Transition through mathematical tasks

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The transition to university level mathematics is often problematic for students. Clark & Lovric (2008) have written about some of the differences between mathematics at school and at university, including the type of mathematics taught and the way mathematics is taught. Students at this stage also have to contend with social and cultural changes. As part of a project on task design, ten first year students at two different universities in Ireland were interviewed. In this paper, we will discuss their experiences of mathematics at school and university. In particular, we will consider the differences in the types of mathematical tasks encountered at both levels and the students' views of the influences of such tasks.

Keywords: transition; task design; calculus

Introduction

A selection of tasks designed by the authors was trialled in a first-year calculus module in each of two Irish universities. Towards the end of the calculus modules five students from each university were interviewed. The data from these interviews form the basis of this paper. We will present the students' views on the differences between the types of mathematical tasks encountered at school and at university and on the effects of the tasks on their ways of working and on the promotion of understanding. We will then discuss the effect of the tasks on the students' experience of the transition process.

Literature review

The transition from school to university has been the object of much research in recent years. Clark and Lovric (2008) described the secondary-tertiary transition as a 'rite of passage' and discussed the changes that students experience as they commence their mathematical studies at tertiary level. These include changes in the type of mathematics taught and changes in the way mathematics is taught. They contend that mathematics at university involves an increased emphasis on conceptual understanding, advanced mathematical thinking, abstract concepts and reasoning, the central role of proof, multiple representations of mathematical objects, and precision of mathematical language, when compared with mathematics taught at school. De Guzman, Hodgson, Robert, and Villani (1998) also reported on the difficulties that first year university students faced including students' ability to develop connections between concepts and to organize their mathematical knowledge.

Gueudet (2008) aimed to collect different views of the transition from secondary to tertiary level mathematics in order to investigate how they contribute to an overall understanding of the complexity of such a transition. In doing so, she observed that many studies on transition compare the practices of students with those of mathematicians. For instance, she remarked that Lithner (2000) found that first year university students often rely heavily on past experience when solving mathematical problems, while mathematicians usually display more flexibility in their thinking and reasoning. Gueudet (2008) reported that in order to deal with this issue, researchers have called for changes in the teaching methods both at school and at university, proposing, for instance, that a wider range of tasks should be used to allow students to develop autonomy and flexibility. Boesen, Lithner and Palm (2010) also contend that the types of tasks assigned to students affect their learning and the use of tasks with lower levels of cognitive demand leads to rote-learning by students and a consequent inability to solve unfamiliar problems or to transfer their mathematical knowledge to other areas competently and appropriately.

In Ireland, research at secondary level has shown that teaching in Irish mathematics classrooms tends to be focussed on the use of algorithmic procedures, with very little emphasis on conceptual understanding, and that students appear unable to apply techniques learnt in unfamiliar contexts (for example, Lyons, Lynch, Close, Sheerin, & Boland, 2003). Despite recent changes in the mathematics curriculum which aim to change this, O'Sullivan's (2014) analysis of mathematical tasks in three textbook series currently used at senior cycle of secondary schools in Ireland show that the emphasis is still on procedural, well-practised exercises with little opportunity for creative reasoning or to engage with unfamiliar problems.

The Task Design Project

Both authors are mathematics lecturers in different third level institutions in Ireland. In the academic year 2011/12, both were teaching first year differential calculus modules. Given the procedural nature of mathematics instruction at second level in Ireland, we endeavoured to design a series of unfamiliar non-procedural tasks in an effort to give students opportunities to develop their thinking skills and their conceptual understanding. (In this paper, the National Research Council's (2001) description of conceptual understanding as the "comprehension of mathematical concepts, operations and relations" (p.116) has been adopted.) An 'unfamiliar task' is one for which students have no algorithm, well-rehearsed procedure or previously demonstrated process to follow. Following Lithner's (2000) observation that students often rely heavily on past experience when solving problems, we hoped, by presenting the students with unfamiliar tasks, to discourage such reliance and help them to develop the flexibility in their thinking and reasoning characteristic of mathematicians.

The tasks designed in this project required students to make use of definitions, generate examples, generalise, make conjectures, analyse reasoning, evaluate statements, or use visualisation. Samples are shown below:

<u>*Question A*</u>: Find examples of the following:

(a) Polynomials $P(x)$ and $Q(x)$ such that $P(4) = 0 = Q(4)$ and $lim_{x\to 4}\frac{P(x)}{Q(x)} = 0$.

(b) Polynomials $P(x)$ and $Q(x)$ such that $P(4) = 0 = Q(4)$ and $lim_{x\to 4}\frac{P(x)}{Q(x)} = 1$.

(c) Polynomials $P(x)$ and $Q(x)$ such that $P(4) = 0 = Q(4)$ and $lim_{x\to 4}\frac{P(x)}{Q(x)} = -2$.

(d) Polynomials $P(x)$ and $Q(x)$ such that $P(4) = 0 = Q(4)$ and $lim_{x\to 4}\frac{P(x)}{Q(x)} = \infty$.

(e) Polynomials $P(x)$ and $Q(x)$ such that $P(4) = 0 = Q(4)$ and does not exist.

<u>*Question B*</u>: Suppose $f(x)$ is a function with natural domain R. Decide if each of the following statements is sometimes, always or never true:

(i) There are two different real numbers a and b such that $f(a)=f(b)$.

(ii) There are three different real numbers *a, b, c* such that *f(a)=b* and *f(a)=c*.

A selection of other tasks designed along with a rationale for the task framework used can be found in Breen and O'Shea (2011). Each problem set (and the final examination) contained unfamiliar non-procedural tasks as well as some more procedural tasks. For example, the following procedural task (taken from Larson, Hostetler and Edwards (2008)) appeared on the same problem set as question A above:

Question C: Find the limit (if it exists): $lim_{x\to 3} \frac{x^2-x-6}{x^2-5x+6}$.

The designed tasks were assigned to the students either as homework (for students to work on independently) or as tutorial problems (for students to work on in small groups). Towards the end of the module, students in a subset of tutorial groups were invited to volunteer to be interviewed. Eleven students volunteered and ten were interviewed by a research assistant who was working on the project but was not involved in the teaching of the module nor in the design of the homework and tutorial tasks. Timetabling restrictions prevented the other volunteer from participating. The interviews were semi-structured and lasted between 13 and 25 minutes; they were audio-recorded and fully transcribed. The students were assigned pseudonyms A-J.

Students were asked about their impressions of mathematics at university, how their experience of mathematics at school differed from that at university, how their study habits or ways of working had changed and about the tasks that they had worked on. Here we will report on the students' views in relation to the differences between mathematics and mathematical tasks encountered at school and university, and their views on how the tasks assigned during the Calculus module impacted on their practices, learning and conceptual understanding in the transition to tertiary level.

Results

Students' views on differences between mathematics in school and university

All the students interviewed described a change in emphasis from a focus on instrumental understanding or procedural fluency in school to a focus on relational or conceptual understanding at university. They all mentioned the importance of procedures in school. For example, student B said:

> Student B: It's just procedure…You learn the methodologies [sic] rather than learning why you are doing what you are doing;

while student F said "you'd always have to use the step and you always had to work out a couple of things". The students also spoke about working with formulae at second level in a procedural manner. Furthermore, three students (A D, and F) also felt that there was a lack of linkages between different topics in the second level curriculum:

> Student D: For the [senior cycle of secondary school] they were separate questions and they didn't really tie together at all.

This was in contrast to the ten students' perception of the importance of conceptual understanding and connections between topics at university. For instance, Student F compared the different approaches in school and university:

> Student F: Because, I remember [the lecturer] at the start of the year, she used to explain to us, you know, we are going to go through why you are doing this and how this graph relates to something else. That's a big thing. We didn't cover anything like that in 6th year [final year of secondary school].

All of the interviewed students spoke about the emphasis on conceptual understanding, for example:

> Student C: The emphasis in the college course is about actually understanding the principles
>
> Student D: It kind of explains why you were doing it before

Student J emphasized the different language used at university level but also appreciated that there was an expectation that students make sense of material for themselves, while Student F spoke about this aspect of the transition as moving from accepting to explaining. However, he pointed out that this is not an easy transition to make and can leave students feeling confused:

> Student F: It takes a while to get used to, ya. Because you don't know whether to accept it as it is or use the new kind of concept.

The interviewees also noticed an increased emphasis being placed on connections or relations between mathematical topics and ideas in university. Both Student A and Student D asserted that links between concepts were made explicit in their university mathematics course:

> Student D: It kind of ties together really everything from the [senior cycle of secondary school] … It kind of interlinks them more.

Students' views on differences between mathematical tasks assigned in school and university

The students were asked what they gained from the tasks assigned to them during their Calculus modules, and in particular which types of tasks helped them gain conceptual understanding. Eight of the ten students chose unfamiliar tasks designed for this study in answer to this question. Student B referred to unfamiliar tasks in general:

> Student B: The ones I haven't seen before, definitely… in those ones you have to like completely understand it to get the answer.

Furthermore, she indicated that, although unfamiliar tasks had been assigned at university, she would have been unlikely to encounter such tasks in school:

> Student B: We hadn't done that in class, so we had to try to figure it out for ourselves. Whereas in school the teacher would have done that with you.

The comments of some students indicate that to perform unfamiliar tasks they were forced to apply and find relationships between previously learned concepts. The quote below from Student A arose from her response when she was asked to compare two particular tasks, the first one familiar (evaluating limits) and the second one unfamiliar to the student (an example generation task). Her answer suggests that she has learned more from the unfamiliar problem.

> Student A: So you're kind of bringing together what you know from other things whereas in Question 1 you kind of — you're told what you have to do. So you're literally just kind of following a procedure really...whereas for the second one you kind of actually are more thinking yourself...it is more difficult, ya, but it kind of helps you understand it better. You see the relationship between them.

Her thoughts were echoed by the majority of the interviewees. Two students also spoke of the benefit of unfamiliar tasks for assessing their own understanding of concepts. For example, Student E when referring to an example generation task, stated she found it beneficial because

> Student E: it kind of proves you understand it more.

The interviews give some insight into how the tasks assigned encourage or stimulate thinking practices and ways of working. Some interviewees described certain tasks, which they had identified as unfamiliar or non-procedural, as encouraging habits such as thinking, analysing, questioning or exploring patterns. Eight of the ten students interviewed (A, C, D, E, F, G, H, I) asserted that the unfamiliar tasks made them think more or think for themselves. For instance, Student C said about a conjecturing task:

> Student C: You have to think about it and then, it's not actually a procedure, it's about you analysing the pattern and stuff.

Student B described how performing an analysing reasoning task prompted her to ask herself questions:

> Student B: Ahm, because it makes you analyse the proof and ask yourself questions like why you do things like that. Whereas if you were just given the proof, the correct one, you just take it for granted that that was correct.

Student H spoke about exploring his example space when working on an example generation question:

> Student H: But you really have to think more about these and understand the concepts and the different – ahm - possible solutions that may be there and why one solution isn't going to work.

A number of the students (A, B, C) also mentioned that, when they encounter an unfamiliar task, they refer to the basic definitions or theorems on the course and think about what they know (in relation to the task) and how they can apply it. One student (A) explained that she approached an example generation exercise by breaking it down or taking it step-by-step but also "drawing on other things" that she knew and bringing them together. Student C described using the following techniques (sometimes in combination) when confronted with various unfamiliar tasks: sketching a graph, examining different cases, generating examples, generalising, working backwards.

Discussion

The responses of the students interviewed here show that they are aware of different requirements for mathematics learning at school and university. Their views reflect the findings of research literature on the transition from second to third level mathematics in which there seems to be general agreement on a requirement at university level for more relational and conceptual understanding and more flexibility in thinking or approaching mathematical problems in comparison to second level mathematics (for example, Clark & Lovric, 2008; De Guzman et al., 1998). Moreover, most of the interviewees welcome this change in focus. It is worth underlining that the comments of the students interviewed suggest that the tasks assigned to them may not only have the effect of stimulating the development of conceptual understanding, but may also have some potential to raise an *awareness* that more than instrumental understanding is required at university, thereby easing their transition to university practices.

All of the interviewees stated that some of the unfamiliar tasks assigned to them in this project encouraged them to think more, or to analyse or question the information given. Though they reported having struggled with the unfamiliar tasks, they found such tasks beneficial regarding the development of conceptual understanding and their learning more generally. Some of the interviewees also explicitly described how these tasks led them to connect ideas met previously. Various authors have made recommendations about the type of tasks that should be assigned to undergraduate students in order to gain the required relational or conceptual understanding and flexibility (for example, Geuedet, 2008; Boesen et al., 2010) and it would seem that the students here believe that the tasks they were assigned go some way towards addressing this recommendation.

However, it should be noted that some of the interviewed students also described significant advantages of familiar and/or procedural tasks on their learning and confidence.

Acknowledgement

The authors would like to acknowledge the support of a NAIRTL grant.

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