College Attendance, Tuition and Family Income*

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Preliminary- Please do not quote

October 1, 2002

*I am grateful to Arnaud Chevalier, Paul Devereux, Aedin Doris, Donal O'Neill and seminar participants at NUI Maynooth and the Irish Economic Association conference in Mullingar for helpful comments regarding this work. I would also like to thank the Irish Social Science Data archive at ISSC for providing me with the data.

1 Introduction

There is broad agreement in the literature about the fact that there is a positive relationship between parental income and college attendance. However there is less agreement about why this relationship exists. The explanations for this relationship fall broadly into two camps. The first explanation is that richer parents find it less costly to send their children to college. Cameron and Heckman distinguish between what they call long-term credit constraints and short-term credit constraints. A short term credit constraint means that parents cannot afford to send their children to college. Long term credit constraints refer to the notion that poorer parents have less resources to spend on their children's education throughout their life and thus they are less likely to be in a position to attend college later on in life. The alternative explanation is that children from richer parents have a lower distaste for education and thus acquire more education. It is important from a policy point of view to distinguish between these explanations. If children were short-term credit constrained then the government might want to introduce a system of loans or grants to encourage children from lower income backgrounds to attend college. The problem with being credit constrained is that children will have money once they graduate but they do not have the funds before they go to college. If it was due to taste factors then access programs for example might be more beneficial. To distinguish between these two factors we would ideally like to conduct an experiment. This might involve taking a group that is identical in all respects and giving one group increased loans to attend college. This is not really feasible. However in 1995 the Minister for Education in Ireland announced that undergraduate fees would be abolished. In 1995/96 students paid half fees and in 1996/97 undergraduate fees were completely abolished. The press release from the Department of Education on February 8 1995 stated:

'Today's decision on abolishing undergraduate fees aims at providing universal access to third level education. The psychological impact of today's decision will encourage and allow people to consider pursuing a third level education as a very realisable option in their life choice... The abolition of third level fees is a major step forward in improving access to higher education - fees are no longer a barrier'[1]

[1] In conjunction with this removal, the Minister also announced changes in the tax relief

This policy will certainly relax the budget constraint. [2] . However this policy alone does not affect the disutility associated with attending college.

In this paper I examine the effect of this policy on college attendance by social class. In the Section 1, I outline a simple model which examines the theoretical predictions of the effect of such a change in fees. In Section 2, I discuss the data set and the methodology used for the empirical work. Section 3 provides preliminary results and discussion.

2 *Model*

Consider a model of educational attainment, similar to Dynarski (2000) and

Card (2000) in a world where there are no means tested grants and everyone pays fees. Individuals differ by parental income and tastes for education which is correlated with parental income.

$$V(S, c(t)) = \int_0^s (U(c(t)) - \gamma(t))e^{-\rho t}dt + \int_s^\infty (U(c(t))e^{-\rho t}dt$$

C is consumption. γ reflects disutility of schooling which depends on parental income. We assume that $\frac{d\gamma}{dy_p} < 0$. As income increases disutility falls. γ also depends on an individual effect u_i where u_i is iid Normal (0,1).

Students face a given interest rate, $r(y_p)$, where r is the opportunity cost of acquiring funds. It depends on parental income. $\frac{dr}{dy_p} < 0$. It is assumed that wealthier parents have easier access to capital markets and face a lower interest rate.

A student also faces direct costs of schooling $T(1 - A(y_p))$ where T is tuition costs and $A(y_p)$ is the proportion of aid which the student gets from the state. $0 \leq A \leq 1$. We assume that aid is means tested. Let us consider a simple system.We assume that children who's parents earn below a certain

arrangements for covenants.

[2]Ideally we would like to maintain lifetime incomes constant and simply give them a loan so that we could distinguish between wealth effects and credit constraints.

threshold value of income get more aid than those above the threshold.We assume that the relationship between A and y_p is such that for $y_p < y^t$(some threshold value) $A = A^h$ and for $y_p > y^t$ $A = A^l$ where $A^h > A^l$.This implies that starting with a y_p below the threshold, for small changes in y_p $\frac{dA}{dy_p} = 0$. For y_p above the threshold, $\frac{dA}{dy_p} = 0$ for all changes in y_p. For changes in y_p that cross the threshold then $\frac{dA}{dy_p} < 0$. This implies overall that $\frac{dA}{dy_p} \leq 0$. A student will invest in schooling until the marginal cost of acquiring funds=marginal rate of return. Let $y(s,t)$ denote earnings by an individual in period t with s years of schooling.

Then intertemporal budget constraint is:

$$\int_0^\infty (c(t))e^{-rt}dt = -\int_0^s (T(t)(1 - A(y_p))e^{-rt}dt + \int_s^\infty (y(s,t)e^{-rt}dt$$

$$MB(S) = \int_0^\infty \frac{\delta y(s,s+\tau)}{dS}e^{-r\tau}d\tau$$

$$MC(S) = y(s,s) + T(s)(1 - A(y_p)) + \frac{1}{\lambda}e^{-(\rho-r)s}\gamma(s)$$

If we assume that post-school earnings do not change over the life-cycle then so that y(s,t) = f(s) then

$$MB(S) = f'(s)/r$$

An individual's optimal level of school is where the MB(S) = MC or re-written:

$$\frac{f'(s)}{f(s)} = r\{1 + \frac{T(1-A(y_p)}{f(s)} + \frac{\frac{1}{\lambda}e^{-(\rho-r)s}\gamma(s)}{f(s)}\}$$

The left hand side measures the proportional increase in earnings per year associated with a change in schooling. The right hand side is the annuitized marginal cost of the Sth unit of schooling expressed as a fraction of foregone earnings. Using Card (2001), if we assume that u(c(t))=log c(t), that tuition costs are constant over the time in school, that the disutility of schooling does not change over time and that tuition costs are small relative to lifetime earnings then the right hand side is approximately equal to:

$$r(y_p)\{1+\frac{T(1-A(y_p))}{f(s)} + e^{-\rho s}\gamma(y_p, u_i)\}$$

This implies that optimal schooling s* is a function of:

$$s^* = f(T, A, \rho, \gamma(y_p), r(y_p), u_i)$$

$$\frac{dS}{dA} = \frac{-rT}{f''(s) - rf'(s) - r\gamma f'(s)e^{-\rho s} + r\gamma\rho f(s)e^{-\rho s}} > 0 \tag{1}$$

This implies that as aid increases optimal schooling increases. Appendix 2 provides more detail.

When there is no disutility associated with schooling , i.e. $\gamma = 0$, then

$$\frac{dS}{dA} = \frac{-rT}{f''(s) - rf'(s)} > 0 \tag{2}$$

Examining the effect of parental income on schooling:

$$\frac{dS}{dy_p} = \frac{-g'(y_p)}{g'(s)} \tag{3}$$

where:

$$g'(s) = f''(s) - rf'(s) - r\gamma f'(s)e^{-\rho s} + r\gamma\rho f(s)e^{-\rho s} \tag{4}$$

and

$$g'(y_p) = -\frac{dr}{dy_p}[f(s) + T(1 - A) + e^{-\rho s}\gamma f(s)] - \frac{d\gamma}{dy_p}re^{-\rho s}f(s) + rT\frac{dA}{dy_p} \tag{5}$$

Since $\frac{dr}{dy_p}$ and $\frac{d\gamma}{dy_p} < 0$,the first two terms are positive. The first term reflects credit constraints. Wealthier parents can borrow at lower rates. The second term reflects the disutility of schooling, children of wealthier parents have a lower distaste for schooling. The third term however is less than or equal to zero. It is negative if the changes in y_p cross the threshold, otherwise it is equal to zero. This implies that the sign of $g'(y_p) > 0$ unless the negative sign on the third term in the above equation outweighs the positive effect of the other two terms. With increases in parental income, schooling will generally increase as both r will fall and γ will fall. However if by increasing parental income, government aid is cut there is some chance that these children which are just above the threshold value of income have lower schooling than those below the threshold assuming that the decrease in aid is substantial and outweighs any credit constraint or disutility effect.

That is

$$\frac{dS}{dy_p} = \frac{-g'(y_p)}{g'(s)} > 0 \; if \quad g'(y_p) > 0$$

Under the above assumptions, as parental income increases, optimal schooling increases.

The expression for the term $\frac{dS}{dy_p dA}$ is more complicated. If we take the simple case where we assume that A is exogenous, $\gamma = 0, f''(s) < 0,\ f'''(s) = 0$, then of $\frac{dS}{dy_p dA}$ is:

$$\frac{dS}{dy_p dA} = \frac{-\frac{dr}{dy_p} T[(f''(s))^2 - rf'(s)f''(s) + r^2 f''(s)[(f(s) + T(1-A)]}{[f''(s) - rf'(s)]^3} < 0$$

$$\frac{dS}{dy_p dA} < 0.$$

As aid increases, the gap in optimal schooling levels by parental income falls.

It becomes more difficult to sign this term with the added assumptions that $\gamma > 0$ and $\frac{dA}{dy_p} \neq 0$.

Next, we want to examine the proportions attending college. The signs on this will depend on what we got above.

Let us assume that we can write s* as
$s^* = f(T, A, \rho, \gamma(y_p), r(y_p)) - u_i$

We assume that attending college involves achieving at least a level of s, s^c.

Therefore:
$P(s^* > s^c) = P(s^* = f(T, A, \rho, \gamma(y_p), r(y_p)) - u_i > s^c)$

$P(u_i < f(T, A, \rho, \gamma(y_p) - s^c) = \Phi(u_c)$

How does income affect the proportion of people attending college?

$\frac{d\Phi(u_c)}{\delta y_p} = \phi(u_c)\frac{\delta u_c}{\delta y_p} > 0$ if $\frac{\delta s^*}{dy_p} > 0$

As income increases, the proportion of people going to college increases. The cutoff u_c increases with income ($\frac{\delta u_c}{\delta y_p} > 0$ if $\frac{\delta s^*}{dy_p} > 0$) . Under these assumptions, the model predicts that a larger proportion of people attend college from higher income groups for two reasons. Firstly they have a lower cost of paying tuition and also they have a lower distaste for college.

What happens the gap in the proportions, attending college by income group,as A increases? What is the sign of $\frac{d^2\Phi(u_c)}{\delta y_p \delta A}$? For example does the difference in the proportion going to college by income group narrow or widen?

$\frac{d^2\Phi(u_c)}{\delta y_p \delta A} = +\phi(u_c)\frac{\delta u_c}{\delta y_p \delta A} + \phi'(u_c)\frac{\delta u_c}{\delta y_p}\frac{\delta u_c}{\delta A}$

If we assume that $\frac{\delta s^*}{dy_p} > 0 \frac{\delta s^*}{dA} > 0$ and $\frac{\delta s^*}{dy_p \delta A} < 0$.

The first term (term 1) in this expression, is negative. The term $\frac{\delta u_c}{\delta y_p \delta A}$ shows what happens the gap in the cutoffs between the low and high income people as A increases. In this case, this has a negative sign which says that the gap narrows as A increases. The sign of the second term(term2) $\phi'(u_c)\frac{\delta u_c}{\delta y_p}\frac{\delta u_c}{\delta A}$ is determined by the sign of $\phi'(u_c)$. If $\phi'(u_c) < 0 (> 0)$ then term 2 is negative(positive). If $\phi'(u_c) < 0$ (> 0), then u_c is located to the right (left) of zero.

This would mean that the gap the between the lower and higher income group in the proportion going to college should narrow as aid increases. However if $\phi'(u_c) > 0$ then the result is ambiguous. From this model we cannot predict exactly whether a higher proportion of low or high income people will go to college as result of the policy change. The introduction of free fees coincided with the abolition of tax relief for covenants. If as the department's press release stated 'covenants benefit the better off disproportionately', then this suggests that the introduction of free fees for the better off was likely to be offset by the changes in tax regulations so their budget constraint may be unaffected.

In Ireland, there are some students who always had their tuition paid by the state based on their family income or on what specific course they were taken. I am going to ignore the second group for the moment. Let us suppose that there are three income groups, low middle and high. We can assume that A=1 for the low income group and that A<1 for the other two groups. With the introduction of free fees for everyone, we would expect no change in the lower income group and the proportion attending college in both of the other two groups to increase. However as we said already, it is not clear whether the gap in the proportions attending college between the group 2 and 3 actually widens or narrows.

3 Proposed Methodology and Data

Since the predictions from the theoretical model are ambiguous. To examine the impact of the policy, I propose to use the data on school leavers and the difference-in-difference estimator to examine the impact of the policy. This can be calculated using a limited dependent variable model such as a probit or logit model.

$$c_{it}=\beta_0+\beta_1(y_p med)+\beta_2(Y_p high)+\beta_3 free+\beta_4(free*y_p med)+\beta_5(free*y_p high)+\delta(X)$$

Where c_{it} is a dummy variable which equals 1 if the person i goes to college from cohort t, zero otherwise. Let us assume for convenience that there are three levels of parental income, low, medium and high. We will assume that individuals from the lower level of income are already entitled to free fees. Free is a dummy variable =1 if free fees have been introduced for everyone,zero otherwise. X reflects other control variables.

The coefficients $\beta_1(\beta_2)$ reflects the difference in average attendance rates between the low and medium(high) income levels before the free fees was introduced. The coefficient β_3 reflects average changes in attendance rates for the lower income group before and after the policy was initiated. $\beta_4(\beta_5)$ measures the change in attendance by the medium(high) income group over and above the low income group. This is a difference in difference estimator. If we assume 1) that this change in policy did not affect lower-socioeconomic groups and 2) this was the only policy change that took place over this time period then we can interpret this as the true effect of the policy change. There may be some problems with assumption 2). It is possible over this time period that there were other policies aimed specifically at increasing attendance from lower socio-economic groups.[3] We may also have to control for changes in the quality of students over time.

Students begin second level education in Ireland at about the age of 12. Second level education consists of a junior cycle which lasts 3 years and a senior cycle which lasts for 2 to 3 years. At the end of the senior cycle, students take a state exam called the Leaving Certificate. Access to third level courses depends on the results obtained. If a student gets the required number of points for a particular course in their Leaving Certificate, they are automatically entitled to enter the course.The data set which I use is the School Leavers survey available annually from 1980-1999[4]. It surveys about 2000 students each year about one year after leaving school and has information about their school experience, what they did after school, information on college attended, course taken , employment record etc. One of the drawback of the data set is that it does not provide information on parental earnings. However there is information on parental occupation. I use this occupational coding information to form 7 social classes. I have a separate category for farmers. These are described in the Appendix. I do not know whether a given student would have been entitled to a grant. I only know for those who attended whether they received a grant or not. The probability of receiving a grant increases with social class number. Farmers have quite a high probability of receiving a grant also.(See Appendix table A1 for details). I limit my sample to those who left with their Leaving Cert completed.

[3] For instance the targeted initiative was launched by the HEA in 1996/1997 see Osborne and Leith(2000)

[4] I do not have the 1997 and 1999 data yet.

4 Preliminary Results and Discussion

Table 1 shows the result from the probit estimates. From the first column we see that there is a significant difference in college attendance rates across social classes. The Professional and Farmers groups have a high probability of attending. However looking at the second column, there is not a significant change in these proportions over time. This would indicate that the removal of the fees barrier did not result in significant changes in college attendance. There are many possible reasons for this result . Firstly 1998 may be too early to look at the effects of the policy change or I may not have enough data to identify a true effect. Secondly fees may represent a very small component of the cost of attending college. The opportunity cost and other direct costs such as maintenance costs could be significant. I have looked at students who lived in cities. It seems reasonable that their maintenance costs would be smaller so that a reduction in fees might be important. For Dublin city, there is a significant negative correlation between living in this area and college attendance but this does not change significantly after 1998.This suggests for those where fees make up a large proportion of their direct costs of college, the attendance rate did not seem to change. For the other cities there are no significant effects. Over this period, Ireland would have experienced a boom so that the opportunity cost of schooling might have increased a lot. If this affected all classes equally then the difference in difference estimator should control for this. However if particular social classes were affected differently then I cannot identify this affect. Another possible reason why there appears to be no significant changes in college attendance after free fees were introduced relates to selectivity. I have limited my sample to those who had completed their Leaving Cert in each year. I need to consider the likely effects of this. Firstly, if policy makers have been more successful at encouraging students to complete their Leaving Cert and these were marginal students then we would expect the proportion of students going to college from this larger base might actually fall.This may not necessarily be a bad thing as we are encouraging more people to stay on in school. From Table 3, over the period 1995-1998,there does not appear to be a large changes in differences in Leaving Cert completion by social class. The other problem which I have not controlled for is Leaving Cert performance. Of course not everyone who wants to go to college can go, they must meet certain points requirements. The School-Leavers survey does have information on Leaving

Cert grades. I used this to calculate the total points. However there is a lot of missing grades for 1998 so I am not sure how reliable the data is for this period. Table 2 shows the results for the probit estimates using 1994 and 1995. We see that part of the gap in college attendance by social class goes away once we control for points. This could reflect long-term family income problems, where wealthier parents have more resources to spend on their children over their life-time. The Minister for Education in her press release stated that 'the abolition of undergraduate fees is just one strand of the... overall strategy... A wide range of initiatives and improvements have been implemented at first and second level. These policies will take much longer to take effect and may explain why there has not been a significant change in the gap in college attendance by social class.

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Table 1

Proportion Attending College 1994,1995,1998

n	4748	4748
constant	-.64(.07)	-.67(.11)
sclass1	1.10(.09)	1.12(.11)
sclass2	.84(.08)	.93(.10)
sclass3	.59(.08)	.61(.11)
sclass4	.20(.08)	.24(.10)
sclass5	.12(.10)	.17(.12)
sclass7	.77(.08)	.78(.11)
y98	-.05(.04)	.06(.15)
sc1*y98		-.04(.19)
sc2*y98		-.23(.17)
sc3*y98		-.07(.18)
sc4*y98		-.14(.18)
sc5*y98		-.15(.22)
sc7*y98		-.01(.18)

Table 2: Proportion Who Attended College

1994 & 1995 with information on total points				
n	2931		2931	
	coefficients	prob	coefficients	prob at mean points
constant	-.55(.09)		–2.56(.13)	-
sclass1	1.07(.12)	70	.45(.14)	.56
sclass2	.86(.11)	.62	.36(.13)	.53
sclass3	.56(.11)	.51	.27(.13)	.49
sclass4	.19(.11)	.36	.09(.13)	.42
sclass5	.07(.12)	.32	.04(.15)	.40
sclass6	-	.39		.39
sclass7	.72(.11)	.54	.38(.11)	.54
points*10			.008(.0003)	

Table 3

Proportion of Students Leaving School with Completed Leaving Certs by Social Class.

	1994	1995	1996	1998
total	.79	.82	.81	.81
sc1	.89	.92		.91
sc2	.88	.90		.89
sc3	.87	.89		.85
sc4	.75	.76		.76
sc5	.66	.72		.68
sc6	.52	.66		.61
sc7	.89	.87		.90

5 Appendix 1

Key for Social Class

sc1	higher professional, managerial, proprietors
sc2	lower professional, lower managerial
sc3	other non-manual
sc4	skilled manual
sc5	semi-skilled manual
sc6	unskilled manual
sc7	farmers

Table A1
Proportion Attending College who Receive Grants by Social Class

	1994 & 1995
total	
sc1	.25(.03)
sc2	.39(.02)
sc3	.55(.03)
sc4	.78(.02)
sc5	.75(.05)
sc6	.92(.04)
sc7	.67(.03)

Table A2
Average Total Points of Those who did Leaving Cert by Social Class

	1994	1995	1996	1998(?)
total	284	287	291	310
sc1	352	343		378
sc2	321	327		327
sc3	280	288		299
sc4	235	250		274
sc5	228	232		264
sc6	227	217		257
sc7	303	293		330

Table A3

Proportion Attending College by Social Class

	1994	1995	1996	1998
total	.47	.48	.44	.46
sc1	.67	.67		.68
sc2	.62	.58		.53
sc3	.47	.49		.47
sc4	.29	.38		.30
sc5	.30	.31		.27
sc6	.29	.21		.27
sc7	.55	.53		.56

6 Appendix 2

The First order condition is:

$\frac{f'(s)}{f(s)} = r\{1 + \frac{T(1-A)}{f(s)} + e^{-\rho s}\gamma(y_p, u_i)\}$

This can be re-written as:
$g(s, A, \gamma, y_p) = f'(s) - rf(s) - rT(1-A) - re^{-\rho s}\gamma f(s) = 0$

The 2nd order condition for a maximum requires that $g'(s) < 0$.
This implies that

$g'(s) = f''(s) - rf'(s) - r\gamma f'(s)e^{-\rho s} + r\gamma\rho f(s)e^{-\rho s} < 0$

$g'(A) = rT$

To find the relationship between aid and schooling.
$\frac{dS}{dA} = \frac{-g'(A)}{g'(s)} = \frac{-rT}{f''(s)-rf'(s)-r\gamma f'(s)e^{-\rho s}+r\gamma\rho f(s)e^{-\rho s}}$

This is positive since $g'(s) < 0$.

As aid increases, optimal schooling increases.

When $\gamma = 0$, that is, there is no disutility of schooling then the relationship between S and A is :

$\frac{dS}{dA} = \frac{-g'(a)}{g'(s)} = \frac{-rT}{f''(s)-rf'(s)}$

Now we examine the relationship between parental income and optimal schooling. Remember that γ, r and A depend on parental income. $\frac{dr}{dy_p} < 0, \frac{d\gamma}{dy_p} < 0$. We assume that children who's parents earn below a certain threshold value of income get more aid than those above the threshold.We assume that the relationship between A and y_p is such that for $y_p < y^t$(some threshold value) $A = A^h$ and for $y_p > y^t$ $A = A^l$ where $A^h > A^l$.This implies that starting with a y_p below the threshold, for small changes in y_p that $\frac{dA}{dy_p} = 0$. For y_p above the threshold, $\frac{dA}{dy_p} = 0$ for all changes in A. For changes in y_p that cross the threshold then $\frac{dA}{dy_p} > 0$

$$g(s, A, \gamma, y_p) = f'(s) - rf(s) - rT(1 - A) - re^{-\rho s}\gamma f(s) = 0$$

$$g'(y_p) = -\frac{dr}{dy_p}f(s) - \frac{dr}{dy_p}T(1 - A) + rT\frac{dA}{dy_p} - \frac{dr}{dy_p}e^{-\rho s}\gamma f(s) - \frac{d\gamma}{dy_p}re^{-\rho s}f(s)$$

$$g'(y_p) = -\frac{dr}{dy_p}[f(s) + T(1 - A) + e^{-\rho s}\gamma f(s)] - \frac{d\gamma}{dy_p}re^{-\rho s}f(s) + rT\frac{dA}{dy_p}$$

Since $\frac{dr}{dy_p}$ and $\frac{d\gamma}{dy_p} < 0$,the first two terms are positive. The first term reflects credit constraints. Wealthier parents can borrow at lower rates. The second term reflects the disutility of schooling, children of wealthier parents have a lower distaste for schooling. The third term however is less than or equal to zero. It is negative if the changes in y_p cross the threshold, otherwise it is equal to zero. This implies that the sign of $g'(y_p) > 0$ unless the negative sign on the third term in the above equation outweights the positive effect of the other two terms. With increases in parental income, schooling will generally increase as both r will fall and γ will fall. However if by increasing parental income, government aid is cut there is some chance that that these children which are just above the threshold value of income have lower schooling than those below the threshold assuming that the decrease in aid is substantial and outweighs any credit constraint or disutility effect.

$$\frac{dS}{dy_p} = \frac{-g'(y_p)}{g'(s)} > 0 \ if \quad g'(y_p) > 0$$

The expression for the term $\frac{dS}{dy_p dA}$ is more complicated. Even if we take the simple case where we assume that A is exogenous, $\gamma = 0, f''(s) < 0$, $f'''(s) = 0$, then the sign of $\frac{dS}{dy_p dA}$ is ambiguous. In this case it is equal to:

$$\frac{dS}{dy_p dA} = \frac{-\frac{dr}{dy_p}T[(f''(s))^2 - rf'(s)f''(s) + r^2 f''(s)[(f(s) + T(1-A)]}{[f''(s) - rf'(s)]^3}$$

$-\frac{dr}{dy_p}T > 0$ since $\frac{dr}{dy_p} < 0$ and $[f''(s) - rf'(s)]^3 < 0$
Taking what is inside the square bracket of the numerator:

We have:$[(f''(s))^2 - rf'(s)f''(s) + r^2 f''(s)[(f(s) + T(1 - A)]$

collecting terms we have:

$f''(s)\{f''(s) - rf'(s) + r^2(f(s) + T(1 - A)\}$

$f''(s)\{f''(s) - r(f'(s) - rf(s) - rT(1 - A)\}$

From the first order condition we know that $(f'(s) - rf(s) + rT(1 - A) = 0$

so we are left with($f''(s))^2$ which is positive. The numerator is positive and the denominator is negative so we have,

$\frac{dS}{dy_p dA} < 0.$

The terms become more complicated with the added assumptions of $\gamma > 0$ and $\frac{dA}{dy_p} \neq 0$.