# A Review of Non-Linear Approaches for Wave Energy Converter Modelling

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***Abstract*—The wave energy industry has grown considerably over the past two decades, developing many different technologies, but still has not achieved economic viability. Economic performance can be significantly assisted through the optimisation of the device intelligence, implemented as a control algorithm, including precise hydrodynamic models able to reproduce accurately real device motions. Wave energy converters (WECs) are designed to maximize power absorption with large motions, suggesting significant non-linear behaviour, and so precise non-linear models are imperative. Although similar modelling philosophies could be applied to the wide variety of WECs, the dominant physical phenomena vary from one device to another. As a consequence, different non-linear effects should be considered in each system, but only those being relevant should be chosen to avoid extra computational costs and potential numerical problems. This paper studies different nonlinear hydrodynamic effects and their influence on the various types of WECs, and suggests a critical classification of different methods to articulate nonlinear effects.**

***Index Terms*—Wave Energy, Nonlinear Modelling, Computational Fluid Dynamics, Boundary Element Methods, State-Space Models**

## I. Introduction

Many wave energy device concepts have been developed based on different principles, but none have been commercially completed yet, which requires maximising economic return in the form of energy/electricity and converted wave power across the full range of sea states, to compete with other technologies within the energy market.

The full range of sea states include highly nonlinear and extreme sea conditions that some devices manage by switching from power production mode to survival mode, to avoid potential structural damage. However, during power production mode alone, there are still a wide range of wave conditions. Although nonlinear dynamics are assumed in survival mode, there is some uncertainty as to how relevant nonlinear dynamics are in power production mode.

Some evidence [1]–[5] suggest the need for nonlinear models over some sea conditions and demonstrate not only how linear models can overestimate power production, but also their inaccuracy in reproducing the behaviour of wave energy converters (WECs) over the full range of sea conditions.

This evidence leads to challenge typical approaches to modelling wave energy converters and suggests a scenario divided into three different regions: linear region, nonlinear region and highly nonlinear region, as Figure 1 illustrates. Thus, nonlinear models will also be essential within the second region for accurate simulation of device motion, power production assessment and model-based control design. This paper focusses on Region 2, in order to shed some light on nonlinear modelling methods by presenting some works from the literature, introducing different hydrodynamic nonlinearities and analysing different existing models considering nonlinear effects.

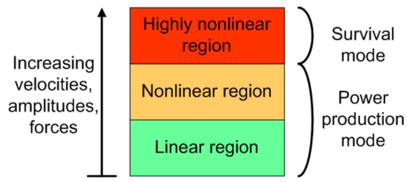


Fig. 1. Different operating regions for wave energy devices

The paper is organized as follows: Section II identifies most relevant nonlinear effects and analyses their relevance for each type of device while Section III suggests a critical classification of the existing methods to articulate nonlinear effects. Finally, conclusions are drawn in Section IV.

## II. Separation of forces and their relevance

In this section, the hydrodynamic modelling problem is subdivided into different forces, to analyse their characteristics and relevance. Each force can be a source of nonlinearities and so is important to analyse it not only independently, but also its influence in the whole system. Figure 2 illustrates this problem in a block diagram, where each block can be nonlinear and where the bi-directional arrows reflect the interdependence of components.

When analysing nonlinear systems, time domain models are required. Equation (1) represents all the forces acting on a wave energy device, without specifying the way they are connected

$$M\ddot{X}(t) = f(F_g, F_{FK}(t), F_{diff}(t), F_{rad}(t), \\ F_{vis}(t), F_{PTO}(t), F_{moor}(t), F_{add}) \quad (1)$$

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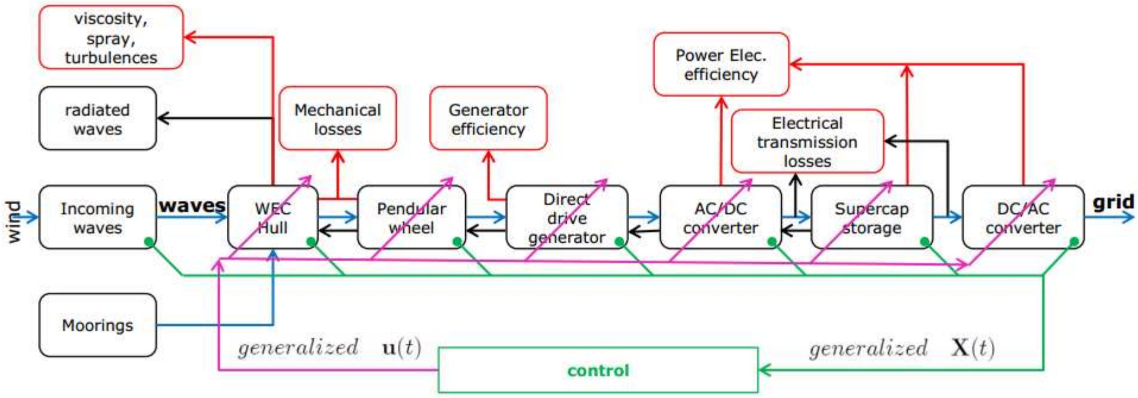


Fig. 2. Block diagram of different aspects participating on the SEAREV device [6]

where $F_g$ is the gravity force, $F_{FK}$ is the Froude-Krylov force, $F_{diff}$ is the diffraction force, $F_{rad}$ is the radiated force, $F_{vis}$ is the viscous force, $F_{PTO}$ is the force acting on the structure due to the power take off (PTO) system, $F_{moor}$ the force due to the mooring system and $F_{add}$ is the force corresponding to any other additional force, such as drift, wind, tidal or other body-water interactions.

In some studies, fully nonlinear codes are used [2], [7], so there is no need to combine forces externally. In other works, linear codes are extended to compute nonlinear effects [3], [8] and, in some others, a nonlinear force is modelled separately and then added to the initially linear Cummins equation to get partially nonlinear results [4], [9], [10].

The following subsections analyse different forces separately.

### A. *Incoming waves*

Waves, implemented by the first block in Figure 2, can be represented in many different ways, from linear monochromatic waves to irregular and fully nonlinear (including viscous effects) waves.

Regular, linear waves can be extended to nonlinear waves using, for example, higher-order Stoke's water waves or Rienecker-Fenton's theory [11]. The most established way of describing real sea-states is the Fourier analysis of records taken in different sites, using different spectra such as Pierson-Moskowitz [12] or the JONSWAP [13] spectra.

While Fourier analysis is well established, other methods may offer more insight into the wave physics, especially in non-stationary and highly non-linear conditions. [14] presents a method, where the key part is the empirical mode decomposition (EMD), and [15], [16] present another method, based on a fast iterative algorithm, to compute the Dirichlet to Neumann operator, able to generate numerical schemes for simulation of fully nonlinear, non-breaking, three dimensional waves, to be implemented in a numerical wave tank.

### B. *Froude-Krylov force*

The Froude-Krylov (FK) force is the load introduced by the unsteady pressure field generated by undisturbed waves. It is generally divided into static ($F_{FK_{stat}}$) and dynamic ($F_{FK_{dyn}}$) forces. The static part represents the relationship between gravity and buoyant forces in a static situation with a still ocean, while the dynamic part represents the force of the incident wave.

Linear codes compute the FK force over the mean wetted surface of the body, while nonlinear computation requires the integration of the incident wave pressure and the hydrostatic force over the *instantaneous* wetted surface at each time step, as Eq. (2) shows.

$$F_{FK} = F_{FK_{stat}} + F_{FK_{dyn}} = - \iint_{S(t)} (P_{stat} + P_{dyn}) dS \quad (2)$$

where $S(t)$ is the instantaneous wetted surface of the body, $P_{stat}$ the static pressure and $P_{dyn}$ the dynamic pressure.

The FK representation using mean wetted surface loses accuracy when analysing large motions, where wetted surface varies considerably over time. Froude-Krylov forces over the instantaneous wetted surface have been addressed in different ways by different authors. [17], [18] calcule the nonlinear Froude-Krylove forces based on the water pressure computed at the center of each wet panel of the mesh, i.e. panels being submerged. However, when estimating the instantaneous wetted surface, panels in the border of the free-surface could be partly submerged and partly out of the water, which lead to a misestimation of the wetted surface. That is why [2] computes the pressure over the instantaneous wetted surface, as in Eq. (2), using a very fine mesh to precisely estimate the instantaneous wetted surface, while [2], [3], [19] use a remeshing routine, modifying those panels being partly submerged and partly out of the water.

The linear computation of Froude-Krylov forces can be accurate for small body motions, or even for situations where the device behaves as a wave follower in big waves. Nonlinear dynamic Froude-Krylov forces may then be neglected, as shown in [20], where experimental data is well reproduced just by considering a nonlinear restoring force.

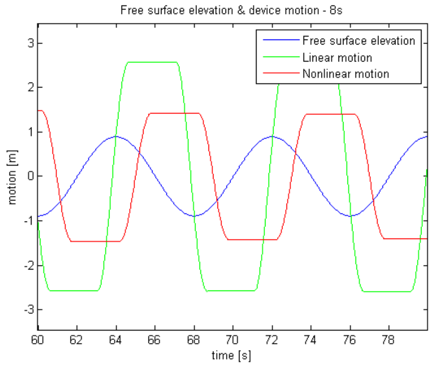


Fig. 3. Device motion for linear and nonlinear simulations with latching control strategy, [21]

However, when the relative motion between the device and the free-surface is big enough, for example when the device resonates due to a control strategy, the influence of the nonlinear dynamic Froude-Krylov on the dynamics of the system becomes important [21]. In such situations, linear models are not able to reproduce the behaviour of the device: they tend to overestimate the motion of the device and, as a consequence, the power absorption. Furthermore, control strategies based on linear models lose performance, as shown in Figure 3.

There is, however, a geometric factor to consider, as pointed out in [21]. Nonlinear Froude-Krylov effects are important, even for small and flat waves, in the case where the cross-section is non-uniform, such as a sphere, while the linear model representation appears to be accurate for the case where the cross-section is uniform.

### C. Diffraction force

The diffraction force is the load associated with the action of the diffracted wave. This disturbance is introduced into the wave system by the presence of the floating bodies, given as:

$$F_{diff} = -\int_{-\infty}^{\infty} K_{diff}(t-\tau)\eta(\tau)dS \qquad (3)$$

where $K_{diff}$ is the impulse response function for diffraction forces and $\eta$ the free-surface elevation.

FK forces, together with diffraction force, make up the total non-viscous forces acting on a floating body. Therefore, linear analysis computes the dynamic FK and diffraction force together as an excitation force, using a convolution product with an impulse response function for the excitation force. [22] mentions that neglecting the diffraction term might be a reasonable approximation to the excitation force, in particular if the body is very small in comparison to the wavelength. In addition, it is computationally convenient to use this approximation.

### D. Radiation force

The radiation force is the hydrodynamic force associated with the motion of the floating body, also expressed by a convolution product (4), according to Cummins equation [23]:

$$F_{rad} = -\mu_{\infty}\ddot{X} - \int_{-\infty}^{\infty} K_{rad}(t-\tau)\dot{X}(\tau)d\tau \qquad (4)$$

where, $\mu_{\infty}$ is the added mass at infinite frequency and $K_{rad}$ the impulse response for the radiation force. In general, a linear approach for the radiation force is reasonably good for devices which are much smaller than the wavelength, but a more precise computation of diffraction-radiation time-derivative terms is possible, by expansion to second-order, using, for example, a Taylor's series expansion of the total potential, as shown in [8].

Some works in the literature [3], [19] have proven that nonlinear computation of radiation-diffraction forces do not make a significant difference for point absorbers (PAs). This is achieved by comparing an extended linear model based on potential theory that computes nonlinear FK forces, diffraction and radiation forces, to another considering only nonlinear FK forces. However, nonlinear diffraction and radiation forces may not be neglectable for all kind of devices, an area which needs further study.

### E. Viscous force

Viscous effects are generally neglected within linear models, as fluid is considered homogeneous, incompressible, inviscid and with an irrotational flow.

However, the nature of viscous drag is nonlinear and can be identified when using wave tank experiments [2], [24]–[26] or fully viscous modelling methods that generally are based on Navier-Stokes equations: Numerical wave tank (NWT) simulations implemented in computational fluid dynamic (CFD) codes [1], [27] or specific hydrodynamic codes [2], [28]. As a consequence of viscous effects, unusual behaviours have been identified in the literature with different devices by using these techniques, which would be unpredictable with traditional linear models.

Another way to consider viscous effects is via the semi-empirical Morison equation [29], by identifying a viscous damping coefficient from experiments or simulations considering viscosity, corresponding to a viscous force

$$F_{vis} = -\frac{1}{2}\rho A C_D \dot{X} \mid \dot{X} \mid - \rho V C_l \ddot{X}, \qquad (5)$$

where $\rho$ is water density, $A$ the cross-sectional area, $C_D$ the viscous drag coefficient, $V$ the volume of the body and $C_l$ the inertia coefficient. Viscous effects can be more or less significant depending on the type and amplitude of the motion and the shape and dimension of the body.

*1) OWC:* Fixed OWC converters are located on the coast, where waves have nomally already broken. Therefore, waves arriving at the chamber with high components of turbulence create shedding vortices around the outer wall of the chamber. A similar phenomenon appears in the case of floating devices, although waves are not broken in that case. [30] demonstrates this phenomenon by simulating the fluid around, and inside, a fixed OWC converter.

In addition, free surface motion inside the OWC chamber cannot be accurately reproduced by linear models and as a consequence, the predicted pressure differential in the chamber cannot be accurately estimated. Usually, a piston model, based on a homogeneous motion of the free surface inside the chamber, is used to predict the pressure differential in commercial codes such as WAMIT [31], AQWA [32] or Aquaplus [33]. However, free surface elevation is rarely constant in the chamber due to viscous effects such as sloshing or vortex generation, and so the linearised model under- or over-predicts the pressure [34].

Viscous effects are therefore relevant for OWC devices, and viscous forces can be inlcuded by models that already incorporate them automatically as in [28], or can be included externally through a calibration process by using experimental data or fully viscous simulations as in [35].

*2) Heaving PA:* Viscous effects appear to have a low influence in small heaving devices [20], [27]. Vortex shedding is generated by the motion of the body relative to the surrounding fluid, but this shedding is not powerful enough to produce significant changes in the behaviour of the body and its power production capacity, as shown in Table I.

*3) Pitching PA:* Motion with large velocity and amplitude encourage the relevance of viscous effects, the SEAREV device [36] being a good example with pitching angles between -10° and 20°. [2] has proven nonlinear hydrodynamic behaviour in some wave tank experiments, such as parametric roll or slamming, which could not be predicted beforehand by linear numerical models. Such nonlinear behaviour can be predicted by nonlinear models which include viscosity effects.

*4) Surging converter:* Surging devices normally consist of large bodies with significant movement relative to the water, which makes the viscous drag force important. Different studies have analysed viscous effects for such devices and important nonlinear behaviour has been observed, such as slamming, as in the case of pitching devices. Turbulent vortices around surging devices is normally strong and has a significant impact on the motion [1]. In addition, due to the slamming phenomenon, a water jet is created as the device re-enters the water [26]. This water jet travels up the face of the flap and is finally ejected when the flap enters the water.

[1], [27] analyse heaving PAs and surging converters and their viscous effects, comparing absorbed power production (APP) rates considering and neglecting viscous effects for each type of device. Table I shows a summary of APP rates with and without viscous terms for heaving PAs and surging converters, and offers good evidence of the relevance of viscous effects for each device.

TABLE I
RELEVANCE OF VISCOUS EFFECTS IN POWER PRODUCTION

| WEC type | Power output | Without viscous term | With viscous term |
|---|---|---|---|
| Heaving PA | APP | 58 kW | 56 kW |
| Surging converter | APP | 114 kW | 74.4 kW |

## F. Power take off force

Different PTO systems are under development for wave energy devices, such as turbines, high-pressure hydraulic systems or direct electrical drives [37]. In many studies, the PTO systems are modelled as a linear spring and damper in parallel, or even just as a single linear damper, consciously avoiding nonlinear effects, which does not mean the relevance of nonlinear effects in PTO systems is low.

*1) Air-turbines:* OWC turbines (Wells turbines being the most popuar) are specially designed for OWC wave energy converters due to the demanding conditions of reciprocating and highly variable flow. Therefore, peak-efficiency for such rectifying turbines is lower (about 0.7-0.8) than for regular turbines, and drops dramatically with movement from the peak efficiency area. [38] compares the pressure drop and flow rates from experimental tests and numerical models as a model validation exercise, and a relation between the pressure drop and flow rate was found as

$$\frac{\Delta p^{\frac{1}{2}}}{q} = K, \tag{6}$$

where $\Delta$p is the pressure drop, $q$ the flow rate and $K$ is a damping parameter.

However, in order to get the maximum pneumatic power, it is important to not only individually optimise the chamber and the turbine, but also the chamber-turbine coupling, which likely includes more nonlinearities.

*2) Hydraulic-turbines:* Hydraulic PTO systems are based on a very well known technology, as they have been used for many years in different industries, especially in hydropower. Likewise in small hydroelectric plants [39], turbines are located between the reservoir of an overtopping device and the mean sea level [40]. In general, the most relevant parameter that drives the choice of the typology of turbine is the head of the plant. For low-head plants, i.e. the Wavedragon device, axial reaction turbines are used, such as Kaplan turbines. In contrast, high-head impulse turbines, such as Pelton turbine, are sometimes used in oscillating devices as alternatives to hydraulic motors at the end of high-pressure hydraulic systems, described in the following subsection.

*3) High-pressure hydraulics:* The high-pressure hydraulic PTO requires both a high-pressure (HP) and a low-pressure (LP) system, along with high- and low- pressure accumulators to feed the hydraulic motor with the working fluid. The hydraulic circuit guarantees energy storage (accumulating working fluid in high pressure) and allows to maintain a constant flow to the hydraulic motor, hence to generate regular power.

In order to describe the nonlinearities of the hydraulic circuit, the model have to consider time-varying gas volume and pressure in the accumulators, the flow rate supplied to the hydraulic machine (motor or turbine), losses on the valves (leakages) and losses on the circuit (viscosity) [9].

*4) Direct drive:* A direct drive generator generally consists of three parts: the armature, the translator and a set of springs attached between the seabed and the armature. [41] shows different available concepts, such as the Snapper concept, the linear permanent magnet synchronous machine, the linear air-cored permanent magnet synchronous machine or the slot-less tubular permanent magnet synchronous machine.

The Snapper is a new concept developed particularly for wave energy devices, whose armature and translator present respectively a series of permanents magnets along thier length, mounted with alternating polarity. The relative motion between the armature and the translator, hence between the two series of magnets is used to generate power. As the translator moves, a magnetic attraction is induced in the armature that moves with the translator. [42], [43] describes different forces, some of which can lead to nonlinear behaviours, that participate in the operation within the machine: interaction of the two sets of magnets, electromagnetic damping due to the current-carrying coils and forces due to losses like eddy currents in armature and translator, saturation and reactive field losses and losses due to the proximity effects.

### *G. Mooring system force*

WECs are subject to drift forces due to waves, currents and wind, therefore they have to be kept on station by moorings. The offshore industries provide examples of many different mooring systems that have been already analysed as moorings for wave energy devices [44].

There are several factors relevant to choose the optimal mooring system: type and dimensions of the device, number of devices and the installation site (seabed depth or conditions and environmental loads). Close to the shore, devices can be tightly moored, while slack moorings are necessary where the seabed is deepe. Nonlinear effects seem to be in general much more significant in the case of slack moorings (whose mooring cables are approximately modelled as catenary lines [45]) than in the case of tightly moored devices [46].

A common modelling approach for mooring system relies on the quasi-static assumption. [45] shows a clear nonlinear behaviour for surge motion of a slack mooring line. In [47] for the slack mooring system and in [46] for the tightly moored system, two-dimensional quasi-static approaches are used in time-domain analysis to consider nonlinear mooring forces.

However, dynamic effects such as cable inertia, viscous drag forces or slow varying forces are ignored with the quasi-static approximation. [48] presents some measurements where the relevance of dynamic effects is demonstrated by comparing experimental results with two different simulations: run by the fully-dynamic software OrcaFlex [49] and a quasi-static simulation presented in [50].

## III. Different nonlinear modelling options

A critical classification of the different modelling options is presented in this section, based on the method used to model the nonlinear problem. Hence, the existing models are organised in three groups (fully nonlinear models, potential flow models and system identification models), where different models are presented and described in each of the groups.

Among the studies in the literature, none of the analyses the whole nonlinear problem as a whole, except the works using fully nonlinear models. However, as mentioned before in Section II, as important as analysing nonlinear effects independently is analysing how to combine them, since due to the nonlinear nature of nonlinear forces, application of linear approaches, such as superposition theory, should not be considered obvious.

### *A. Fully Nonlinear models*

Fully nonlinear models are supposed to consider all the effects of the wave and the wave-body interaction, including viscous effects, studying the fluid flow by solving a set of differential equations known as Navier-Stokes equations. The fundamental basis of almost all problems are governed by the transfer of mass, momentum and heat, described by the following equations: continuity equation (7), equation of motion (8) and conservation of energy (9).

$$\frac{\partial \rho}{\partial t} + \bigtriangledown(\rho u) = 0 \tag{7}$$

$$\frac{\partial u}{\partial t} + (u\bigtriangledown)u = -\frac{1}{\rho}\bigtriangledown p + F + \frac{\mu}{\rho}\bigtriangledown^2 u \tag{8}$$

$$\rho(\frac{\partial \epsilon}{\partial t} + u \bigtriangledown \epsilon) - \bigtriangledown(K_H \bigtriangledown T) + \rho \bigtriangledown u = 0 \tag{9}$$

where $\rho$ is the fluid density, $u$ the velocity vector, $p$ the pressure field, $F$ the external force per unit mass, $\mu$ the fluid viscosity, $\epsilon$ the internal energy, $K_H$ the heat conduction coefficient and $T$ the temperature.

However, equations 7, 8 and 9 cannot be solved analytically, and so numerical discretization will be necessary to obtain a solution. It is here where computational codes enter into play, where complete Navier-Stokes equations are implemented using different discretization methods, turbulence models and solving-algorithms in order to achieve a numerical solution.

In the case of wave energy converters, these computational codes allow simulating wave tank experiments or real-sea tests numerically, by using NWT simulations [51], implemented in CFD codes. These numerical simulations have been used for decades in offshore and ocean engineering for the fluid-body interaction analysis.

Different fully nonlinear codes are presented in the following subsections:

*1) Computational fluid dynamics:* CFD is a branch of fluid mechanics that uses numerical methods and algorithms to solve fluid flow problems. As mentioned in Section III-A, discretization of the problem is essential to solve the equations numerically and lies in three main steps: mesh generation, space discretization and time discretization. The whole numerical procedure is normally divided into three different processes: pre-processing, solving and post-processing.

During the pre-processing, the geometry of the problem is defined and the volume of fluid is discretized generating the mesh, physical modelling is defined and boundary and initial conditions are set. That way, continuous differential equations are discretized into a system of linear algebraic equations, breaking continuity into finite portions in time, by using time-steps, and in space, by using different methods.

Several discretization methods can be found in the literature: the finite element method, the finite volume method, the finite difference method, the spectral element method or the boundary element method among others. Generally, each CFD code uses one of these methods and a specific algorithm to solve the problem. More details about numerical discretization methods are given in [52].

Before the simulation starts, turbulent model needs to be chosen, being one of the most important decisions to make in the pre-processing part, and again several models are available. [53] presents a classification of different models and a description of each of them. Depending on the objective of the analysis, a different model could be the optimal one. Reynolds-average Navier-Stokes (RANS) model is the most widely used method, due to the high computational requirements of other methods like the large eddy simulation (LES) or direct numerical simulation (DNS).

As two different fluids (air and water) take part in the problem, using Navier-Stokes equation-based methods for modelling hydrodynamics of WECs involves the calculation of the free surface in a NWT and the simulation of the turbulent flow. Therefore, a free-surface modelling technique is required and two different techniques have been applied in the analysed works: the tracking method and the interface-capturing method. The tracking method models the free surface as a sharp boundary [54], while the interface-capturing method includes water and air in the mesh, being the volume of fluid (VOF) method [55] the most used technique.

NWT simulations implemented in CFD codes have some advantages and disadvantages when comparing to real wave tank tests, but both appear to be essential in the process towards an optimal design of a WEC. For wave tank experiments, one needs, first of all, a real device to be tested, which is normally a scaled device due to the dimensions of the tank. Despite the great experience designing scaled prototypes based on different similitude coefficients [56], results need to be analysed carefully due to the scale effects. NWT simulations avoid the complexity and costs of creating a real prototype and the scale effects of the tank tests, as full scale devices can be easily simulated numerically. In addition, reflection effects from tank walls can be controlled effectively and a large variety of situations can be studied easily.

[57] presents main advantages and drawbacks of of using CFD methods in the design process of a WEC, being the high computational requirement the main weakness of such methods.

*2) Hydrodynamic approaches for Wave Energy:* CFD codes are normally very general codes used for many and very different applications where fluid-flows are considered. Nevertheless, other specific models especially created for the analysis of wave-structure interactions and based on Reynolds-averaged Navier-Stoke (RANS) equations are also available. Two of the specific models found in the literature are presented in Sections III-A2a and III-A2b: *spectral wave explicit Navier-Stokes equation (SWENSE)*, developed by the Hydrodynamic and Ocean Engineering group of the Ecole Centrale de Nantes (ECN) [58] and *IH2VOF* developed at IH Cantabria [59].

*a) SWENSE:* The SWENSE approach based on RANS equations combines the advantages of potential and viscous solvers by solving each physical problem with the right tool: propagation of the waves with the potential solver and diffraction-radiation with the viscous solver [60].

*b) IH2VOF:* The IH2VOF model was initially created for coastal structures and includes realistic wave generation, second order generation and active wave absorption. It solves the 2-Dimensional wave flow by the resolution of the volume-averaged Reynolds averaged Navier-Stokes (VARANS) equations, based on the decomposition of the instantaneous velocity and pressure fields and the k-$\epsilon$ equations for the turbulent kinetic energy (k) and the turbulent dissipation rate ($\epsilon$) [61].

Regarding to wave energy applications, many different codes based on Navier-Stokes equations have shown their capacity to reproduce wave-structure interactions and viscous effects reasonably well even in the case of large motions [2], [62], [63], but their computational requirements are still too high, which makes difficult to use CFD codes for long simulations. However, fully nonlinear hydrodynamic codes could be very useful for identification purposes, as shown later in Section III-C or to estimate viscous effect coefficients to be included into other hydrodynamic models based on other methods by using, for example, the Morison equation presented in Section II-E.

*3) Smoothed-particles hydrodynamics:* The smoothed-particles hydrodynamics (SPH) method is a purely Lagrangian technique that divides the fluid into a set of discrete elements or particles, instead of using a mesh, and has been successfully applied to a broad range of problems [64], [65]. The discrete elements or particles are transported with the local velocity and they carry the information of the field, such as pressure or density.

In order to define continuous fields, field variables are smoothed by using a kernel function. Thus, the physical quantity of any of the field variables can be obtained by summing the relevant properties of all of the particles within the range of the kernels. This means, for example, that pressure at any position $r$ depends on the pressure of all the particles within a radial distance $2h$ of $r$, being $h$ the smoothing length.

The contribution of all the particles within the radial distance $2h$ is not the same. This contribution is weighted related to the distance between the analysed particle and the contributor particle, and their density, and is mathematically governed, again, by the kernel function. As a consequence, the field variable is known at a discrete set of points within this radial distance and so the field variable can be defined as follows at the position $r$:

$$A(r) = \sum_{j}^{N} \frac{m_j}{\rho_j} A(r_j) W(r - r_j, h) \tag{10}$$

where $A(r)$ is any field variable at any position $r$, $N$ the number of contributor particles, $m$ and $\rho$ are respectively the mass and the density associated to the particle and $W$ the kernel function.

Different SPH techniques can be applied, as shown in [66], depending on the characteristic of the flow and the problem to be studied.

### B. Potential flow models

Potential flow models, also known as boundary element methods (BEMs), are based on potential theory, where the potential flow describes the velocity flow as the gradient of the velocity potential. This potential of the incident flow can be split into three different parts in order to study the water-structure interaction: undisturbed incident potential ($\Phi_I$), diffracted potential ($\Phi_{diff}$) and radiated potential ($\Phi_{rad}$), being their sum the total potential of the incident flow ($\Phi_{tot}$). Hence, the pressure of the total incident flow acting on the body can be obtained by deriving this total potential in the Bernoulli's equation.

Some well known hydrodynamic codes, such as WAMIT [31], AQUAPLUS [33] or NEMOH [67] in frequency-domain, or ACHIL3D [68] in time-domain, are based on potential flow theory, being all of them linear codes.

In order to analyse nonlinearities, specific nonlinear considerations can be added to the linear hydrodynamic model, resulting in partially nonlinear models in time-domain. First to be presented is the linear method, to which different improvements are added to consider nonlinear effects.

*a) Linear model:* The linear model accepts the assumptions of inviscid fluid, irrotational and incompressible incident flow, constant wetted surface and small body motion amplitudes.

Pressure acting on the body can be presented as a sum of different potentials (undistrubed, diffracted and radiated), dividing the total force acting on the body into different forces, where second order diffraction-radiation terms are also considered.

$$\begin{aligned} P = &-\rho g z - \rho\frac{\partial \phi_I}{\partial t} - \rho\frac{\mid \nabla\phi_I \mid^2}{2} \\ &- \rho\frac{\partial \phi_{diff}}{\partial t} - \rho\frac{\mid \nabla\phi_{diff} \mid^2}{2} - \rho\frac{\partial \phi_{rad}}{\partial t} - \rho\frac{\mid \nabla\phi_{rad} \mid^2}{2} \\ &- \rho \nabla \phi_I \nabla \phi_{rad} - \rho \nabla \phi_I \nabla \phi_{diff} - \rho \nabla \phi_{diff} \nabla \phi_{rad} \end{aligned} \tag{11}$$

One can observe some nonlinearities, such as quadratic and second-order radiation-diffraction terms, in Eq. (11). For the linear case, all the nonlinearities are disregarded by neglecting quadratic and second order terms and computing pressure over a constant wetted surface as follows,

$$\begin{aligned} M\ddot{X} = F_g - K_H X - \int_{-\infty}^{\infty} K_{Ex}(t-\tau)\eta(0,0,\tau)d\tau \\ - \mu_\infty \ddot{X} - \int_{-\infty}^{\infty} K_{rad}(t-\tau)\dot{X}(\tau)d\tau \end{aligned} \tag{12}$$

Thus, diffraction force and dynamic FK forces are summed into excitation force, by using the excitation force kernel $K_{ex}$. $K_H$ is the hydrostatic stiffness that gives the relation between the gravity force and the static pressure. In the case of the radiation force, it is expressed as a convolution product based on Cummins equation [23], where $\mu_\infty$ is the infinite frequency added mass parameter and $K_{rad}$ the reduced radiation impulse-response function.

*b) First nonlinear extension:* From this linear basis, the model can be extended to consider nonlinear effects. In this first extension, nonlinear FK forces are considered by integrating the pressure over the instantaneous wetted surface. This means that wetted surface needs to be estimated and remeshed at each time-step, as described in Section II-B. This time, static and dynamic FK forces are summed into the instantaneous FK force, while other forces, such as radiation or diffraction, remain linear and are computed separately.

$$\begin{aligned} M\ddot{X} = F_g - \int_{S(t)} (P_{stat} + P_{dyn})\overrightarrow{n} dS \\ - \int_{-\infty}^{\infty} K_{diff}(t-\tau)\eta(0,0,\tau)d\tau \\ - \mu_\infty \ddot{X} - \int_{-\infty}^{\infty} K_{rad}(t-\tau)\dot{X}(\tau)d\tau \end{aligned} \tag{13}$$

*c) Second nonlinear extension:* The second extension is another step, but not the last, towards a fully nonlinear model. In this case, a more precise computation of radiation-diffraction forces is presented. Gilloteaux, [8], proofs that Taylor series extension of the total potential, including not only quadratic but also second-order terms, can be performed over the mean position of the body.

First extension improves considerably the results by considering nonlinear FK forces, while the improvement of the second extension is quite low. In addition, this second model requires a recalculation of hydrodynamic parameters at each sampling instant, resulting in a high computational overhead.

In any case, linear or nonlinear, the time-domain solution of a BEM simulation is based on Cummins equation [23] as shown in Eq.(12) and Eq.(13), which needs to solve the convolution integral by using any of the following techniques: direct convolution [69], truncation of the convolution [20] or state-space approximation in frequency- [70], [71] or time-domain [72], [73].

### C. System identification models

Other alternative modelling approaches are system identification models, which are well established in the control system community, where really complex models are determined by input/output data. System identification models use statistical methods to build mathematical models of dynamic systems from measured data, being particularly interesting for very complex systems, where the physical principles can be too complicated to formulate.

Every identification method consists basically of

- selecting a series of representative data of the system to be reproduced,
- determining the structure of the model (model type and order) and
- defining the fitting criteria.

The choice of the structure of the model is probably the key point in order to create a representative model. There are several possibilities of model structures, from a structure based on the knowledge of the physical principals (white-box) of the system to a structure completely regardless of the physical principals (black-box) [74]. Parameters of the selected model are estimated by using the specified fitting criteria.

One of the difficulties of theses models is that input/output data needs to represent the system significantly. In addition, it is not always easy to isolate the required data from the hydrodynamical simulations (either BEM or CFD), where there is little transparency between the physical system and the model.

If the system is considered linear, then an autoregressive with exogenous input (ARX) model can be chosen, while if the system is nonlinear, the way to analyse the nonlinearity (the form) and its complexity have to be selected. Many different options, from block-structured systems [75] to neural networks [76], are available.

Some models that can be used for nonlinear modelling of the wave energy converters are presented in the following subsections:

*1) Hammerstein and Feedback block-oriented models:* A nonlinear static block is added to the linear block in order to consider nonlinear effects. In the case of the Hammerstein model, the static bloc $r$ is connected in cascade, while in the case of the feedback block-oriented model, the structure is characterized by a negative feedback [77]. Both models are shown in Figure 4.

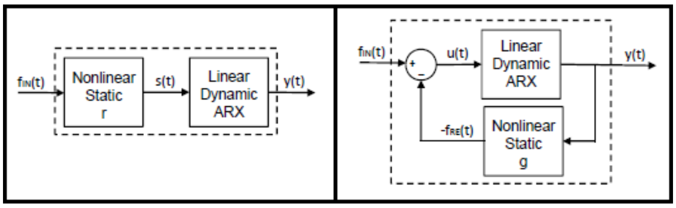


Fig. 4. Block diagram of the Hammerstein model on the left and the feedback block-oriented model on the right [75]

Both blocks of the structures in Hammerstein and feedback models, the ARX block and the static block, are linear in the parameters and they are identified by using linear regression and least squares. It is interesting that a physical meaning was found for the static block in Hammerstein and Feedback models by [75]. For the Hammerstein model, the$r(f_{IN}(t))$ was found to be interpreted as the inverse of the restoring force, while in the feedback model $g(y(t))$ was the negative of the restoring force. Therefore, the static block in both models can be identified separately from the linear dynamic block, from the knowledge of the restoring force as a function of the body position.

*2) Volterra model:* The Volterra method is a black-box model having a nonlinear input-output relationship, but it is always convergent and remains linear in parameters.

[78] presents a Volterra-type model that uses a polynomial nonlinearity to model the relationship between free-surface elevation and excitation force in wave energy devices.

*3) Artificial neural networks:* Artificial Neural Networks (ANNs) are statistical learning algorithms used to estimate or approximate complex systems or functions that depend on several and generally unknown inputs.

ANNs are generally presented as systems of interconnected neurons, simple artificial nodes, consisting of sets of adaptive weights. ANNs are capable of approximating nonlinear functions of their inputs, where adaptive weights are connection strengths between neurons.

There is not a single definition or model of ANNs and there exist different structures. Hence, through the modelling procedure, different conditions, such as the number of layers and the number and type of neurons in each layer, must be determined to define the required structure of the ANNs.

[76] proposed to use a multi-layer perceptron (MLP) form including delay elements at the network input layer and uses an optimisation algorithm to determine parameters of the structures. NWT is used to produce hydrodynamic data, where three identification tests are performed: one wave excitation (irregular sea-state) and two types of direct force inputs (chirp and random amplitude random period (RARP).

## IV. Conclusion

Different aspects of nonlinear modelling are analysed in this paper. Section II studies different nonlinear forces or effects that affect WECs and their relevance:

- Nonlinear waves are an important step in the way towards the 'real' sea state simulation and their influence on the behaviour of the device is also significant.
- Computation of nonlinear FK forces seems to be essential for controlled point absorbers, where the big amplitudes of the relative motion between the device and the free-surface makes the variation of the wetted surface important. In addition, control strategies need to be adapted to the nonlinear models.
- Nonlinear diffraction or radiation forces seem to be negligible, at least for devices much smaller than the wavelength.
- In the case of viscous effects, there is big uncertainty. Some studies show that for flap-type surging converters or pitching point absorbers, viscous effects not only are relevant, but also lead to nonlinear behaviours such as slamming or parametric roll, that would never be observed in linear models. Viscous effects for heaving point absorbers nonetheless, seem to be weak, even if effects like shedding vortex generation exist.
- Nonlinear effects of any of the PTO systems are significant enough not to be simply modelled as linear dampers.

- Dynamic effects of mooring lines seem to be important enough to be considered, instead of using simple quasi-static approximations, especially within the design process.

Regarding the different modelling methods presented in Section III, CFD codes are probably the most realistic ones, but their computational costs make them prohibitive. Nevertheless, CFD can be very useful for parameter identification, since viscous effects are also computed. SPH methods seem to be adequate for extremely nonlinear effects, such as impacts or slamming, but not as a complete alternative to CDF codes, due to their bigger computational weight.

BEM, or potential theory based codes, are able to compute some nonlinear forces accurately, but they are not able to compute viscous effects, although viscous effects could be added by using the Morison equation. However, the computational load can become heavy when nonlinearities are analysed.

Another useful option for nonlinear modelling is based on Cummins equation with a superposition of different forces, where nonlinear effects are studied independently using any of the methods shown in the paper and introduced to the model at a later stage as additional loads. However, the possibility of using superposition with nonlinear terms need further work, especially with several nonlinear terms.

Finally, system-identification models still need to be further developed, but they have already been used in different fields with great success and they could be an option in the future, as the few results available in the literature promise.

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