

# DESIGNING RICH SETS OF TASKS FOR UNDERGRADUATE CALCULUS COURSES

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*Research has shown that the types of tasks assigned to students affect their learning. Studies have found that many mathematical tasks created for second and third level students promote instrumental rather than relational understanding, and imitative rather than creative reasoning. In this paper, we describe some frameworks that aim to guide teachers when writing tasks. From these frameworks a set of task types that are deemed appropriate for undergraduate students, and that foster mathematical habits of mind, have been selected: these are evaluating mathematical statements; example generation; analysing reasoning; conjecturing; generalising; visualisation; and using definitions. We report on an effort to design a set of tasks for an introductory calculus course using these principles and give examples of exercises on the topic of continuity. The teacher's role in the implementation of such tasks is also discussed.*

## INTRODUCTION

Skemp (1976) identified two types of understanding: relational and instrumental. Relational understanding refers to conceptual understanding and involves knowing both what to do and why to do it. Skemp explained instrumental understanding as 'rules without reasons' (p. 21) and conjectured that for many students and even teachers 'the possession of such a rule, and ability to use it, was what they meant by understanding' (p. 21). The main goal of a mathematics lecturer is to foster mathematical understanding in their students; however, many authors have expressed the view that mathematics at third level suffers from an over-emphasis on procedures and memorisation. Dreyfus (1991) asserted that many students learn a large number of standardised procedures in their university mathematics courses but fail to gain insight into how or why these procedures were developed. Cuoco, Goldenberg and Mark (1996) remarked that curricula are usually given in terms of the content of mathematics courses, but that the thought processes that created the mathematical results studied in the courses are rarely mentioned. They advocated inclusion of 'mathematical habits of mind' in order to 'give students the tools they will need in order to use, understand and even make the mathematics that does not yet exist' (p 376).

Closely related to mathematical understanding is the type of thinking or reasoning that students are expected to employ. Lithner (2008) has characterised reasoning into two main categories: imitative reasoning, and creative mathematically-founded reasoning. The term imitative reasoning refers to the use of memorisation or well-rehearsed procedures, while creative mathematically founded reasoning involves new (to the student at least) arguments that are mathematically well-founded. Boesen, Lithner and Palm (2010) used Lithner's framework to analyse the relation between the types of tasks assigned to high-school students and the types of reasoning employed by the students in solving these tasks. They found that when faced with familiar tasks students employed imitative reasoning and, in contrast, used

creative mathematically-founded reasoning to tackle unfamiliar tasks. Boesen et al. (2010) claim that the solutions to familiar tasks required little or no conceptual understanding and they conjecture that exposure to these types of tasks alone limits the students' ability to reason and gain conceptual understanding. In other words, the use of familiar tasks fosters instrumental understanding but not relational understanding.

Studies carried out in universities in the UK (Pointon and Sangwin, 2003) and in Sweden (Bergqvist 2007) have found that, at least in introductory mathematics courses, students are mostly required to carry out procedural calculations and rarely need to use any higher order thinking skills. The consequences of this phenomenon were explored by Selden, Selden, Hauk and Mason (2000) in a study of second year calculus students. The students were given two tests. The first asked them to demonstrate familiar techniques; the second required them to use these techniques to solve non-routine problems. The study found that more than half of the students could not solve any non-routine problems although the first test had shown that they were technically competent. It seems that these students had instrumental but not relational understanding of the material. Selden et al. (2000) recommend that lecturers should regularly assign non-routine problems to students in order to develop their thinking skills.

### **The Irish Context**

We are not aware of any research that has studied the types of tasks assigned to students in introductory calculus courses in third level institutions in Ireland. However much has been written recently on the teaching of mathematics at second level, and in particular on the examination system. Since the majority of our first year undergraduates proceed directly from second level education to third level, we outline here some studies that we feel are relevant to understanding the mathematical background of our freshman classes.

Students undertake a state examination in Ireland at the end of their time in post-primary school, called the Leaving Certificate examination, the results of which determine entry into third level education. The Chief Examiner's Report (2005) on the Leaving Certificate Mathematics Examination reported that students' strengths were 'most evident in procedural questions where a definite sequence of familiar steps was required' (p. 48) and that weaknesses related to 'inadequate understanding of the mathematical concepts and a consequent inability to apply familiar techniques in anything but the most familiar of contexts' (p. 49). Unwarranted levels of difficulty were caused by questions requiring students to display an understanding of underlying concepts. Problems were also encountered due to students' under-developed problem-solving and decision-making skills.

Elwood and Carlisle (2003) noted that the way in which questions were structured on the Leaving Certificate examination papers year-on-year seemed to reward the learning of rules and their application in familiar mathematical contexts, and that the traditional style of questions appearing on the papers possibly closely reflected the items pupils were exposed to from their textbooks. They suggested that this predictability of the examination questions was likely to influence not only how the students prepared for these examinations but also how they were taught. Conway and Sloane (2005) speak of this as the 'backwash effect' examinations have on curriculum, 'shaping both what is taught and how it is taught' (p. 32)

by determining what is deemed to be valuable knowledge. They describe how ‘teaching to the test’ can enable students to perform well on examinations without engaging in higher levels of cognition.

Lyons, Lynch, Close, Sheerin and Boland (2003) carried out a video study of twenty post-primary mathematics lessons in ten schools in Ireland. They found that teaching was strongly didactic, and that a procedural rather than a conceptual approach prevailed. Hourigan and O’Donoghue (2007) also observed two second-level senior-cycle Irish classrooms over 10 weeks and found them to be focused on examinations and on the mastery of algorithmic procedures as isolated skills, with ready-made mechanisms provided for the pupils to aid memorisation. Teachers tended to do the pupils’ thinking for them and a ‘learned helplessness’ in the pupils was repeatedly reinforced. Hourigan and O’Donoghue (2007) contended that the inability of such students to successfully make the transition to tertiary level mathematics education lies in the substantial mismatch between the nature of their pre-tertiary mathematics experiences and their subsequent mathematics-intensive tertiary level courses. They described mathematics-intensive courses at third level as requiring students to be independent learners, displaying conceptual skills and the ability to solve unfamiliar problems. However, the fostering of such skills was not observed in the second level classrooms studied.

Mason (2002) explains how students who can perfectly complete routine problems are not necessarily prepared to ‘tackle non-routine problems which require juxtaposing ideas, adapting familiar techniques, or are simply more complex than the ones with which they are familiar’ (p. 33). In order to enable our students to overcome their possible weaknesses in applying techniques in unfamiliar contexts and to develop ‘mathematical habits of mind’, it is necessary to focus primarily on tasks which foster relational understanding. Skemp (1976) describes the development of instrumental understanding as requiring the learning of an increasing number of fixed ‘plans’ whereby students can find their way to the answers of a fixed number of questions. Whereas relational understanding can be explained as relying on the learners’ building of a conceptual structure from which he/she can answer an unlimited number of questions. It is such relational understanding that will combat ‘learned helplessness’ and enable students to think independently. To this end, in this paper we will consider some frameworks which aim to guide the creation of tasks that will foster good mathematical habits of mind and aid the development of relational understanding. As many of these frameworks were designed to be used at second level we will report on the task types that we feel are most useful for third level courses. We will illustrate these task types by giving examples of tasks on the theme of continuity.

## **TASK FRAMEWORKS**

Before we consider task frameworks we will explain what is meant by ‘mathematical habits of mind’. Cuoco et al. (1996) proposed that students need to conjecture, experiment, visualise, describe, invent and generalise. They also advocate the precise use of mathematical language. Bass (2005) claimed that these habits of mind are precisely those used by research mathematicians, and that these practices can, and should, be fostered at all educational levels.



## Mason's Framework

How then can we design tasks which will allow our students to develop these habits of mind? Mason and Johnston-Wilder (2004a) suggest that questions posed to students should involve the practices employed by mathematicians when conducting research. To this end they propose that the following words be used when designing tasks: 'exemplifying, specialising, completing, deleting, correcting, comparing, sorting, organising, changing, varying, reversing, altering, generalising, conjecturing, explaining, justifying, verifying, convincing, refuting', (p 109). Watson and Mason (1998, p. 7) grouped the words in this list and gave a table of questions and prompts that could be used to generate the thinking processes in each section. For example, the prompts associated with the processes of exemplifying and specialising, and generalising and conjecturing are shown in Table 1.

Exemplifying	Give me one or more examples of ...
Specialising	Describe (show, choose, find, ...) an example of ... Is ... an example of ...? What makes ... an example? Find a counter example of... Are there any special examples of ...?
Generalising	What happens in general?
Conjecturing	Of what is this a special case? Is it always, sometimes, never ...? Describe all possible ... as succinctly as you can. What can change and what has to stay the same so that ... is still true?

**Table 1: Watson and Mason's (1998) questions and prompts**

## Swan's Framework

Swan (2008) described his framework of five tasks types which he selected in order to promote conceptual understanding amongst secondary school students. These task types encourage the development of skills such as classifying, interpreting, comparing, evaluating and creating. The five task types are: classifying mathematical objects; interpreting multiple representations; evaluating mathematical statements; creating problems; analysing reasoning and solutions. In the first of these, students devise or apply classifications. In doing this, they gain an appreciation for the properties of objects and for the importance of definitions and mathematical language. In the second task type, interpreting multiple representations, students must link different representations of the same object. Swan claims that this process leads to the development of new mental images. The third type of task involves students deciding whether mathematical statements are always, sometimes, or never true and justifying their

reasoning. These tasks encourage the use of examples and counter examples. The fourth task type asks students to create problems for their classmates to work on. This naturally involves the notions of doing and undoing. Students are asked to compare solutions or to diagnose errors in the last task type. This allows them to appreciate the possibility of more than one solution to a problem and enhances their reasoning skills.

Swan (2007) trialled the use of these task types with teachers on a professional development course. It is interesting to note that, during the study, many of the teachers changed their beliefs and practices and moved from a teacher-centered to a student-centered approach to teaching.

### Pointon and Sangwin’s Framework

Sangwin (2003) approached the chore of categorising mathematical tasks used in university courses from a different perspective. Taking into account the backwash effect of examinations, he took the view that ‘any attempt to elaborate on what is meant by mathematical skills must be based on an analysis of what *in reality* we ask students to do’ (p. 814) through the assessment criteria for a particular course. Together with Pointon (Pointon & Sangwin, 2003), he developed a taxonomy with eight classes of mathematical questions, distributed over two levels as shown in Table 2.

1. Factual recall	5. Prove, show, justify (general argument)
2. Carry out a routine calculation or algorithm	6. Extend a concept
3. Classify some mathematical object	7. Construct an instance
4. Interpret situation or answer	8. Criticize a fallacy

**Table 2: Pointon and Sangwin’s (2003) mathematical question taxonomy**

Tasks falling into categories 1-4 are said to be those of ‘adoptive learning’ which involves students applying well-understood knowledge in bounded situations. On the other hand, tasks which typically require higher cognitive processes, such as those described by categories 5-8 (and sometimes 4), are deemed to require students to behave in a more sophisticated way mathematically and are characteristic of ‘adaptive learners’.

### Comparison of Frameworks

There is some overlap evident in the frameworks outlined above, for instance, Mason and Johnston-Wilder (2004a) speak of ‘exemplifying’ and ‘specialising’, while Pointon and Sangwin (2003) use the term ‘construct an instance’ to describe the same mathematical skill. However, due to the difference in the motivation or perspective of the authors involved, some of the frameworks focus on different aspects of mathematical thinking than others. Recall that Pointon and Sangwin were interested in categorising the types of mathematical skills actually

assessed in first year undergraduate mathematics courses, while Mason and Johnston-Wilder aspired to describe the types of questions students *should* be asked in order to facilitate the development of mathematical habits of mind, and Swan (2008) focussed on task types teachers could use to foster conceptual understanding (in particular) in their students. Nonetheless, one thing many of those discussed have in common is the broad categorisation of the task types they comprise into two levels. This is also true of the theories used to describe types of mathematical understanding and reasoning. Skemp (1976) wrote of ‘instrumental’ versus ‘relational understanding’, Pointon and Sangwin (2003) use the terms ‘adoptive’ and ‘adaptive learning’, while Lithner (2008) speaks of ‘imitative’ and ‘creative reasoning’. Similarly, learners are described as being in ‘accepting’ or ‘asserting mode’ by Mason and Johnston-Wilder (2004a). Each of these descriptions can be facily understood as distinguishing between a student behaving mathematically as a competent practitioner or as an expert. As a competent practitioner, a student can succeed on familiar, routine tasks, but in order to construct and reconstruct mathematics for himself, a student must have developed the skills of an expert. The latter typically require higher level cognitive processes such as reflection, creativity and criticism and stem from the mathematical habits of mind advocated by Cuoco et al. (1996) and Bass (2005).

## TASKS DESIGNED

During the academic year 2010/2011, both authors taught courses on differential calculus to groups of first year students in their respective third level institutions. The courses were delivered to large mixed-ability groups of students and were calculus rather than analysis courses. The emphasis in this type of introductory calculus module is often on procedures and calculations.

In redesigning a set of homework tasks for our first year calculus courses, we realised that some of the task types discussed in the previous section seemed more appropriate for our groups of students than others, as most of the frameworks discussed above concern the creation of tasks for secondary school pupils. In drawing on these frameworks, we wanted to help our students make the transition from thinking about mathematics in a procedural or instrumental manner to using the habits of mind of a research mathematician. In particular, we wanted our students to gain experience of behaving like an expert. Thus, for example, instead of asking students to prove that a mathematical statement is true, we sometimes asked them to decide if it was true or not and to justify their answer. The act of making a decision about a mathematical statement was novel and challenging to our students (Breen & O’Shea 2011).

From the variety of types of tasks and prompts discussed earlier, the seven task types that we felt were most useful in first year undergraduate calculus courses in an Irish context were: **evaluating mathematical statements**; **example generation** (constructing an instance); **analysing reasoning**; **conjecturing**; **generalising**; **visualisation**; **using definitions** (to classify a mathematical object, for instance). We do not include tasks that ask students to prove a result here, not because we do not think these tasks are useful, but because we feel that they are present in most mathematics courses.

In order to illustrate these task types we will concentrate on the topic of continuity and give some tasks under the headings above.

### Examples

#### *Evaluating Mathematical Statements*

Suppose  $f$  is continuous and never zero on  $[a,b]$ .

Is it possible that  $f(z) < 0$  for some  $z$  in  $[a,b]$  and  $f(w) > 0$  for some  $w$  in  $[a,b]$ ? Explain.

#### *Example Generation*

Give an example of the following:

1. A function  $f$  which is continuous at  $x = 5$ .
2. A function  $f$  which is not continuous at  $x=5$  because  $f(5)$  is not defined.
3. A function  $f$  which is not continuous at  $x = 5$  because  $\lim_{x \rightarrow 5} f(x)$  does not exist.
4. A function  $f$  which is not continuous at  $x = 5$  because  $\lim_{x \rightarrow 5} f(x) \neq f(5)$ .

#### *Analysing reasoning*

Consider the following argument. Decide whether the reasoning used is satisfactory, justifying your answer.

*Statement:* Let  $f$  and  $g$  be functions that are continuous everywhere, then  $f/g$  is continuous everywhere.

*Proof:* Let  $c$  be a real number. Since  $f$  and  $g$  are continuous at  $c$  we know that  $\lim_{x \rightarrow c} f(x) = f(c)$  and that  $\lim_{x \rightarrow c} g(x) = g(c)$ . Thus  $\lim_{x \rightarrow c} f(x)/g(x) = f(c)/g(c)$  and so  $f/g$  is continuous at  $x=c$ . Since  $c$  is arbitrary,  $f/g$  is continuous everywhere.

#### *Conjecturing*

Suppose  $f$  is a continuous function on  $[a,b]$  and suppose that  $f(a) < 0$ , and that  $f(b) > 0$ .

What can you say about the number of times that the graph of  $f$  crosses the x-axis?

What about the number of times the graph of  $f$  touches the x-axis?

#### *Generalising*

The function  $f(x) = (x^2 - 1)/(x - 1)$  has a removable discontinuity at  $x = 1$ .

(a) Describe a family of functions which have a removable discontinuity at  $x = 1$ .

(What can change and what must stay the same?)

(b) Describe two families of functions which have a removable discontinuity at  $x = c$ .



### *Visualisation*

Draw a rough sketch of the graphs of the following functions:

1. A function  $f$  which is continuous everywhere except at  $x=-3$  and  $x=4$ .
2. A function  $g$  which is continuous everywhere except for removable discontinuities at  $x=-3$  and  $x=4$ .
3. A function  $h$  which is continuous everywhere except for a removable discontinuity at  $x=-3$  and a non-removable discontinuity at  $x=4$ .

### *Use of definitions*

Let  $[x]$  denote the integer part of  $x$ . Is the function  $f(x) = [x]$  continuous on the interval  $(0,1)$ ? What about on the interval  $[0,1]$ ? How about on the interval  $[0,1)$ ?

(Further tasks and students' reactions to them can be found in Breen & O'Shea (2011).)

In designing calculus tasks we were mindful of the need to create a rich variety of tasks as it was imperative that the coursework should not become predictable nor a particular type of task become over-familiar. Mason and Johnston-Wilder (2004a) advocate what they term a 'mixed economy' (p. 6) of tasks as no single strategy or task type has proved to be universally successful in developing mathematical thinking. Moreover, Schoenfeld (1992) explains how any one task could not be expected to foster all types of mathematical thinking and thus, he emphasizes the need for balance in designing a set of tasks, so that as many different 'dimensions' of mathematical thinking as possible can be addressed. To ensure that students encounter a range of different possibilities, Bell (1993) also advises that the structure and format of tasks presented to students should be varied.

## **IMPLEMENTATION OF TASKS**

Conditions for effective learning are complex: designing a rich set of tasks is not sufficient in itself and there is no guarantee that the learner will successfully develop the desired mathematical skills on the completion of such tasks. Researchers have drawn a distinction between the intended, implemented and attained curricula (Mason & Johnston-Wilder, 2004a). It is particularly important to be aware of the 'didactic tension' present in a classroom or teaching situation by which the more clearly the behaviour expected from learners in undertaking a task is expressed in the instructions accompanying it, the more the teacher deprives the learners of the conditions needed for learning and understanding (Mason & Johnston-Wilder, 2004b). Henningsen and Stein (1997) warn that it is not enough for a teacher to select and appropriately set-up worthwhile mathematical tasks: he/she must also provide support for students' cognitive activity in order to ensure that the complexity and cognitive demands of the tasks are not eroded during implementation. Furthermore, Stein, Grover and Henningsen (1996) found that those mathematical tasks which were designed to



be the most cognitively demanding (e.g. involving conjecturing, justifying, interpreting) were those most likely to decline into somewhat less-demanding activities in implementation. Their research also found that tasks which built on students' prior knowledge were most likely to maintain high-level cognitive engagement.

Bell (1993) recommends that tasks should be attempted by students initially and only when their responses have been given should the teacher intervene to offer hints or help towards a solution. He also asserts that learning takes place more effectively when the learning situation is 'managed' by the teacher by, for instance, adjusting the challenge of a task presented to keep it at an appropriate level for all learners. Lampert (1990) advocated a public demonstration of mathematical thinking in the classroom, through engaging in mathematical arguments with her students, encouraging the students to make conjectures and to muster appropriate evidence to support or challenge each others' assertions in a discursive manner. She believed that students would not learn a different way of thinking about what it means to do mathematics simply by being told what to do or having mathematical problems explained to them. Mason (2002) also suggests that possibly what 'students need most is to be in the presence of someone who is 'being mathematical''(p. 4). In short, the practices of the teacher matter.

## CONCLUDING REMARKS

In this paper, we have drawn on the work of Mason and Johnston-Wilder (2004a), Swan (2008), and Pointon and Sangwin (2003) to gather together a set of task types that are appropriate for use in an undergraduate course. The task types have been chosen to promote the mathematical habits of mind described by Cuoco et al. (1996).

Mason (2002) contends that 'in a sense, all teaching comes down to constructing tasks for students... This puts a considerable burden on the lecturer to construct tasks from which students actually learn' (p. 105). In doing so, a balance must be struck between sufficient and excessive challenge in order to avoid disillusioning students and destroying their confidence. Keeping this in mind, tasks similar to those outlined above were interspersed with more familiar, sometimes procedural, tasks on the homework sets distributed to students. Some preliminary feedback from students describing their reactions to a small number of routine and non-routine tasks and their understanding of the purposes of the tasks is described elsewhere (Breen & O'Shea, 2011). Responses gathered from the students indicated that they had a mature understanding of the purposes of different tasks. We hope to undertake further analysis of this data and to gather further data to examine aspects of these tasks more closely.

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