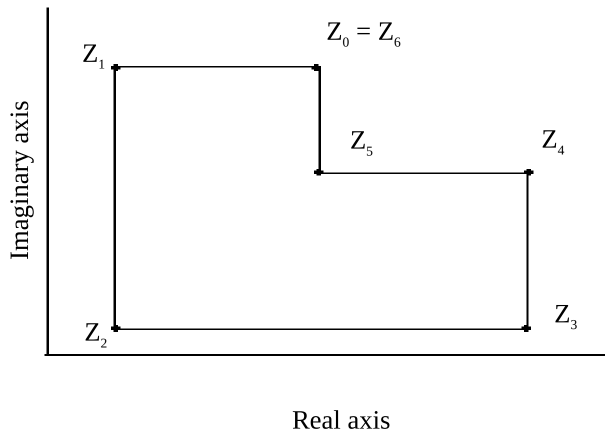


**Figure 1**: Representation of a shape using complex numbers.

$Z_0 - Z_6$ are the points defining the vertices of the object boundary. Traversal of the boundary is represented by the direction of the arrows. In order to represent traversal at a constant speed, it is necessary to interpolate equidistant points around the boundary. Traversing the boundary more than once results in a periodic function. The Fourier transform of a continuous function of a variable x is given by the equation:

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx \qquad (1)$$

However, when dealing with discrete images the Discrete Fourier Transform (DFT) is used. So equation (1) transforms to:

$$F(u)=\left(\frac{1}{N}\right)\sum_{x=0}^{N-1} f(x)e^{\frac{-j2\pi ux}{N}} \qquad (2)$$

N is the number of equally spaced samples. The variable *x* is complex, so by using the expansion e [ -jA ] = cos (A) – j.sin (A), equation (2) becomes:

$$F(u)=\left(\frac{1}{N}\right)\sum_{x=0}^{N-1} f(x+jy)).(\cos(Ax)-j.\sin(Ax)) \qquad (3)$$

where A = 2.π.u/N.

The DFT of the sequence of complex numbers, obtained by the traversal of the object contour, gives the Fourier descriptor values of that shape.

### 2.3 Normalisation

The DFT is a reversible linear transformation so it retains all the information in the original boundary. This information is therefore in a form that allows different factors to be isolated and, if required, eliminated. In this way, the Fourier descriptor values can be normalised to make them independent of translation, scale and rotation of the original shape.

Translation of the shape by a complex quantity having x and y components, corresponds to adding a constant $x + jy$ to each point representing the boundary. The DFT is such that this constant only effects the coefficient F(0) in the Fourier series. Therefore normalisation of position is achieved by setting the term F(0) to be equal to zero.

Scaling a shape is achieved by multiplying all co-ordinate values by a constant factor. The DFT results in all members of the corresponding Fourier series being multiplied by the same factor. So by dividing each coefficient by the same member, normalisation for size is achieved. It can be shown that when the contour is traced in the counter-clockwise direction and the contour is non-overlapping that F(1) will always be the largest coefficient. Normalisation for scale is achieved by dividing each F(n) by the magnitude of F(1) resulting in all remaining terms being fractional.

Rotation normalisation is achieved by finding the two coefficients with largest magnitude and setting their phase angle equal to zero. As stated earlier, F(1) is the coefficient of largest magnitude. Let the coefficient of the second largest magnitude be F(k). This F(k) coefficient has a normalisation multiplicity m where m = |k - 1|. Therefore the requirement of F(1) and F(k) to have zero phase angle can be satisfied by m different orientation and starting point combinations. In the case where k =2, then the orientation and the starting point of the contour are defined uniquely [17].

However, it is generally not the case that F(2) is the second largest coefficient of magnitude.

## 3. Fourier descriptors of cartographic shapes

Fourier descriptors have mainly been used to classify particular shapes, for example of a particular type of aircraft [14]. One of the aims of this project is to test the Fourier descriptor technique for general categories of cartographic shapes, for example houses or roads. The data used was extracted from data sets representing large-scale (1:1250) plans of the Isle of Man [18]. The data was pre-processed to extract closed polygons from lines with the same feature codes. Unlike shapes extracted directly from aerial images, cartographic shapes are of a known scale, which can be used as an additional classification descriptor.

After extracting the required polygonal data from the maps, an interpolation procedure was applied to sample the boundary at a finite number (N) of equi-distant points. These points are stored as a series of complex numbers and then processed using the Fourier transform resulting in another complex series also of length N. If the formula for the discrete Fourier transform were directly applied, each term would require N iterations to sum. As there are N terms to be calculated, the computation time would be proportional to $N^2$. So the algorithm chosen to compute the Fourier descriptors was the Fast Fourier Transform (FFT) for which the computation time is proportional to NlogN. The FFT algorithm requires the number of points N defining the shape to be a power of two. In the case of this project it was decided to use 512 sample points.

The FFT algorithm is applied to these 512 coefficients. The list is normalised for translation, rotation and scale. This results in the first two terms always having the values 0 and 1.0 respectively which makes them redundant for classification. Calculation of the Fourier Spectrum builds a new list and disposes of the Fourier transform list. The result is 510 Fourier descriptor terms.

Given two sets of Fourier descriptors, how do we measure their degree of similarity? An appropriate classification is necessary if unknown shapes are to be compared to a library of known shapes. If two shapes, A and B, produce a set of values represented by a(i) and b(i) then the distance between them can be given as c(i) = a(i) – b(i). If a(i) and b(i) are identical then c(i) will be zero. If they are different then the magnitudes of the coefficients in c(i) will give a reasonable measure of the difference. It proves more convenient to have one value to represent this rather than the set of values that make up c(i). The easiest way is to treat c(i) as a vector in a multi-dimensional space, in which case its length, which represents the distance between the planes, is given by the square root of the sum of the squares of the elements of c(i).

## 4 Results

In this section a sample of the results produced by the application of the Fourier descriptors is presented to evaluate their usefulness in shape discrimination of general cartographic features. Figure 2 plots the average values of FD(2), FD(3) and FD(4) for five categories of object from the sample maps.

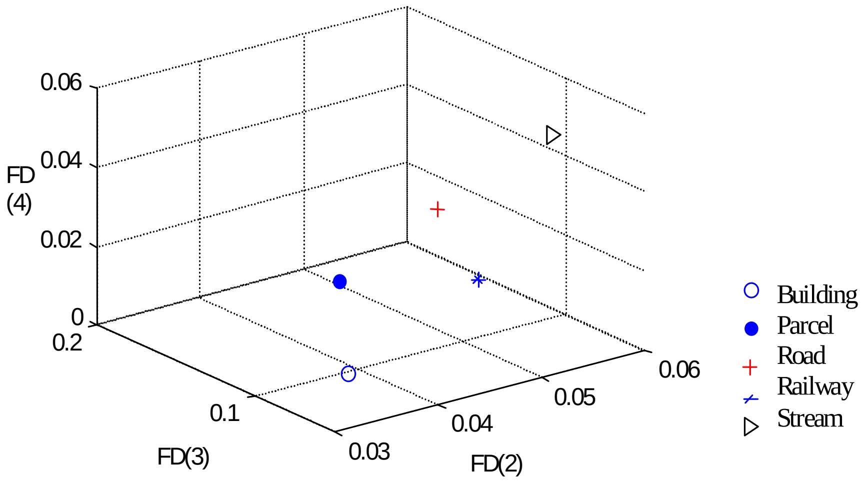


**Figure 2**. Average FDs of five sample classes.

In order to classify shapes with any degree of certainty, the variation within classes must be less than that between classes. As an example Figures 3 and 4 show respectively building and parcels on a portion of one map. The Fourier descriptor sets are computed from the 512 equally spaced points along the boundary of each test shape using the formulae derived earlier. The following table is an example of the first 16 low-order Fourier descriptors obtained for a house shape, where FD(0) represents the first descriptor value.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1.0000 | 0.0466 | 0.0933 | 0.0375 | 0.0221 | 0.0326 | 0.0078 |
| 0.0107 | 0.0150 | 0.0052 | 0.0066 | 0.0083 | 0.0047 | 0.0001 | 0.0023 |

Figure 5 shows a plot of the first 16 FDs for a sample of 200 shapes. From inspection of the values produced for each polygon, most of the shape information is described by the first few descriptors and so only the first 16 terms were used for comparison, remembering that due to the normalisation procedures, FD(0) and FD(1) are redundant.

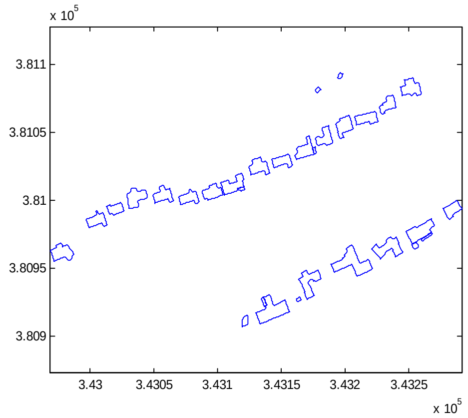


**Figure 3**: Sample data representing house shapes.

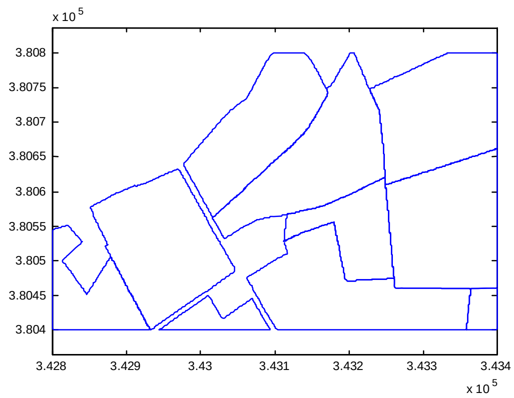


**Figure 4**: Sample land parcel shapes.

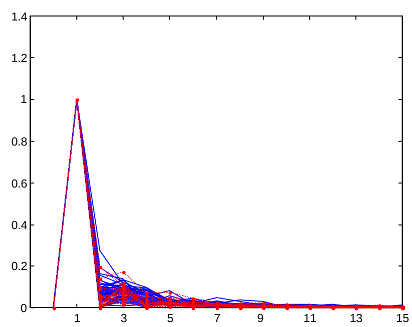


**Figure 5**: Plot of Fourier Descriptor values of 200 sample shapes.

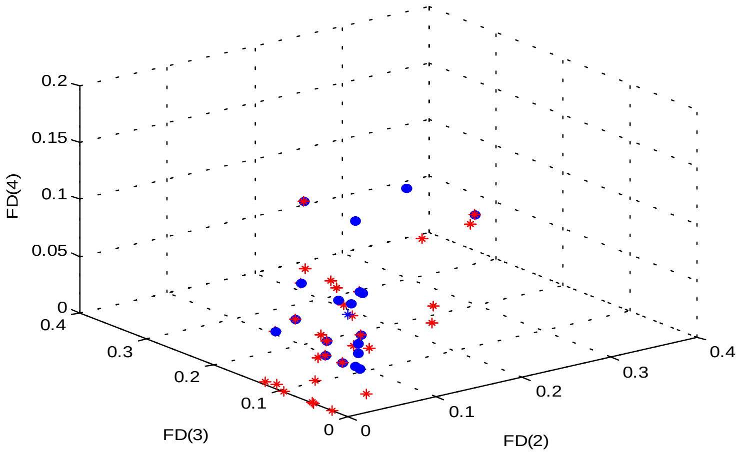


**Figure 6**: Clustering of the polygon shapes in three-dimensional space of the features FD(2), FD(3) and FD(4) for a small representative sample of buildings and land parcels.

Figure 6 shows the degree to which these two sets cluster in (FD(2),FD(3),FD(4)) space. As can be seen, these two sets are not distinct. This evidence therefore indicates that normalised Fourier descriptors are not very good for use in shape description where the data sets are of a very general shape. As these three descriptors contain most of the general shape information, the two classes do not get more distinct with the inclusion of more FD terms.

Because the polygon shapes are of a known scale, it was decided to carry out the experiment when using a Fourier descriptor technique that is not normalised for scale, that is, $FD(1) \neq 1$. This means that FD(1) is no longer redundant as a comparison and can be used as a further descriptor. Computing the new set of Fourier descriptors for the interpolated test images, the following are examples of the results obtained. Taking the same house shape as above, the first 16 low-order Fourier descriptors beginning with FD(0) become:

1.0e+003*

0 4.2482 0.1981 0.3961 0.1592 0.0941 0.1384 0.0332
0.0455 0.0635 0.0222 0.0279 0.0353 0.0198 0.0003 0.0100

The first few descriptors again, mostly describe the shape information. The same classification method was applied. The two data sets were plotted in the three-dimensional space, this time using the features (FD(1),FD(2),FD(3)) to observe how well they separated or to see if they did separate using the modified Fourier descriptor method. Figure 7 shows the degree to which the two data sets cluster in (FD(1),FD(2),FD(3)) space. The resulting clusters are much more significant than in the previous part of the experiment. This result indicates that using the Fourier descriptor technique without scale normalisation gives an improvement in object discrimination.

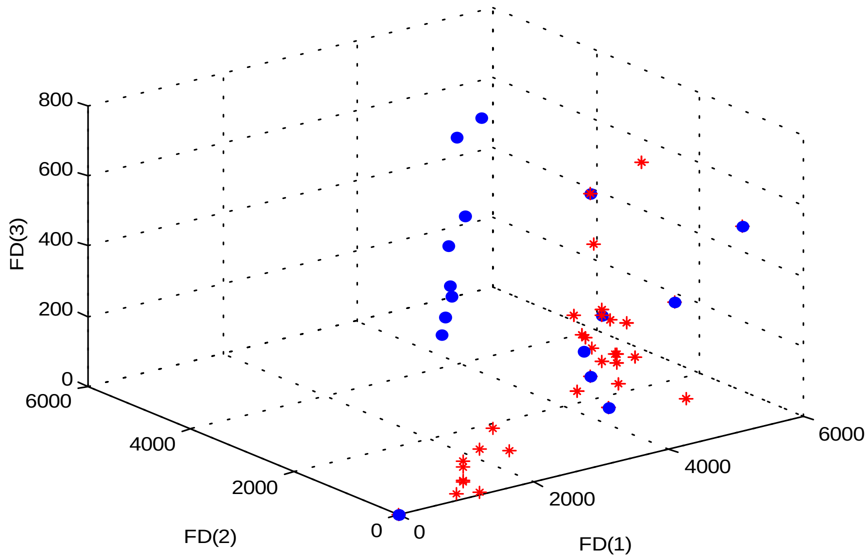


**Figure 7**: Clustering of the polygon shapes in three-dimensional space of the features FD(1),FD(2) and FD(3).

## Conclusion

As a shape descriptor technique, the evidence to date is that Fourier descriptors are very good features to use when dealing with particular types of shapes such as aircraft or alphanumeric characters [15]. The aim of this paper was to investigate the usefulness of Fourier descriptors for the shape description of general shapes on maps, for example houses, roads, parcels etc. When tested for the more generalised cartographic shapes, Fourier descriptors do not appear to be as conclusive and successful as hoped.

Table 1 shows measurements for a sample of building and land parcel shapes. As can be seen, the repeatability of the measurements for each class, represented as three times the standard deviation, is considerably larger than the distance between the mean values for the two classes of feature

| | Buildings | Land parcels |
|---|---|---|
| **Number of polygons analysed** | 537 | 1095 |
| **Mean FD values** | FD(2) = 0.0422<br>FD(3) = 0.0795<br>FD(4) = 0.0416 | FD(2) = 0.0489<br>FD(3) = 0.0672<br>FD(4) = 0.0279 |
| **Variance in FDs ($\sigma^2$)** | FD(2) = 0.0073<br>FD(3) = 0.0067<br>FD(4) = 0.0049 | FD(2) = 0.0088<br>FD(3) = 0.0030<br>FD(4) = 0.0016 |
| **Repeatability ($3\sigma$)** | FD(2) = 0.2562<br>FD(3) = 0.2457<br>FD(4) = 0.2100 | FD(2) = 0.2814<br>FD(3) = 0.1644<br>FD(4) = 0.1200 |
| **Distance between means for buildings and parcels** | FD(2) = 0.0067<br>FD(3) = 0.0123<br>FD(4) = 0.0137 | |

**Table 1**: Comparison of repeatability within feature classes and distance between classes.

However when the same technique was applied using Fourier descriptors without scale normalisation, the end results appeared to be more significant and successful. These results indicate that using scale as a further descriptor could prove useful when using Fourier descriptors for general forms of shapes.

The work in this paper is part of ongoing research and the results obtained are to date being used only as an indication of the capabilities of the Fourier descriptor technique. Future work will involve performing the same experiment on much larger data sets extracted from large-scale maps with more classes of feature. It is envisaged that Fourier Descriptors will be only one of several techniques of object recognition [19] that will be combined to produce the optimal results.

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