Tarek Mohamed Khadir* and John Vincent Ringwood

Higher Order Predictive Functional Control Versus Dynamical Matrix Control for a Milk Pasteurisation Process: Transfer Function Versus Finite Step Response Internal Models

Abstract: Predictive functional control (PFC), a model predictive control algorithm, has been proven to be very successful in a wealth of industrial applications due to its many laudable attribute, such as its simplicity and intuitive appeal. For simple single input single output processes, PFC applications use a first-order plus delay internal model and, as long as such models improve the control over classical control strategies, then their use remains justified. In this paper, a higher order internal PFC model is considered in order to reduce any possible plant-model mismatch, where the internal model is formulated as a series of cascaded or parallel first-order systems. The control approach is compared to a more conventional over parameterized dynamical matrix control (DMC) approach, used extensively for Multi-Input Multi-Output systems in the petrochemical industry. This paper demonstrates the benefits of the PFC higher order formulation for a typical milk pasteurisation plant, with significant improvements in the variances of both controlled and manipulated variables when compared to a first-order PFC. In this aspect, the higher order controller competes well with DMC performances, however, using a much more simpler and compact internal model form.

Keywords: model predictive control, predictive functional control, milk pasteurisation

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1 Introduction

Model predictive control (MPC) grew substantially in popularity and its field of application diversified substantially since its first applications in the refining and petrochemical industry [1, 2]. It is also reported in Ref. 3 that MPC has been used in over 2,500 industrial applications in the chemical, pulp and paper and food processing industries, from a total of 4,500, aside from the traditional refining and petrochemical sector. The goal behind the emergence of using MPC, and predictive functional control (PFC) in particular, is to reduce occurring variance in the controlled variable (CV). This may allow lowering of the control setpoint target, therefore reducing cost and saving energy. MPC was found to be very effective in tackling such control problems, due to the ability of prediction, given by the embedded model, as well as effective constraint handling.

The milk pasteurisation procedure may be considered as an appropriate case for highlighting MPC benefits, since higher setpoints are frequently specified to avoid, in the case of disturbances, any violation of the typical 72°C pasteurisation temperature limit. This, most of the times, results in heating as high as 76°C. Heating at such temperatures may alter the milk constituents, especially if the temperature variance is positive. The goal for an MPC controller is then to decrease the variance in the CV and shift the setpoint target towards lower temperatures, as shown in Figure 1. Note that, when the milk is at the pasteurisation temperature, it has to be held for at least 15 s in order to ensure the destruction of all unwanted microorganisms and bacteria. The pure time delay introduced, as well as the existence of input constraints on the manipulated variable (MV) (generally a steam valve to heat the medium), furthermore encourages the use of MPC, as it is suitable for such control problems [4].

Having established the motivation for the use of MPC in milk pasteurisation control, we are left with a range of

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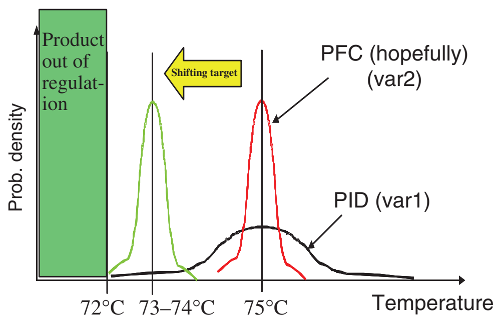

Figure 1 Goal of MPC: squeeze the variance shift the target.

MPC algorithms to choose from. In this paper, PFC is considered for the following reasons:

- PFC was originally designed for improving the control efficiency of single input single output (SISO) processes and has been successfully tested with these types of models for milk pasteurisation control in
- PFC has proven its efficiency in dealing with heat exchange processes, including in the dairy industry (i.e. chemical reactors, spray dryers, climate control, furnaces, etc. [3], [5], [16]).
- PFC has a relatively simple and robust formulation, which makes industrial implementation realizable.

However, other researchers [6] have applied dynamical matrix control (DMC) to milk pasteurisation, but these authors feel that the more compact ARX/ARMAX internal model structure of PFC is more attractive than the over parameterized finite impulse response DMC internal model, especially for the SISO case. The DMC control approach will be developed as a benchmark controller.

The main focus of this paper is to examine the use of higher order internal PFC models, for improved process prediction, in the context of milk pasteurization. The paper continues with a general description of PFC (Section 2), followed by the derivation of the control laws using higher order internal models in parallel and cascaded forms (Sections 3 and 4), since these are not currently widely available in the literature. The benchmark used, DMC control strategy, is developed in Section 5. The resulting control laws (DMC and PFC) are then applied to the pasteurization problem in Section 6, with a range of simulation results used as illustrative performance. Finally, conclusions are drawn in Section 7.

2 Basic principles of PFC

PFC was originally developed in 1993 and subsequently reported in Ref. 7, primarily to tackle industrial SISO processes outside the petrochemical industry. PFC has been used successfully in more than hundred industrial applications in furnaces, aerospace, automotive and food industry [3]. A reason for the success of PFC is its primitive formulation, simplicity and good robustness to process/model mismatch. Like other MPC strategies, PFC is based on four basic principles presented in the following subsection.

2.1 PFC basic principles

2.1.1 Internal model

The internal model is usually “independent model”, where the model output relies only on the past outputs of the process *model*, along with the process MV. In most cases, a first-order plus delay model is used.

2.1.2 Reference trajectory

PFC uses an exponential reference trajectory, as in eq. (1), which requests only one initialisation point, and generally gives responses without overshoot.

$$\lambda = e\left(-\frac{T_S}{T_R}\right) \tag{1}$$

where T_S is the sampling period and T_R is the closed loop response time (CLRT), to be specified.

2.1.3 Computation of the MV

PFC structures the MV by projecting it onto a functional basis [7]. Usually, the elementary case is considered, where the MV is structured by a zero-order basis element (a step input).

2.1.4 Auto-compensation

This involves assessment of the accuracy of the model using the plant output [7], with the difference forming a correction to the prediction model.

2.2 Tuning in PFC

According to the four principles of PFC (Section 2), tuning is a function of the order of the basis constructing the

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MV, the reference trajectory, the control horizon and the CLRT value.

A general idea of the influence of the PFC parameters is given in Table 1, where the influence of various PFC parameters on precision, transient response and robustness are graded between 0 (indicating minimum influence) and 100 (indicating influence).

Table 1 Effect of PFC parameters in tuning.

	SS resp.	Transient resp.	Robustness
Basis function	100	0	0
Reference trajectory	0	100	50
Coincidence horizon	0	50	100

In most cases, an exponential reference trajectory is chosen along with a single coincidence horizon point ($H = 1$) and a zero-order basis function [8]. Considering the known open loop response time (OLRT) of the system, one can choose the CLRT value given by the ratio OLRT/CLRT. This ratio then becomes the major tuning parameter shaping the system output and MV, dictating how much overshoot occurs. For slow processes, for example, heat exchange systems, a ratio of 4 or 5 is found most suitable [9].

2.3 Constraint handling in PFC

PFC uses a simple (but non-optimal) solution to handle constraints. For input constraints, that is, the maximum and minimum steam flow in the case of the pasteuriser, it consists of feeding the model, not with the MV calculated by the PFC algorithm, but with its constrained value. The model output y_M is calculated using the constrained MV u_A, see Figure 2.

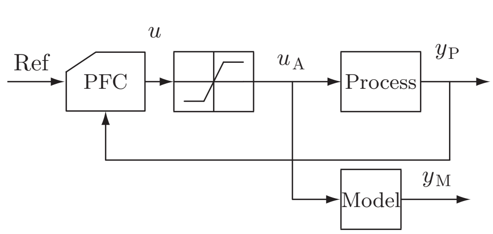

Figure 2 Limitations on the MV.

The model output y_M is computed using u_A, the constrained value of u, calculated at current time, k. Thus, the controller will still see the process as linear, and the prediction is still valid. The control is non-optimal, since the future constraints that can affect the control are not taken into account.

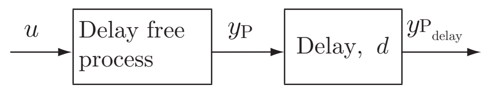

Figure 3 Process with time delay.

2.4 Case of a process with a pure time delay

In the linear case, a process with a pure time delay can be expressed in terms of a delay-free part plus a delay added at the output, as in Figure 3. The value $y_{P_{delay}}$ at time k is measured, but not y_P. In order to take into account the delay in a control law formulation, prior knowledge of the delay value d is needed. y_P can be estimated as:

$$y_P(k) = y_{P_{delay}(k)+y_M(k)-y_M(k-d)} \quad (2)$$

In addition to the basic PFC formulation, the above modification can be applied to the higher order developments in Sections 3 and 4.

3 Higher order PFC using a parallel decomposition

3.1 Decomposition

A generic higher order model, obtained with a zero-order hold (ZOH), $G_M(z)$, can be synthesized by m parallel first-order systems, as shown in Figure 4. The model output is given by y_M, and the outputs of each branch are given by y_1 to y_m. The process is single input, which means that each branch is subject to the same MV, u. The control law obtained for PFC is given in Section 3.2.

If the internal model, $G_M(z)$, is strictly proper, then the discrete parallel decomposition is given in eq. (3).

$$G_M(z) = \sum_{i=1}^{m} \frac{(1-\alpha_i)z^{-1}K_i}{1-\alpha_i z^{-1}} \quad (3)$$

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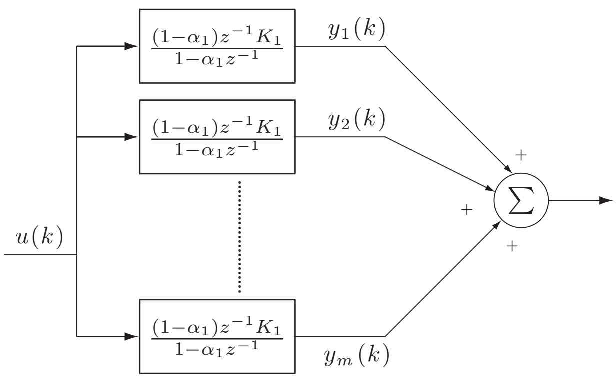

Figure 4 *m*th-order parallel model.

where

- K_i: dc gain of each first-order system.
- u: MV.
- $\alpha_i = e\left(\frac{-T_S}{\tau_i}\right)$, where T_S is the sampling period.
- τ_i is each first-order system time constant in the Laplace form [eq. (4)].

This is equivalent to a generic higher order model on a parallel form in the Laplace domain given by eq. (4).

$$G_M(s) = \sum_{i=1}^{m} \frac{K_i}{1+\tau_i s} \tag{4}$$

3.2 PFC design

3.2.1 Output prediction

From Figure 4, the model output $y_M(k)$ is given by eq. (5).

$$y_M(k) = y_1(k) + y_2(k) + \cdots + y_m(k) \tag{5}$$

The finite difference equation equivalent to the model in eq. (5) is given by:

$$y_i(k) = \alpha_i y_i(k-1) + K_i(1-\alpha_i)u(k-1) \tag{6}$$

$$1 \le i \le m$$

replacing eq. (6) in eq. (5) gives the model output, eq. (7):

$$\begin{aligned} y_M(k) = {} & \alpha_1 y_1(k-1) + \alpha_2 y_2(k-1) \\ & + \cdots + \alpha_m y_m(k-1) + [K_1(1-\alpha_1) \\ & + K_2(1-\alpha_2) + \cdots + K_m(1-\alpha_m)] \\ & u(k-1) \end{aligned} \tag{7}$$

Regrouping terms containing u and terms containing only the auto-regressive part of the model y, as in eq. (8).

$$y_M(k) = \sum_{i=1}^{m} \alpha_i y_i(k-1) + \sum_{i=1}^{m} K_i(1-\alpha_i)u(k-1) \tag{8}$$

The response $y_M(k)$ may be then divided into two parts:

$$y_A(k+H) = \sum_{i=1}^{m} \alpha_i^H y_i(k) \tag{9}$$

and

$$y_F(k+H) = \sum_{i=1}^{m} K_i\left(1-\alpha_i^H\right)u(k) \tag{10}$$

where $y_A(k+H)$ is the future auto-regressive prediction (free response), and $y_F(k+H)$ is the predicted forced response. Only the non-realigned nature of the internal model, inherent to PFC, permits such an easy decomposition [10].

3.2.2 Reference trajectory formulation

The future process output is specified by the reference trajectory, initialised on the real process output, y_P. The reference trajectory used in PFC is generally an exponential given by:

$$y_R(k+H) = C(k) - \lambda^H(C(k) - y_P(k)) \tag{11}$$

where λ is given in eq. (1).

3.2.3 Predicted process output

At the coincidence horizon H, the estimated process output, $\widehat{y}_P$, is set equal to the reference trajectory.

$$y_R(k+H) = \widehat{y}_P(k+H) \tag{12}$$

where the process output estimate is given by:

$$\widehat{y}_P(k+H) = y_M(k+H) + (y_P(k) - y_M(k)) \tag{13}$$

Replacing $y_M(k+H)$ with the expression from eq. (8), with $k = k+H$, we obtain:

$$\widehat{y}_P(k+H) = \sum_{i=1}^{m} y_i(k+H) + \left(y_P(k) - \sum_{i=1}^{m} y_i(k)\right) \tag{14}$$

3.2.4 Computation of the control law

At the coincidence point $y_R(k+H) = \widehat{y}_P(k+H)$, and using eqs (9), (10) and (14), we obtain:

$$\begin{aligned} & C(k)(1-\lambda^H) - y_P(k)(1-\lambda^H) + y_1(k)(1-\alpha_1^H) \\ & + y_2(k)(1-\alpha_2^H) + \cdots + y_m(k) \\ & (1-\alpha_m^H = (K_1(1-\alpha_2^H) + K_2(1-\alpha_2^H) + \cdots \\ & + K_m(1-\alpha_m^H)u(k) \end{aligned} \tag{15}$$

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Rewriting the expression in eq. (15), we end up with the control law given in eq. (16):

$$u(k) = \frac{(C(k) - y_P(k))(1-\lambda^H)}{\sum_{i=1}^{m} K_i(1-\alpha_i^H)} + \frac{\sum_{i=1}^{m} y_i(k)(1-\alpha_i^H)}{\sum_{i=1}^{m} K_i(1-\alpha_i^H)} \tag{16}$$

Proper systems can also be handled by performing a further decomposition, as illustrated in Ref. 11.

3.3 Handling of added disturbances

For the ARMAX case (inclusion of a output disturbance), a decomposition of the same form as above can be specified as:

$$y(s) = \sum_{i=1}^{m} \frac{K_i}{1+\tau_i s} u(s) + \sum_{i=1}^{m} \frac{K_i'}{1+\tau_i s} \upsilon(s) \tag{17}$$

Following the steps of Section 3.2, the corresponding PFC control law is given as:

$$u(k) = \frac{(C(k) - y_P(k))(1-\lambda^H)}{\sum_{i=1}^{m} K_i(1-\alpha_i^H)} + \frac{\sum_{i=1}^{m} y_i(k)(1-\alpha_i^H)}{\sum_{i=1}^{m} K_i(1-\alpha_i^H)} - \frac{\sum_{i=1}^{m} K_i'(1-\alpha_i^H)}{\sum_{i=1}^{m} K_i(1-\alpha_i^H)} \upsilon(s) \tag{18}$$

Specification of the disturbance dynamics in a parallel from is not crucial to the determination of the controller solution, as long as disturbance is subtracted in a feed-forward manner [as in eq. (18)]. However, the choice as in eq. (17) leads to a particularly elegant control solution.

4 Higher order PFC using a cascaded decomposition

4.1 Decomposition

A second possibility is that the internal model is given in a cascaded form. However, in general:

$$\begin{aligned} Z_{ZOH}[G_1(s)G_2(s)\ldots G_m(s)] &\neq Z_{ZOH}[G_1(s)]Z[G_2(s)]\ldots \\ Z[G_m(s)] &\neq Z[G_1(s)]Z[G_2(s)]\ldots Z[G_m(s)] \end{aligned} \tag{19}$$

where $Z_{ZOH}(.)$ represents taking the ZOH equivalent of $(.)$, and $Z(.)$ is just the normal impulse invariant transformation.

The exception to eq. (19) is where there is a sampler between each $G_i(s)$ block [12], which is not present in a cascade expansion of the mth-order internal process s-domain model. Therefore, the only possibility is to obtain the ZOH equivalent of the intact mth-order $G_M(s)$, as $G_M(z)$, which is subsequently decomposed into cascaded first-order blocks (illustrated in Figure 5) as:

$$G_M(z) = \prod_{i=1}^{m} \frac{K_i(1+\beta_i z^{-1})}{1-\alpha_i z^{-1}} \tag{20}$$

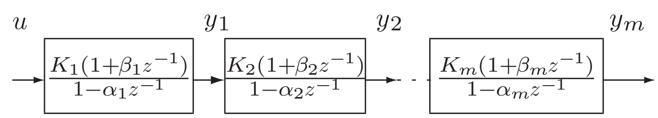

Figure 5 mth-order cascaded model.

4.2 PFC design

In the time domain, the model output, $y_m = y_M$, may be determined from:

$$\begin{aligned} y_i(k) &= \alpha_i y_i(k-1) + K_i y_{i-1}(k) \\ &\quad + K_i\beta_i y_{i-1}(k-1) \end{aligned} \tag{21}$$

$$2 \leq i \leq m$$

and

$$y_1(k) = \alpha_1 y_1(k-1) + K_1 u(k) + K_1\beta_1 u(k-1) \tag{22}$$

The free (auto-regressive) and the forced responses are given (by eqs (23) and (24), respectively) as:

$$\begin{aligned} y_A(k+H) = \alpha_m^H y_m(k) &+ K_m\beta_m y_{m-1}(k-1) \\ &+ K_m[\alpha_{m-1}^H y_{m-1}(k) \\ &+ [K_{m-1}\beta_{m-1} y_{m-2}(k-1)(k) \\ &+ K_{m-1}[\cdots[\alpha_1^H y_1(k) \\ &+ K_1 u(k) + K_1\beta_1 u(k-1)]\cdots]]] \end{aligned} \tag{23}$$

$$y_F(k+H) = K_m K_{m-1} \cdots K_1 u(k) \tag{24}$$

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The control law, following the steps in Section 3.2, is derived as:

$$u(k) = \frac{(C(k) - y_P(k))(1-\lambda^H)}{K_m K_{m-1} \cdots K_1} + \frac{y_m(1-\alpha_m^{H-1}) - (K_m \alpha_{m-1}^H + K_m \beta_m) y_{m-1}(k) -}{K_m} \frac{(K_m K_{m-1} \alpha_{m-2}^H + K_m K_{m-1} \beta_{m-1}) y_{m-2}(k) -}{K_{m-1}} \frac{(K_m K_{m-1} \cdots K_2 \alpha_1^H + K_m K_{m-1} \cdots K_2 \beta_2) y_1(k) -}{K_{m-2}} \frac{K_m \cdots K_1 \beta_1 u_{k-1}(k)}{K_{m-3} \cdots K_1} \tag{25}$$

5 DMC

DMC was developed by Cutler and Ramaker in 1980 for The Shell oil company. The control algorithm is originally developed for linear internal controller models of finite step response (FSR) type. DMC was firstly designed to take into account petrochemical Multi-Input Multi-Output systems [2].

For a SISO model on its FSR form, given in eq. (26),

$$y(k+1) = \sum_{i=1}^{M} a_i \Delta u(k-i+1) + y_0 + d(k+1) \tag{26}$$

where y_0 is the initial output, $\Delta u(k)$ is the variation of the MV at time *k*, *d*(*k*) the non-modelled perturbation and/or model process mismatch on *y*(*k*), a_i step response system coefficient and M the number of samples required by the system to reach steady state. For any $i \geq M$, the step system coefficient $a_i = a_M$.

Assuming that the present time sample is given by k, the predicted outputs y for the N_h next samples $(k+1|k)$ to $k+N_h|k))$ is given by $y(k+l|K))$, eq. (27).

$$y(k+l|k)) = \sum_{i=1}^{l} a_i \Delta u(k+l-i|k)) + y_0 + \sum_{i-l+1}^{M} a_i \Delta u(k+l-i|k)) + d(k+l) \tag{27}$$

where different effects on $y(k+l|k))$ are given as follows:

- Effect of future MVs:

$$\sum_{i=1}^{l} a_i \Delta u(k+l-i|k))$$

- Effect of past MVs:

$$y_0 + \sum_{i=l+1}^{M} a_i \Delta u(k+l-i|k))$$

- Predicted disturbances:

$$d(k+1)$$

for simplificative reasons let us define:

$$y^*(k+l|k)) = y_0 + \sum_{i=l+1}^{M} a_i \Delta u(k+l-i|k)) \tag{28}$$

as the contributions of $y(k+l)$ due to past MVs $\Delta u(k-i)$ a $\Delta u(k)$. The term $y^*(k+l)$ may be calculated at any time using past outputs and MVs.

$y(k+l)$, eq. (27), may be formulated on the following matrix form, with l going from 1 to N_h and Δu from 1 to $N_c - 1$, respectively, the prediction and control horizon.

$$\begin{bmatrix} \hat{y}(k+1|k)) \\ \vdots \\ \hat{y}(k+N_h|k)) \end{bmatrix} = \begin{bmatrix} \hat{y}^*(k+1|k)) \\ \vdots \\ \hat{y}^*(k+N_h|k)) \end{bmatrix} + A \begin{bmatrix} \Delta u(k|k)) \\ \vdots \\ \Delta u(k+N_c-1|k)) \end{bmatrix} + \begin{bmatrix} d(k+1|k)) \\ \vdots \\ d(k+N_h|k)) \end{bmatrix} \tag{29}$$

avec:

$$A = \begin{bmatrix} a_1 & 0 & \cdots & 0 \\ a_2 & a_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ a_{N_h} & a_{N_h-1} & \cdots & a_1 \\ \vdots & \vdots & \vdots & \vdots \\ a_M & a_{M-1} & \cdots & a_{M-N_h+1} \\ \vdots & \vdots & \vdots & \vdots \\ a_M & a_M & \cdots & a_M \end{bmatrix} \tag{30}$$

A is called the dynamic matrix of the system. Note that only the first N_c values of Δu are calculated: $\Delta u(t) = 0$ pour $k > k + N_c|k)$

Replacing the values of Δu after $k + N_c$ affects positively the controller stability. According to Garcia and Morshedi [13], a choice of $N_h = M + N_c$ ensure the control stability in most cases.

As for unknown values of *d*(*k*), the best that can be done remains an estimation.

5.1 Derivation of the control law

From eq. (26), and for $k > k-1|k)$, as well as eq. (27), we obtain:

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$$y(k) = y^*(k) - d(k) \tag{31}$$

$d(k)$ may then be estimated using past measured values of $y_m(k)$ as well as the result of passed MVs. In the absence of complementary information on future values of $d(k)$, the predicted disturbance is assumed equal to the present disturbance $d(k)$.

$$d(k+l|k) = d(k|k) = y_m(k) - y^*(k|k) \quad \text{for} \quad l = 1, \cdots, N_h \tag{32}$$

Now, the future predicted output values are formulated on vector form, starting from eq. (29) as:

$$\widehat{Y} = A\Delta U + \widehat{Y}^* \tag{33}$$

$$\text{Avec } \widehat{Y} = \begin{bmatrix} \hat{y}(k+1|k)) \\ \vdots \\ \hat{y}(k+N_h|k)) \end{bmatrix},$$

$$\widehat{Y}^* = \begin{bmatrix} \hat{y}^*(k+1|k)) \\ \vdots \\ \hat{y}^*(k+N_h|k)) \end{bmatrix} \text{ and}$$

$$\Delta U = [\Delta u(k|k) \cdots \Delta u(k+N_c-1|k)]^T \tag{34}$$

where N_c is the control horizon.

The goal of every predictive control formulation is to find the sequence of future N_c MVs ΔU, minimizing the sum squared error between $y(k+l|k)$ and the reference setpoint y_w, given by the quadratic criteria equation (35).

$$J = \sum_{j=1}^{N_h} (\hat{y}(k+j|k) - y_w(k+j))^2 \tag{35}$$

if:

$$\begin{bmatrix} y_w(k+1) - \hat{y}^*(k+1|k) - d(k|k) \\ \vdots \\ y_w(k+N_h) - \hat{y}^*(k+N_h|k) - d(k|k) \end{bmatrix} = \mathbf{e}(k+1) \tag{36}$$

where $\mathbf{e}(k+1)$ is a vector of dimension N_h, giving the deviations with respect to the setpoint, then replacing eq. (33) in eq. (35) we obtain:

$$J = A^T A \Delta U^2 + \mathbf{e}(k+1)^2 + 2A\mathbf{e}(k+1)\Delta U \tag{37}$$

Minimizing J with respect to ΔU gives:

$$\frac{\partial J}{\Delta U} = 2A^T A \Delta U + 2A\mathbf{e}(k+1) \tag{38}$$

the optimal solution is then given by:

$$\frac{\partial J}{\Delta U} = 0 = 2A^T A \Delta U + 2A\mathbf{e}(k+1) \tag{39}$$

Giving the final analytical control law:

$$\Delta U = (A^T A)^{-1} A^T \mathbf{e}(k+1) \tag{40}$$

Usually, DMC uses the first value of the ΔU sequence, $\Delta u(k)$.

6 Application to a milk pasteurisation process

6.1 Pasteurisation model

The pasteuriser model describes an industrial plant based on a Clip 10-RM plate heat exchanger, with two brazed heat exchangers of type CB76 from the constructor Alfa Laval. The validated first principles model [9], [15] is given in eq. (41), where the numerical values for the parameters a_i, b_i and c_i are given in Table 2.

$$T_{op1}(s) = \frac{b_0 + b_1 s + b_2 s^2}{a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4} T_{op3}(s) + \frac{c_0 + c_1 s}{a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4} e^{-12s} F_{v1}(s) \tag{41}$$

Note that the pure time delay of the system (i.e. 12 s), given by the term e^{-12s} in eq. (41), is excluded from the internal model formulation and is dealt with as explained in Section 2.4.

Table 2 Continuous time pasteurisation model parameters.

	Parameter subscript, i				
Parameters	**0**	**1**	**2**	**3**	**4**
a_i	0.085	30.86	880.1	8,100	23,625
b_i	28.44	199.1	–		
c_i	0.08505	2.7	20.25		

6.1.1 Parallel form

The model formulation equation (41) can be rewritten as the sum of four first-order systems, as in eq. (42):

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$$T_{\mathrm{op1}}(s) = \left(\frac{K_1}{1+\tau_1 s} + \frac{K_2}{1+\tau_2 s} + \frac{K_3}{1+\tau_3 s} + \frac{K_4}{1+\tau_4 s}\right) F_{v1}(s) + \left(\frac{K_1'}{1+\tau_1 s} + \frac{K_2'}{1+\tau_2 s} + \frac{K_3'}{1+\tau_3 s} + \frac{K_1'}{1+\tau_4 s}\right) T_{\mathrm{op3}}(s) \qquad (42)$$

where the model parameters τ_i, K_i and K_i' are given in Table 3.

Table 3 Parallel model parameters.

Time const.	τ_1	τ_2	τ_3	τ_4
	6.2109	8.8422	15.2091	332.5593
Gain (MV)	K_1	K_2	K_3	K_4
	1.3173	8.8799	−34.9245	359.1188
Gain (Dist.)	K_1'	K_2'	K_3'	K_4'
	−0.0639	0.0580	0.0112	0.9947

6.1.2 Cascaded form

The ZOH equivalent of the pasteurisation first principles model given in eq. (41) may be decomposed into a cascaded form (i.e. a gain-pole-zero decomposition), as in eq. (20), as:

$$\begin{aligned} T_{\mathrm{op1}}(z) = & \frac{K_1(1+\beta_1 z^{-1})}{1-\alpha_1 z^{-1}} \frac{K_2(1+\beta_2 z^{-1})}{1-\alpha_2 z^{-1}} \frac{K_3(1+\beta_3 z^{-1})}{1-\alpha_3 z^{-1}} \frac{K_4(1+\beta_4 z^{-1})}{1-\alpha_4 z^{-1}} F_{v1}(z) \\ & + \frac{K_1'(1+\beta_1' z^{-1})}{a-\alpha_1 z^{-1}} \frac{K_2'(1+\beta_2' z^{-1})}{1-\alpha_2 z^{-1}} \frac{K_3'(1+\beta_3' z^{-1})}{1-\alpha_3 z^{-1}} \frac{K_4'(1+\beta_3' z^{-1})}{1-\alpha_4 z^{-1}} T_{\mathrm{op3}}(z) \end{aligned} \qquad (43)$$

with the model parameters given in Table 4. Note that such a decomposition, unlike the parallel decomposition, is non-unique.

Table 4 Cascaded model parameters.

Gain (MV)	K_1	K_2	K_3	K_4
	$13.364\ 10^{-4}$	1	1	1
Gain (Dist.)	K_1'	K_2'	K_3'	K_4'
	$3.9998\ 10^{-4}$	1	1	1
α_i	α_1	α_2	α_3	α_4
	0.997	0.9364	0.8931	0.8513
β_i	β_1	β_2	β_3	β_4
	0	3.551	−0.8669	0.2548
β_i'	β_1'	β_2'	β_3'	β_4'
	0	−0.9502	−0.921	0.9325

6.2 PFC controllers results

Two PFC-based pasteuriser controllers are designed with both parallel and cascaded internal models, according to the control laws developed in eqs (18) and (25), respectively. With a single coincidence point $H = 1$, an exponential reference trajectory, as in eq (1), and a CLRT value equal to OLRT/5 (~ 50 s).

Figure 6 shows the comparative performance of the parallel and cascade fourth-order controllers for a selection of setpoint changes and an input milk disturbance variance of 8°C. From Figure 6(a), it is seen that the regulation and transient response of both controllers is very similar, but Figure 6(b) indicates a slightly more aggressive action on the MV (steam valve) in the case of the cascade formulation.

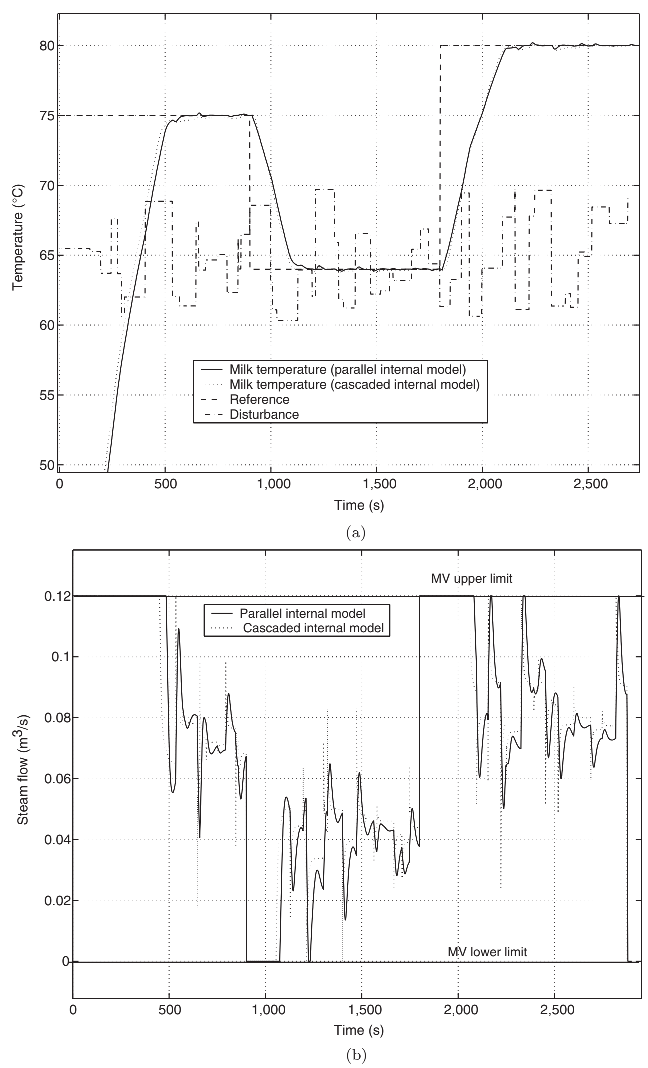

Figure 6 Comparison between PFC using a cascaded and a parallel internal model for the pasteuriser temperature control. (a) CV and (b) MV.

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For comparative purposes, a PFC controller was also designed, based on a first-order approximation to the model in eq. (41), obtained using a balanced reduction algorithm [14] as:

$$T_{\text{op1}}(s) = \frac{288.6}{1 + 422.4s} F_{v1}(s) + \frac{0.909}{1 + 422.4s} T_{\text{op3}}(s) \qquad (44)$$

A ZOH equivalent was subsequently determined for the approximated model, and this was used as the PFC internal model. Figure 7 shows the relative performance of this reduced order controller, compared to a fourth-order PFC controller based on a parallel model decomposition, as an example. Clearly, the transient response of the reduced order controller is inferior (for the same choice of CLRT = 50), and the regulation of the milk temperature also shows a slightly larger variance for the lower order case. In an effort to improve the transient response of the reduced order controller, the CLRT is reduced to 10, with the result also shown in Figure 7. However, although an improvement in transient response is evident (though still not as good as the higher order controller), the control action becomes very aggressive, as shown in Figure 7b.

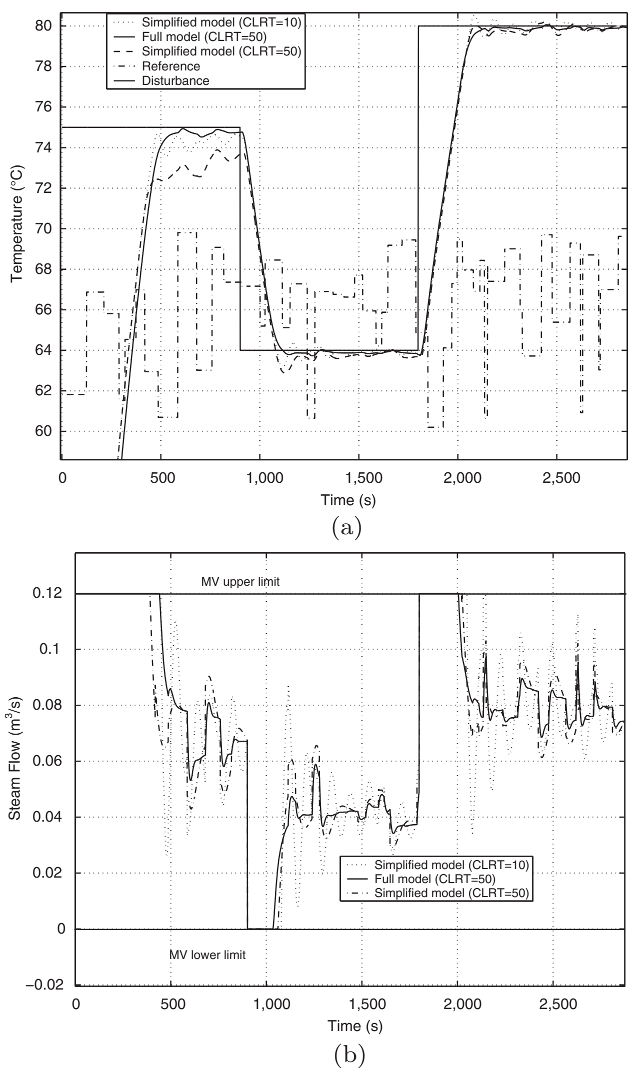

Figure 7 Comparison between a higher order and first-order PFC controller for the pasteuriser temperature control. (a) CV and (b) MV.

These differences in transient and regulation performances are enumerated in Tables 5 and 6, respectively, with the following key:

- MAEcv, mean absolute error on the CV (milk temperature) in °C;
- VAR_{MV}, variance of the MV (steam flow);
- MSSE_{CV}, maximum steady-state excursion of the CV (milk temperature) from the setpoint in °C.

In particular, Table 6 is noteworthy, since these are important performance measures for pasteurisation, where the principal objective is a regulatory one. Specifically, the MSSE_{CV} is vastly superior in the higher order case, which has important implications for lowering the setpoint (refer to Figure 1). Also, excessive control action of the reduced order controller will shorten steam valve life.

Table 5 PFC control performance in the transient case.

Controller and internal model form	MAEcv (°C)	VARMv (m^3/s)	Maximum overshoot (°C)
PFC: parallel fourth order	6.1665	0.0212	0.0180
PFC: cascaded Fourth order	7.1166	0.0276	0.0190
PFC: first order	6.1420	0.0377	0.4900
DMC: step response model	5.9550	0.0483	(1.213) 0.4541

Table 6 PFC control performance in the regulatory case.

Controller and model order	MAE_{CV} (°C)	VAR_{MV} (m^3/s)	MSSe_{CV} (°C)
PFC fourth order (CLRT = 50)	0.1225	0.0040	0.2679
PFC first order (CLRT = 50)	0.3001	0.0064	0.9945
PFC first order (CLRT = 10)	0.1455	0 0082	0.4585
DMC fourth order	0.3657	0.0620	0.3682

6.3 Benchmark DMC controller results

The DMC controller is implemented on its original form as given by the analytical control law equation (40) with no alteration of the process model, and the constraints are taken into account in an optimal way [13].

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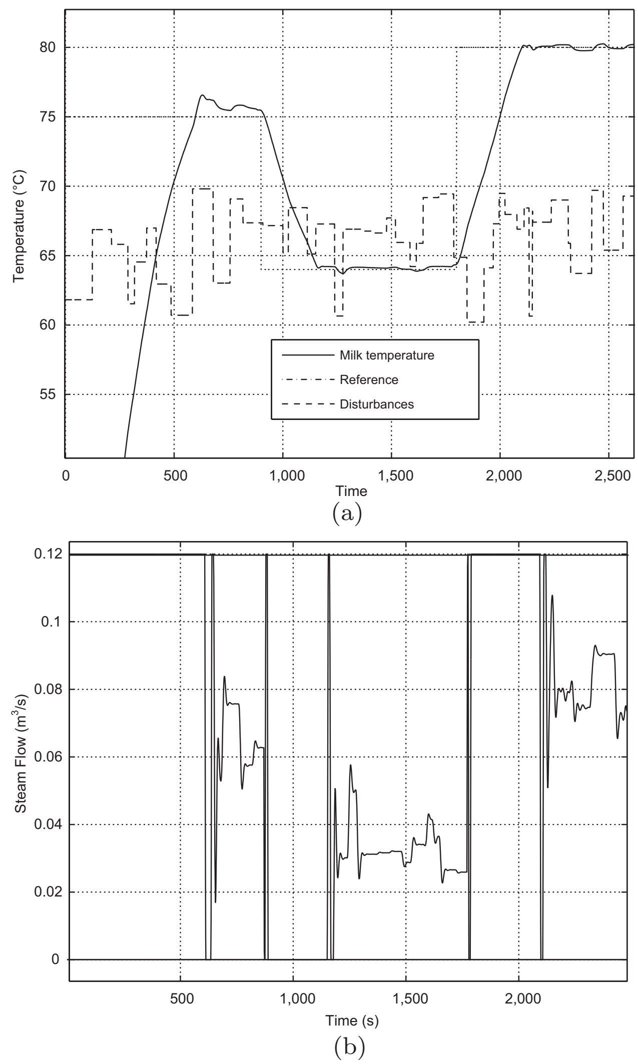

Figure 8 Benchmark DMC controller performances. (a) CV and (b) HV.

The designed DMC controller uses the following parameters: a sampling period of $T_S = 15$, a maximum number of step response coefficients $M = 80$, a prediction horizon $N_h = 70$ and a control horizon $N_c = 2$.

The results are comparatively similar to PFC in terms of time response and regulation error, Figure 8, with however a couple of weakness points. Indeed, Table 5 shows that the transient response obtained by the DMC controller is of inferior quality to the one obtained by PFC in terms of MAE_{CV}, VAR_{MV} and $MSSE_{CV}$. For the latest point, note that when reaching the first setpoint of 75°C, the maximum overshoot is over 1.213°C, it will be reduced sensibly for the second and third setpoint value to around 0.4241. The variation of the MV is also superior to the one obtained with PFC, leading to a more aggressive MV. This is principally due to the fact that DMC does not use, on its original form, an exponential reference trajectory.

In the regulatory case, Table 5, the DMC controller outperforms first-order PFC only in terms of $MSSE_{cv}$. As for the higher order PFC, its results are still better than DMC.

7 Conclusions

From the results shown in Section 6.2, it is clear that there are benefits in extending the internal PFC controller model order in the milk pasteurization control case. This comes at some extra computational expense, but this is offset by the pervading simplicity of the PFC formulation and the availability of cheap computational power. Furthermore, the pasteurization process is slow, with correspondingly low requirements on computational speed.

This paper has demonstrated two possible methods for decomposing the internal controller model in use with a PFC formulation compared to a benchmark classical DMC formulation. Though broadly comparable in terms of performance (regulation, variance of the MV and robustness), the parallel form is preferred from a computational point of view and gives a more elegant controller form. However, there are numerous other decomposition possibilities, including exploitation of the non-uniqueness of the cascade form.

Lower order internal model approximations, in PFC, can give comparable transient response performance (and can sometimes give a faster rise time, due to reduced model latency), there is significant degradation in regulation performance (e.g. output variance) and MV activity (variance and constraint violation). For the pasteurization application, regulation is of the upmost importance, if the process is to be optimised in terms of:

- Reduced recycling of “out of spec” product,
- Lowering of temperature setpoint values, giving:
  - Reduced energy costs
  - Improvement of milk quality, both in taste and in retention of protein content, and
- Reduced valve activity and excursion, with corresponding reductions in wear

On the one hand, first-order PFC, despite its primitive formulation compared to the complete FSR internal model used in DMC, competes well with the latter. This is due to the well-behaved nature of the process, as neither oscillatory nor non-minimum phase behavior are sustained in the milk pasteurisation case. On the other hand, both higher order PFC internal models (parallel

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and cascaded forms) outperform DMC results in terms of MAE, overshoot and maximum variance. This is noticed despite the over parameterized internal model, complicated formulation, optimal constraints handling and computational requirements of the DMC formulation. These may be seen as favourable attributes to a better control, even if in the case of pasteurisation they do not contribute to a better control. While these design advantages of PFC do not obviously manifest themselves in the pasteurisation application results, they result in a more compact system representation and control solution, with resulting improved computational and numerical properties.

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