# AM/FM DAFX

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## ABSTRACT

In this work we explore audio effects based on the manipulation of estimated AM/FM decomposition of input signals, followed by resynthesis. The framework is based on an incoherent mono-component based decomposition. Contrary to reports that discourage the usage of this simple scenario, our results have shown that the artefacts introduced in the audio produced are acceptable and not even noticeable in some cases. Useful and musically interesting effects were obtained in this study, illustrated with audio samples that accompany the text. We also make available Octave code for future experiments and new Csound opcodes for real-time implementations.

## 1. INTRODUCTION

Analysis-synthesis methods are a common way of implementing digital audio effects. The Vocoder [1] is a good example of this approach. In this ubiquitous audio effect, the signal spectrum is broken down in a number of sub-bands, and effects are implemented by spectral manipulation followed by resynthesis. The success of the Vocoder paradigm can be assessed by the various applications [2] and the number of refinements [3, 4] proposed since its basic conception.

In this work we explore digital audio effects based on AM/FM decomposition of signals, processing the estimated signals for the envelope and instantaneous frequency, followed by resynthesis using these modified signals. This work follows a previous study where we derived selected psychoacoustic metrics in order to assess the extent of the modifications imposed by different configurations of smoothers acting on the AM and FM estimations of sample waveforms taken from an analog synthesizer [5]. Results showed that there is potential in creating different audio effects using the technique.

Recent surveys, such as the one in [6], have considered the level where the transformations in the signal are applied. Operations directly on the signal, in time-domain, are seen as surface operations, since sound is represented by the waveform samples. Effects like chorus, delay lines and distortion are good examples of such operations. On the other hand, systems based on analysis/-transformation/resynthesis work at a higher level with regards to how far the sound representation is unfolded (transformed). Thus, it can be accurately refined in a new domain, followed by the related inverse transform. AM/FM processing can de understood as a mix of both levels, as the input signal is transformed to a joint time-domain representation and “surface operations” are applied to the new signals. To put it in another way, the AM/FM decomposition unravels the mechanics of a (time-domain) signal in a couple of time-domain signals, and then we apply effects distorting these characteristics. Some examples in this paper are inspired on classic audio effects, but we highlight that the processing is applied in this different AM/FM domain. Hence, the effects obtained are not equivalent to the original ones, but can illustrate some possibilities with the technique. We also expect new kinds of effects to be developed within the framework.

AM/FM analysis is a topic extensively explored in the time-frequency literature. However, most of the studies are based on speech analysis, either with the intention of deriving parameters for analysis of speech, or for compression, transmission and resynthesis goals. Only a few works deal with audio as a broader scope, or specifically with music.

One line of work that explores music signals is Modulation Filtering, which is defined as the process of modifying a sub-band signal by filtering its estimated modulator and recombining the results with the original estimated carrier [7]. The series of works regarding the Modulation Vocoder [8, 9] explore a similar technique. The input signal is separated in bands based on center of gravities (COG), followed by a sub-band AM/FM decomposition. The goal is to achieve selective pitch transposition, which they implement by multiplying the FM envelopes in all bands by a same value. The paper reports supreme quality for the transposition, compared to commercial packages, but an inferior timbre preservation. Pitch transposition is the only effect discussed, and we think there is space for further exploration within this and similar frameworks.

In this paper, however, we study the decomposition of the whole signal into AM/FM as a means for implementing a variety of audio effects. We hope to demonstrate that this simpler approach can yield a number of useful transformations, which are economical in computational terms (if compared, for instance, to the sub-band methods), and can be implemented in real-time. In order to characterise the effects presented here, we employed a short guitar lick consisting of a bend followed by a vibrato. This signal was chosen for its smooth but varying frequency evolution, which can be observed in its spectrogram (Figure 1).

This paper is organised as follows. We first discuss our decomposition scheme, following to a description of new audio effects based on this decomposition. Then we present some reflections regarding the method and conclude the text. Audio examples with the sonorities obtained are available[1] alongside code for implementing the techniques. Octave [10] programs were used during the development of the work, and Csound [11] opcodes were written for real-time implementation of the techniques presented in the paper.

[1] http://www.ime.usp.br/~ag/dl/dafx15.zip

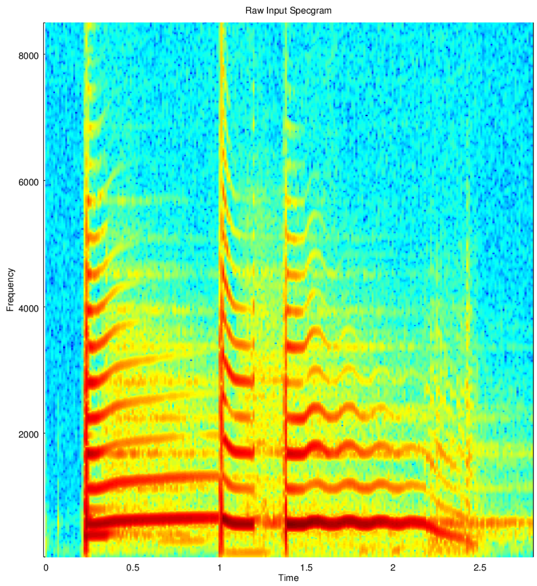


Figure 1: *Evolution of frequencies present in our test signal. Notice the bend through the first second, followed by a vibrato.*

## 2. AM/FM ANALYSIS

The AM/FM decomposition is a powerful method for the analysis of non-stationary signals [12]. For the mono-component case, the scheme presupposes the signal to be processed as a sole sinusoidal tone modulated both in amplitude and frequency. This follows the principle of Dudley's first speech synthesizer [13], in which speech and other non-stationary signals were interpreted as low-frequency processes that modulate higher frequency carriers representing fine structure [7]. This kind of decomposition conceptually involves two separate notions of frequency, that of the sinusoid carrier and another one for the modulating signal [7]. In other words, the decomposition consists of a transform of a one-dimensional broadband signal into a two-dimensional joint frequency representation [14].

Considering a signal

$$x(t) = m\cos(\omega t + \phi), \tag{1}$$

being $m$ the amplitude, $\omega$ the frequency and $\phi$ the initial phase, the argument $(\omega t + \phi)$ is the instantaneous phase, and its derivative $\omega$ is the instantaneous frequency (IF) [15]. We could also have a signal modulated both in amplitude and frequency, as in [16]

$$x(t) = m(t)\cos(\theta(t)). \tag{2}$$

The phase derivative $\dot{\theta}(t)$ is the IF of the signal, and $m(t)$ its instantaneous amplitude (IA). The IF can be described as the frequency of the sinusoid that locally fits the signal at instant $t$ [12].

The AM/FM signal analysis is intended to decompose a signal into functions for the AM (related to the IA signal) and the FM (related to the IF signal). A vast number of techniques exist for that [17, 16]. In this work we assume a mono component signal and apply the decomposition based on the analytic signal (AS). The AS, in turn, is based on the Hilbert Transform (HT) decomposition. The HT of a signal $x(t)$ is given by [12]

$$\hat{x}(t) = x(t) * \frac{1}{\pi t}. \tag{3}$$

This creates a $90^\circ$ phase shifted version of the original, from which we build the analytic signal related to $x(t)$ as

$$z(t) = x(t) + j\hat{x}(t) = |z(t)|e^{j\theta(t)}, \tag{4}$$

where the AM/FM signals $a(t)$ and $f(t)$ are defined as

$$a(t) = |z(t)| \tag{5}$$

and

$$f(t) = \frac{d}{dt}\theta(t). \tag{6}$$

Another possibility for implementing the Hilbert Transform is by using filters, as described in [18]. A discrete-time Hilbert Transform can be approximated by a FIR filter with impulse response given by [18]

$$h(n) = \frac{1-\cos(\pi n)}{\pi n} = \begin{cases} \frac{2}{\pi n} & \text{for } n \text{ odd,} \\ 0 & \text{for } n \text{ even.} \end{cases} \tag{7}$$

As the filter need to be truncated in an appropriate length, an approximation of the theoretical transform will be achieved, and a short delay is necessary to make the filter causal.

Alternatively, it is possible to use a bank of allpass filters [19] for an efficient, latency-free, implementation of the HT. The allpass filters structure output a pair of signals with a $90^\circ$ phase difference over a wide range of frequencies. The *hilbert* opcode[2] in Csound, used for the experiments in this paper, employs this algorithm.

For a signal of the form of (2), the absolute value of its associated analytic signal $z(t)$ can be used as an estimate for the AM portion, $a(t)$; the derivative of the unwrapped $\theta(t)$ can be used to estimate the FM portion, $f(t)$. The analysed signal can then be recreated using the expression

$$x(t) = a(t)\cos\left(2\pi\int_0^t f(\tau)d\tau\right) \tag{8}$$

For non real-time systems, real time systems with the acceptance of a delay long enough for a suitable frequency resolution, or off-line methods (such as our implementation in Octave), the Hilbert Transform can be implemented using the Fourier Transform. If $X(f)$ is the Fourier Transform of a real signal $x(t)$, the Fourier Transform of the analytic signal is given by

$$Z(f) = 2u(f)X(f), \tag{9}$$

where $u(f)$ is the unit-step signal [16]. By inverse transforming $Z(f)$ we get $z(t)$.

We are aware that a lot of criticism has been addressed to incoherent Hilbert based decompositions. Modulation filtering literature reports that the Hilbert based decomposition of arbitrary band-limited signals is often meaningless, as the instantaneous frequency estimation is discontinuous and with a bandwidth greater than the original signal one [7]. In [20] the authors argue that the

[2] `http://www.csounds.com/manual/html/hilbert.html`

model relating the IF to the analytic signal phase might be inadequate, and propose an alternative.

There is also an interesting discussion about what would be the best - or at least a suited - interpretation for the instantaneous frequency [21, 22]. Seeing it as the average frequency at each time is one possibility, but it only holds in the case of two components with the same amplitude [21]. It was shown that for a higher number of components, the equal-amplitude condition is not even sufficient [20]. Intuitively we can think that it makes sense to talk about IF for a mono-component signal, but not for multicomponent ones, even in the two equal amplitude component case.

So we should better understand what this IF "number" might mean. Regardless the complexity of the mathematical description of a signal, in most cases, if we step away from the math and observe the phenomenon we are able to notice an "instantaneous behaviour". For instance, it is easy to visualize this idea in a color changing light [21]. However, if we make a chord using two or more sinusoids we hear more than one frequency; but maybe we also hear this instantaneous metric related to the IF metric we get by applying the decomposition. Maybe by manipulating this quantity to achieve audio effects, more intuition can be achieved around this concept.

Yet another aspect regarding the decomposition of a signal is whether the signal is a mono- or multi-component one. Cohen [23] attempts to characterize this difference based on the instantaneous bandwidth of each ridge (related to the spread of the ridge) in a time-frequency plane description of a signal. He argues that if two ridges are close enough and present enough spread they coalesce and should be considered as a sole partial.

Another way to look at this is to consider the AM/FM decomposition as an analysis process that can be applied to a non-linear distortion synthesis scenario. The resynthesis is effectively a combination of both a heterodyne effect (amplitude modulation) and complex-signal (as opposed to sinusoidal) frequency modulation. It is in this context that we propose a number of possible DAFx.

The analysis-synthesis process is transparent: if we work with an unmodified pair of modulating functions, we are able to recover the original signal. Transforming the IA will lead to different types of modification of the original signal, as already reported in other contexts. While these changes might be undesirable in other applications, here there are no specific limitations beyond the intended aesthetic considerations of a good audio effect. Similarly, our modification of IF, which is behind most of the processes explored here, has a wide-ranging scope.

All of the effects discussed here can be implemented in realtime in Csound. Listing 1 shows how the analysis process is implemented in terms of user-defined opcodes (UDOs). The first UDO in the listing performs the phase differentiation and unwrapping, whereas the second is the actual full analysis process, which yields two outputs, the amplitude and frequency modulation signals. Depending on the input signal, we have observed that the approximation involved in the `hilbert` opcode implementation might sometimes result in some IF estimation errors. In this case, while reconstruction is still good, some artefacts might arise when the frequency modulation signal is scaled. Such issues are overcome, however, by the use of more precise HT calculation methods.

Listing 1: *AM/FM decomposition in Csound*

```
/* diff & unwrap */
opcode Udiff,a,a
        setksmps 1
 asig xin
 asig diff asig
 ksig = downsamp(asig)
 if ksig >= $M_PI then
  asig -= 2*$M_PI
 elseif ksig < -$M_PI then
  asig += 2*$M_PI
 endif
  xout asig
endop

/* AM/FM analysis */
opcode AmFmAnal,aa,a
  asig xin
  aim,are hilbert  asig
  am = sqrt(are^2 + aim^2)
  aph = taninv2(aim, are)
      xout  am, Udiff(aph)*sr/(2*$M_PI)
endop
```

## 3. AM/FM EFFECTS

In this section we show some effects experimented in this study. We focus on those which produced interesting sonorities. The effects are based on famous ones, but applied in this subverted paradigm of coping with the amplitude and frequency modulations descriptions of the signal.

After processing of the signals estimated by eqs.3 to 6 , the resynthesis (as defined in eq. 8) can be implemented in Csound with the code given in Listing 2. Alternatively, a sinusoidal oscillator will suffice.

Listing 2: *Csound resynthesis code*

```
/* AM/FM synthesis */
opcode AmFmSyn,a,aa
   am,aif xin
   xout am*cos(integ(aif)*2*$M_PI/sr)
endop

/* alternatively, just use an oscillator */
asig = oscili(am,aif)
```

### 3.1. OctIFer

The octaver effect mixes a signal with a transposed version of it, either an octave up or down. The OctIFer is an octaver-like process based on applying scaling to the estimated signal for the IF, before the integration for the resynthesis. As common sense might suggest, a simple multiplication of the IF by $0.5$ produces a signal an octave lower than the original. That is indeed the case, but additionally to that a very interesting effect is introduced, that enriches the signal and this case resembles a very clean bass boost in a guitar amplifier. The spectrogram of the signal achieved in such a way is shown in Figure 2.

By doubling the IF, we would expect to get a signal that is an octave higher. However, the perception of the resultant signal is not a transposition, but instead, a beautiful coloration of the higher harmonics (this can be also visualized in the spectrogram in Figure 3). The resultant signal resembles a guitar single note played with a metal slide.

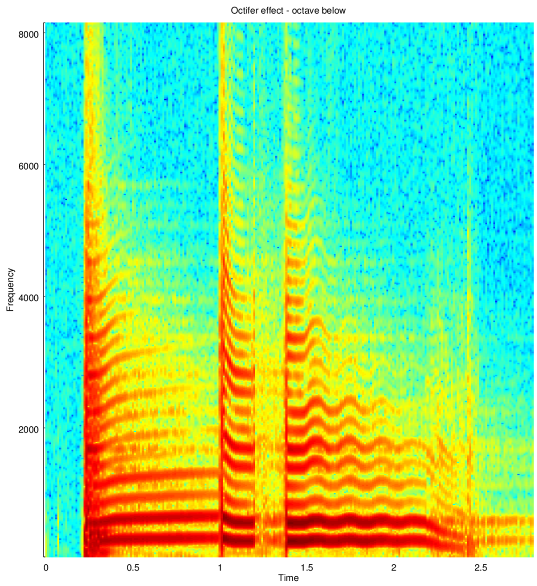


Figure 2: *Spectrogram of the octave below OctIFer effect. The lower partials introduced are clearly evident.*

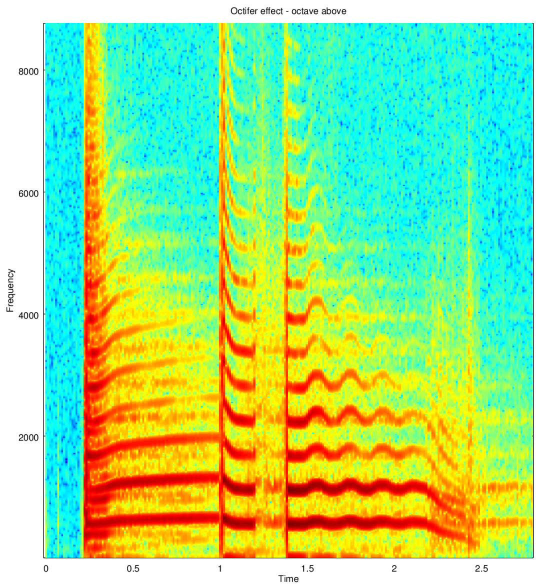


Figure 3: *Spectrogram of the octave up OctIFer effect. In comparison to the original sound spectrogram, a coloration can be observed.*

The reason for this effect is that while the frequency modulation signal is moved one octave, the amplitude modulation is unmodified. Since the synthesis expression is a ring modulation of the FM synthesis and the AM signal, we get a convolution of the two spectra. A similar process is at play in the other IF modification effects.

### 3.2. TransColorIFer

Values different than 2 or 0.5 might also be used to achieve general timbral transformations with the TransColorIFer technique. For instance by multiplying the IF by the relation 3:2 the effect introduced will add a coloration so the resultant signal bears resemblance to the fifth of the original note played. In such a way, a possible application is to build signals which are sums of signals resembling notes of specific chords. Each of the voices will have a different coloration and timbre, gently uncorrelated, leading to pleasant groupings. In the audio examples an example of a dominant (just minor, based on 9:5) 7th chord can give a good taste of that.

This same explanation holds for the case of the up-OctIFer. The original pitch is kept by the unprocessed IA signal, and the coloration only alters the timbre of the sound. In the case of the down-OctIFer, lower components are introduced, while the other harmonics introduced are aligned with the components present in the IA signal.

The expression for the TransColorIFer (and OctIFer) effect is given by

$$x(t) = a(t) \cos\left(r 2\pi \int_0^t f(\tau) d\tau\right), \tag{10}$$

where $r$ is the interval relation intended for the coloration.

### 3.3. HybridIFer

The cross-synthesis effect generates a sound based on the combination of two sound inputs [24], also sometimes referred to as *mutation between sounds*. With our HybridIFer effect we implement mutation between sounds by combining their instantaneous frequencies with complementary weights and keeping only one of the envelopes. Sounds obtained show that for the same two sounds, while using one of the envelopes can result in a noisy reconstruction, simply switching to the other can produce a clean outcome. Mix the two amplitude envelopes, instead of using only one, also tends to result in noisy outcomes.

The mutation types produced with the technique are interesting as they preserve the sonorities and characteristics of both sounds, also introducing a novel coloration. In the audio examples we provide mutations of all the combinations possible with a guitar, a saxophone, and a TB-303 synthesizer sample, using equal contribution of each sound IF estimation ($p = 0.5$). The spectrogram in Figure 4 show the spectrogram of a saxophone signal we used to combine with the guitar sound. The resultant spectrogram is shown in Figure 5.

The expression to implement the HybridIFer is given by

$$x(t) = a(t) \cos\left(2\pi \int_0^t (p f_1(\tau) + (1-p) f_2(\tau)) d\tau\right), \tag{11}$$

where $f_1(t)$ and $f_2(t)$ are the IF for the two signals to be hybridized, $a(t)$ is the amplitude envelope for one of these signals and $0 \leq p \leq 1$.

It is interesting to note that this technique resembles the synthesis scheme of Double FM modulation [25], where a sum of sinusoids modulate a common carrier, where in our case the modulators are IFs. Double FM produces many sum and difference

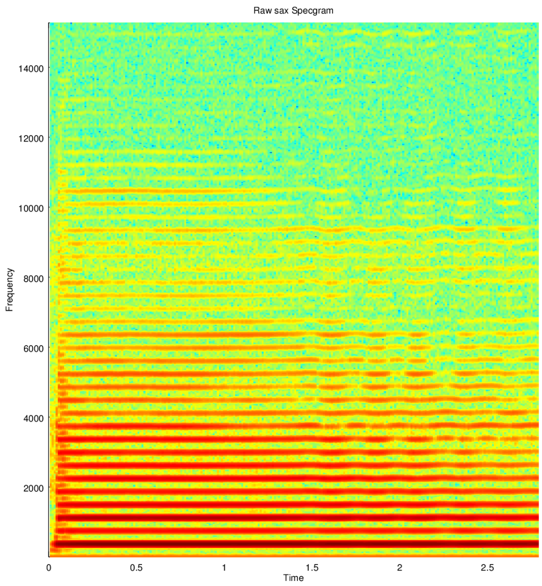


Figure 4: *Spectrogram of a saxophone sample.*

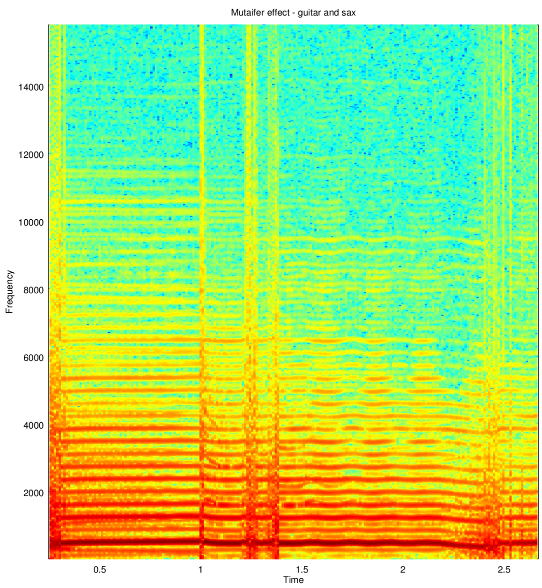


Figure 5: *Spectrogram of the guitar and sax combination using the HybridIFer effect.*

frequency components that are related to the frequencies of the modulators. It has a connection to the well-known AM technique of Ring Modulation [26, 27] that also produces sum and difference outputs.

### 3.4. ChorIFer

Chorusing is an effect based on the sum of slightly delayed and detuned versions of a signal. The resultant signal is perceived as a choir of the original sound. Due to the small differences in frequencies and phases, beatings and modulations in the resultant signal are very evident within this effect. Some guitar pedals in the market are even labeled as chorus/vibrato.

The ChorIFer is our version of the chorus effect, where the detunements are obtained with the method described in Sections 3.1 and 3.2. The resynthesized signal sounds feature a clean vibrato for low detunements values, up to 1% of the IF (this can be visualized in the spectrogram shown in Figure 6). For larger values, partials due to frequency modulation are also introduced and an interesting more dramatic variation is achieved. An effect similar to that is the animation effect in analog subtractive synthesizers, where closely detuned versions of the main oscillator are added [28].

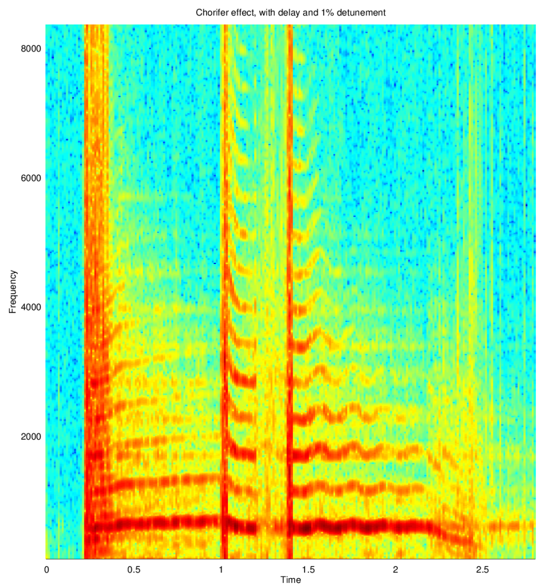


Figure 6: *Spectrogram of ChorIFer effect.*

It is important to be aware that the AM/FM decomposition produces synchronized IA and IF signals. By delaying the IF signal we are desynchronizing the IA and IF, and this can cause the sound to have a noisier attack. Both delayed and non-delayed versions are available in the examples. The delay periods experimented were around 20 milliseconds (the usual range for the chorus is from 10 to 25 ms [24]).

The ChorIFer mathematical expression is given by

$$x(t) = \frac{1}{2N+1} a(t) \sum_{k=-N}^{N} \cos\left(2\pi(1+kD)\int_0^t f(\tau - L_k)d\tau\right), \tag{12}$$

where $2N+1$ is the number of signals added to achieve the effect, $D$ is a detunement value, and $L_k$ is the delay for each version of the signal.

### 3.5. WahIFer

The classic and ubiquitous wah-wah effect is produced by adding to a signal a bandpass version of it, where the center frequency of a narrow bandpass varies over time [24]. Typically the center frequency is foot controlled with a pedal, but an alternative is to use a low frequency oscillator for it. This configuration is known as the auto-wah [24].

In the WahIFer the bandpass filtering is applied to the IF estimated signal. After that this processed IF is used along with the IA for the resynthesis, and added to the input signal. Notice that in the WahIFer the bandpass is not cutting out frequencies directly from an audio signal, but by smoothing the IF function. In such a way, the filter controls the generation of partials resultant of the frequency modulation process. In our experiments, frequencies around 1 Hz lead to the more interesting sonorities. However, even with this low value for the wah-wah filter swap artefacts of frequency modulation are present (Figure 7). Signals produced with higher values features noise and aliasing. Different sets of values are illustrated in the audio examples.

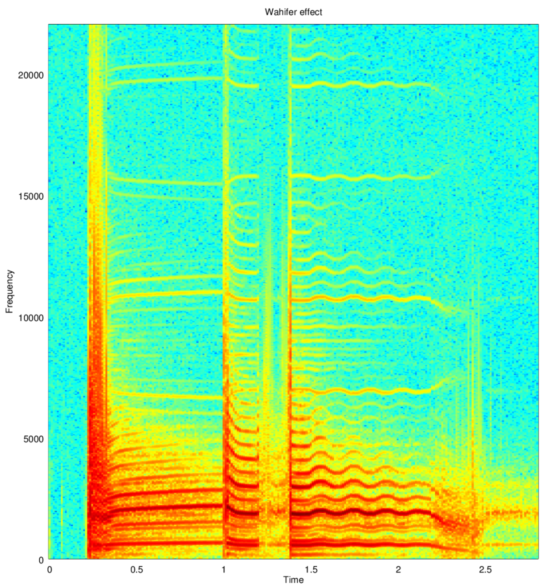


Figure 7: *Spectrogram of WahIFer effect.*

The expression for the WahIFer effect is given by

$$x(t) = a(t) \cos\left(2\pi \int_0^t h(\tau) * f(\tau) d\tau\right), \qquad (13)$$

where $*$ is the convolution operator and $h(t)$ is the impulse response for a wah-wah filter. A comprehensive description of a possible implementation of a wah-wah filter is described in [24]. We used this filter in our experiments.

## 4. CONCLUSIONS

In this work we presented different versions of long known audio effects. Instead of applying the processing directly on the signal, we decompose it into an amplitude envelope and instantaneous frequency using the Hilbert Transform. Then we process the IF signal and resynthesise the signal. The use of AM/FM decomposition within the context of non-linear sound synthesis techniques can open up a variety of avenues for DAFx design, some of which we have presented here. As we are applying these effects to analysis signals, the outcome is different from the application in their usual context as "surface" operations. For instance, in the WahIFer technique, the filter acts in the signal which ultimately is used in a non-linear operation, and thus it has an indirect effect on the output. While most of the effects discussed are achieved by manipulation of the IF, it is possible to envisage a whole suite of transformations that will act on the IA.

While this kind of demodulation has been criticised by the speech processing literature, it yields some interesting possibilities when brought into the context of DAFx for electronic music applications. In this environment, the requirements are not so narrowly defined (for instance, intelligibility is not a factor here), and a more creative approach to design sound transformations can be applied. We have demonstrated that there is enough scope for a variety of useful musical DAFx.

The main aim of this paper was to provide an initial exploration of AM/FM transformations. The ideas presented here can be used within any kind of single or multicomponent decomposition scheme, leading to different results according to the estimated signals. There is scope for the development of these techniques presented here in more complex systems, which would consider sub-band decomposition and coherent detection. In fact, multi-band AM/FM audio effects based on different processing for the AM/FM estimations of each sub-band seem to be a rich tool for the creation of new DAFx and variation of classic ones. We understand that there is still space for further investigation, and encourage the use of the code that accompany the paper, so more AM/FM DAFx can be proposed.

## 5. ACKNOWLEDGMENTS

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