William Rowan Hamilton (1805-1865): Ireland's greatest mathematician

FIACRE Ó CAIRBRE

Introduction

In 1865, shortly before his death, William Rowan Hamilton became the first Foreign Associate to be elected to the newly established National Academy of Sciences of the United States of America. This meant that the Academy deemed Hamilton to be the greatest living scientist.[1] The honour was richly deserved for Hamilton has emerged as a giant in the history of science. He is Ireland's greatest mathematician and is widely regarded as one of the world's most outstanding mathematicians ever. This remarkable Irishman spent much of his youth in Meath and had his early education in Trim.

William Rowan Hamilton was born at midnight, on 4 August 1805, at 29 Dominick Street in Dublin. His father, Archibald Hamilton, was a solicitor in Dominick Street and his mother, Sarah Hutton, came from a coachbuilder's family in Summerhill, Dublin. William had four sisters: Grace, Eliza, Sydney and Archianna. His godfather, Archibald Hamilton Rowan, was a prominent figure from Killyleagh, Co Down. He was a friend of Wolfe Tone and was involved with the United Irishmen. In 1794 he was imprisoned in Dublin for "endeavouring by tumult and by force to make alterations in the constitution and the government and overturn them both".[2] However, he managed to escape with the assistance of William's father and fled into exile in 1795. He returned eleven years later and William's father was the law agent who then acted on his behalf. It was from his godfather that William took his middle name, Rowan.

Early days in Trim

In 1808 William was sent to Trim to be educated by his uncle, James Hamilton, who was a curate and conducted the Meath Diocesan school. William spent his next fifteen years in Trim and later on he looked back on these early days with great fondness. He lived with James, James's wife, Elizabeth, and their children, and James's sister, Aunt Sydney, in a large house that is now called St. Mary's Abbey, which is beautifully situated beside the Yellow Steeple, on the banks of the Boyne, across from the spectacular ruins of Trim castle. The house also served as the school, and in 1809 it had fourteen day scholars. Later on the school also accommodated boarders.[3] The house was built in 1425 by Lord Talbot and was also known as Talbot's castle. Part of the ruins of St. Mary's Abbey, built in 1368, also made up a section of the house. The Yellow Steeple is part of the original tower of the Abbey and William later used it as a gnomon for a huge sundial. He also used it as a site for his own semaphore system.

St. Mary's Abbey offered an ideal environment for nurturing genius as it provided a vibrant and stimulating atmosphere for the young William. James was a well respected educator and an honours graduate in classics from Trinity College, Dublin. He was also a member of the Royal Irish Academy and contributed to its Transactions. James had broad interests and was a very conscientious and effective mentor. His prospectus of a course of instruction for the school was impressive in its diversity of topics.[4] William wrote: "I would cherish the fondness for classical and for elegant literature which was early infused into me by the uncle to whom I owe my education".[5] Aunt Sydney was very perceptive and enthusiastic, and her many letters provide an invaluable account of William's early years. The following is from a letter by Aunt Sydney to William's mother in 1808:

> Your dear little Willy is very well and improving very fast; indeed James pays unremitting attention to him, and Willy is a very apt scholar, and yet how he picks up everything I know not, for he never stops playing and jumping about; I sometimes threaten to tie his legs when he comes to say his lessons.[6]

In the next letter from Aunt Sydney, in 1809, we see that people (in particular, the Vicar of Trim, Mr. Elliot) are beginning to notice William's emerging talents:

> The room was full of gentlemen both belonging to the town and from Kells, Navan ... but Willy was the subject of conversation most of the evening. Mr. Elliot declared that such a child he had never seen and that he was certain

> there was not another such in Ireland, that he not only read well, but was made to understand what he read. I went out to visit the other day and every place I went I was told that Mr. Elliot had been there telling about Willy and that he could talk of nothing else; but if he knew half the things Willy knows, he would indeed be astonished.[7]

The following letter from Aunt Sydney in 1810 portrays William as a playful prodigy:

> There was a Mr. Montgomery with the Elliots the other day; he is a curate and takes a certain number of boys. We were there : they had been talking a great deal of Willy to him; however he looked on it all as nonsense, 'till after tea Mr. Elliot got a Greek Homer and desired Mr. Montgomery to examine him. When he opened the book, he said, "oh this book has contractions, Mr. Elliot, of course the child cannot read it". "Try him, sir", said James. To his amazement Willy went on with the greatest ease. Mr. Montgomery dropped the book and paced the room; but every now and then he would come and stare at Willy, and when he went away, he told Mr. and Mrs. Elliot that such a thing he had never heard of and that he really was seized with a degree of awe that made him almost afraid to look at Willy. He would not, he said, have thought as much of it had he been a grave, quiet child; but to see him the whole evening acting in the most infantile manner and then reading all these things astonished him more than he could express.[8]

Formal schooling began for William in 1816. He had private tuition from James and Sydney before that. In this letter to his father, we catch a glimpse of his mind at the age of fourteen:

> I sometimes feel as if the bottle of my brain were like those mentioned, I think in Job, "full and ready to burst"; but when I try to uncork and empty it, like a full bottle turned upside down, its contents do not run out as fluently as might be expected; nor is the liquor that comes off as clear as could be wished. Perhaps I am not long enough in bottle to be decanted. I fear indeed the vintage of my brain is yet too crude and unripe to make good wine of. When it shall have been more matured, I hope the produce of the vineyard you have planted and watered will afford some cups "to cheer but not inebriate" you.[9]

William's mind did indeed produce copious cups of the very finest wine.

William lost both parents before he reached the age of fifteen: his mother died in 1817 and his father died two years later. Consequently, he was now nominally responsible for his four orphaned sisters and this responsibility had an important impact on his future career decisions. His sisters had been educated at a Moravian Settlement near Manchester. The Moravian connection arose from the fact that William's aunt, Susan Hutton, had married Joseph Willey, who was a minister in the Moravian Brethren near Ballinderry, Co. Antrim. William now began to confide in Eliza who was two years younger than him. She would be his closest sister. He wrote: "Eliza is perhaps the person to whom I have talked most and most freely".[10] She was now living with the Willeys in Ballinderry and William wrote to her:

> The hope of being, if we are spared, useful to my dear sisters will, I trust, stimulate, and the hope of God's blessing in doing so animate, my exertions. Uncle encourages me to hope that with the divine blessing they will be successful.[11]

Shortly after his father's death, William began to write poetry and this interest played a prominent part in the rest of his life. He went on to win the Chancellor's poetry prize twice in Trinity College and publish his literary work in journals and magazines. Eliza also developed a keen interest in poetry and became a poet: she published a volume of her poetry in 1839. Here is an extract from William's poem in celebration of the feast of Halloween in 1822, in which he reveals how much he cherished the occasional reunions with his sisters:

> To me this day has highest charms,
> It gives Eliza to mine arms;
> Again our kindred spirits meet,
> And every joy is doubly sweet;
> And while my life flows smooth away;
> This will have been my happiest day.[12]

From his early days, William enjoyed rambling and playing in the countryside around Trim, and visiting the many ancient monuments in the locality. He used to go for long walks through the countryside and along the banks of the Boyne, sometimes deep in thought. He retained this habit throughout his life and made his most famous discovery on such a walk, along the Royal Canal in 1843. He had an interest in the rich history and archaeology of the surrounding area from an early age. Indeed, the Boyne valley for about 20 miles around Trim, is arguably the most impressive archaeological region in Ireland, and William was well aware and appreciative of this. The following lines from his "Verses on the Scenery and Associations of Trim", written in 1823, capture his rapture and awe for the beauty of his natural surroundings at St. Mary's Abbey:

> Yet lovely all the prospect seems
> And suited to a poet's dream.
> O'er all the verdure of the scene
> Fresh sunbeams fling a brighter green;
> Clouds of every shape and dye
> Are scattered o'er the deep blue sky;
> And melody of many a bird
> In the charmed air is heard;
> Through those boughs so closely twining
> The river's sparkling waves are shining;
> Adown its course, the little bays
> Are glittering in a fuller blaze;
> And as by fits the gentle blast
> So fondly o'er the bosom passed
> Of the bright Naiad in repose,
> Saw you not how new beauties rose?
>
> How well with surrounding bloom
> Contrasts those ramparts' solemn gloom!
> With what a proud and awful frown
> Appear their turrets to look down
> On all beside that meets my gaze,
> On monuments of later days,
> On all that modern art around
> Has reared upon this classic ground!
> O genius of those ruined towers,
> Who lovest to dwell in ivy-bowers,
> Have I not paid thee honour due;
> Have I not kindled at the view
> Of thy majestic walls, surveyed
> While the meridian sun has stayed
> His steeds above them, or his light
> At morn or eve illumed their height,
> Or bright Orion from above,
> Or that fair Vesper, star of love![13]

Interest in mathematics

During his early years, William showed exceptional talent for languages and the classics. His mathematical education was good but he had not yet developed as keen an interest in mathematics as he had in languages and the classics. The turning point came in 1818 when a fourteen-year-old American child prodigy, Zerah Colburn, visited Dublin. Colburn was in Europe exhibiting his extraordinarily quick computational skills and William was considered good enough to be invited to compete against him. William lost, but the loss seems to have sparked his interest in mathematics. He wrote to his cousin, Arthur, in 1822:

> I was amused this morning, looking back on the eagerness with which I began the different branches of Mathematics, and how I always thought my present pursuit the most interesting. I believe it was seeing Zerah Colburn that first gave me an interest in these things.[14]

The two met again when Colburn returned in 1820. Colburn now revealed to William all his tricks for determining factors and finding roots, but William was more interested in understanding why Colburn's methods worked, rather than in the speed of the calculations. William subsequently wrote up a collection of "Remarks", which showed the limitations of Colburn's methods. The methods were heavily dependent on an exceptional memory and thus were quite limited as regards practical use. Apparently, Colburn did not have a genuine talent for mathematics, beyond the ability to calculate at high speed;[15] there is much more to mathematics than mere "calculation".

William's first innovation came in 1820 with his own design of a semaphore system. Semaphores were still of scientific interest on account of the threat of an attack from Napoleon's forces. The famous inventor, Richard Lovell Edgeworth, from Edgeworthstown in Co. Longford, had just tested out a new semaphore on the Hill of Collon, in Co. Louth. However, William's design was much easier to use than Edgeworth's one. Figure 1 shows William's semaphore design.[16]

To communicate the letter "N", one first locates N in the box of letters. N appears in the third horizontal row and the fourth vertical column. Thus, one places the arms in position 3 first and then in position 4. In this way the letter N is transmitted and words can then be formed. With the aid of telescopes, people

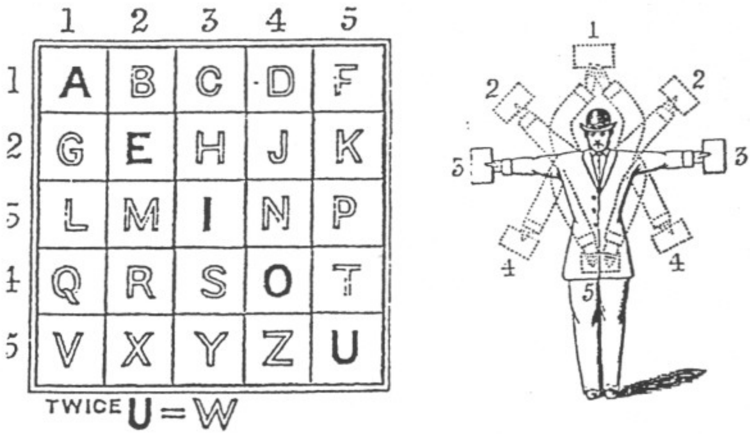


Figure 1.

were able to use semaphores to communicate messages between two heights which were a good distance apart.

In 1820, William was reading Isaac Newton's *Principia*,[17] and in 1821 he solved a problem that his tutor, Charles Boyton, an eminent mathematician in Trinity, was unable to solve. Later in 1821, he corrected an error in Laplace's book, *Mécanique Céleste*, that had not been noticed before. As Laplace was a famous French mathematician at the time, this was quite an achievement for a sixteen-year-old and news spread quickly. When John Brinkley, the Astronomer Royal of Ireland in Dunsink Observatory, heard the news, he expressed a desire to meet William. After their meeting, Brinkley told him that he was welcome to visit Dunsink at any time.

William now had a fervent ambition to excel as a mathematician. We see his aspiration to greatness in a letter to his aunt Mary in 1822:

> Mighty minds in all ages have combined to rear upon a lofty eminence the vast and beautiful temple of Science and inscribed their names upon it in imperishable characters; but the edifice is not yet completed: it is not yet too late to add another pillar or another ornament. I have yet scarcely arrived at its foot, but I may aspire one day to reach the summit.[18]

This youthful aspiration was to be richly realised.

Beauty and practical power

Mathematics has many features. I will discuss some of the features here because they are relevant to Hamilton's story and may be of interest to the general reader.

Firstly, mathematics is an exceptionally powerful practical tool. It provides a language, in which many problems in science, engineering, finance and many other areas, can be formulated. It then provides methods by which these problems can be tackled and possibly solved. In the seventeenth century, Galileo wrote:

> The great book of nature lies ever open before our eyes and the true philosophy is written in it. But we cannot read it unless we have first learned the language and the characters in which it is written. It is written in mathematical language and the characters are triangles, circles and other geometrical figures.[19]

Take the example of motion, which is fundamental to our understanding of the physical world. Whenever an object moves, there are variables like time, speed, acceleration, position, and forces (e.g. gravitation, friction etc.), which play a very important role in the motion. The object could be a car, an aeroplane, a train, a planet, a satellite, an electron, anything that moves. It is essential to know how these variables are related if we want to understand motion. The relationships between the variables are expressed in mathematical language. We then need to use mathematical concepts and techniques (as well as ideas in physics, engineering etc.) to solve the particular problem, which could be the launch of a satellite into orbit or the design of the new engine in Mercedes. These concepts and techniques could involve algebra, geometry, trigonometry and an abundance of more advanced concepts. The list of important mathematical applications is endless: it varies from the designs of the most modern electronic products to the technique of carbon dating in archaeology. The applications of mathematics are all around us: every time we switch on the radio or talk on the telephone we are using the outcome of centuries of deep mathematical research. We might not be aware of this because we see only the physical product, but that product is based on many profound ideas from the history of science. In the physical world, mathematics can also do a lot more than solve problems; it can analyse, predict and prescribe; it can provide deeper insight; and it can generate and explore new ideas. Mathematics has a rich history, and it has played a very significant role in our civilisation.[20, 21, 22]

Secondly, mathematics can have a certain beauty. Some mathematics can be considered as an aesthetic creation of the mind, without any initial motivation from the physical world. This creation of the mind can involve imagination, intuition, symmetry and harmony. The quest for aesthetic pleasure has guided much of the evolution of mathematics. Very often this search for beauty in mathematics has led to new ideas and to discoveries of new theories that are indispensable in the practical world. Thus, the practical power of mathematics is often an offspring of the search for beauty in mathematics.

Science, to Hamilton, meant mathematics.[23] He was a mathematician who searched for beauty and aesthetic pleasure in mathematics, and was not necessarily concerned with the practical uses. Yet, his work has had very significant applications in the physical world. This was no surprise to him, as he believed that any mathematics could be applied to the physical world, even though it did not originate from that world. This was so, he believed, because "the forms of our thought that give us a true science are mirrored closely by the actual events in the physical world; the organisation of our minds and the organisation of the physical world are in happy accord". He believed that this mirroring was the work of God.[24] Hamilton regarded mathematics "as an aesthetic creation, akin to poetry, with its own mysteries and moments of profound revelation".[25] He held that mathematics and poetry both "employed the creative imagination to construct links between the intelligent thinking self and the external world".[26] He wrote to Wordsworth:

> For Science, as well as Poetry, has its own enthusiasm and holds its own communion, with the sublimity and beauty of the Universe.[27]

Once Hamilton had sipped the pleasure of mathematics, he was captivated. He wrote:

> The pleasure of intense thought is so great, the exercise of mind afforded by mathematical research so delightful, that, having once fully known, it is scarce possible ever to resign it.[28]

Theory of system of rays

The germs of Hamilton's first major contribution to science came in 1822 when he was seventeen. In a letter to Eliza in that year, he mentions "some curious discoveries". These discoveries would lead on to a paper entitled "On Caustics".

> I have some curious discoveries – at least they are so to me – to show Charles Boyton when we next meet.... No lady reads a novel with more anxious interest than a mathematician investigates a problem, particularly if in any new or untried field of research. All the energies of the mind are called forth, all his faculties are on the stretch for the discovery. Sometimes an unexpected difficulty starts up, and he almost despairs of success. Often, if he be as inexperienced as I am, he will detect mistakes of his own, which throw him back. But when all have been rectified, when the happy clue has been found and followed up, when the difficulties, perhaps unusually great, have been completely overcome, what is his rapture! Such in kind, though not in degree, as Newton's, when he found the one simple and pervading principle which governs the motions of the universe, from the fall of an apple to the orbit of the stars.[29]

His paper "On Caustics" was submitted to the Royal Irish Academy (RIA) in 1824, while he was still a student in Trinity College. It was rejected, essentially because the examiners could not understand his work.[30] Hamilton continued his mathematical investigations in the same area and his work, "On Caustics", formed the basis for a paper called "Theory of System of Rays", which he submitted to the RIA. This was accepted and published in 1827. It was a major breakthrough in optics, which is the important area of physics that deals with light and vision (e.g. lenses, microscopes, telescopes, mirrors, cameras etc.). His idea was to apply algebra to describe the path of light:[31] he used Descartes' algebraic techniques as well as his own new ideas to solve the geometrical problems in optics.[32] In 1637, Descartes, the famous mathematician and philosopher, had revolutionised mathematics by showing how algebraic equations could describe geometric figures (like lines, circles, spheres etc.) and their properties. Problems in geometry, whose solutions previously required diagrams and geometrical constructions, could now be translated into algebraic problems and potentially more easily solved using algebraic techniques. This approach, by which we translate a problem in one area to an equivalent problem in a seemingly totally different area where it might be more easily solved, is extremely powerful in mathematics. In Hamilton's work, all properties of any optical system are incorporated into just one equation. The solution of this equation gives what he called his "characteristic function". This function gives a complete description of the optical system, because if a ray of light enters the optical system, the function describes how the ray leaves the system.[33]

Hamilton conveys the desire behind his work, in a letter to his friend, the renowned poet and philosopher, Samuel Taylor Coleridge. Hamilton had met Coleridge in 1832 and they corresponded for the next two years, until Coleridge's death in 1834.

> My aim has been, not to discover new phenomena, nor to improve the construction of optical instruments, but with the help of the Differential or Fluxional Calculus to remold the geometry of Light by establishing one uniform method for the solution of all problems in that science, deduced from the contemplation of one central or characteristic relation.... my chief desire and direct aim being to introduce harmony and unity into the contemplations and reasonings of Optics, considered as a portion of pure Science.[34]

Hamilton was primarily interested in achieving mathematical harmony and unity in optics. Yet, his work did have important applications in optics and in the design of optical instruments.

College days

In 1823, Hamilton obtained first place in the entrance examinations for Trinity College. His examination performances were exceptional in Trinity as he won an abundance of awards and prizes in science, classics and poetry. In his first year, he received an "optime" for an examination on Homer. An optime was a mark off the scale, indicating that the student had a total mastery of the subject. No student had received an optime in the previous twenty years, and the distinction had never been awarded to a first-year student.[35] Mr. Montgomery, mentioned in Aunt Sydney's letter back in 1810, would not have been too surprised with Hamilton's optime for Homer! In 1826 Hamilton received an optime in both science and classics. No undergraduate had ever achieved this before.

Hamilton's reputation in Dublin was quickly established and during his college days he frequently socialised with important figures. Many lifelong friendships were formed in this way. Maria Edgeworth, who was then fifty-six, and a leading novelist, was one such celebrity who would form a lasting friendship with Hamilton. Brinkley had told her, before she met Hamilton, that she might be meeting a second Newton.[36]

On 17 August 1824, Uncle James introduced Hamilton to the Disney family in Summerhill, Co. Meath. This was when Hamilton first met Catherine Disney. Hamilton was captivated and instantly fell in love with Catherine.[37] Many years later, in 1861, Hamilton described the event to Catherine's sister, Louisa:

> Wonderful hour! of my sitting, irregularly, from the very first, — beside her; when, without a word said of love, we gave away our lives to each other. She was, as you know, beautiful; I was only clever and (already) celebrated.[38]

The "irregularly", above, refers to the fact that Hamilton escorted Catherine into dinner when he should have led Catherine's mother in.[39] Catherine was of a similar age to Hamilton and the two met several times afterwards. Hamilton made his feelings for Catherine explicit in a Valentine ode in 1825:

> Forgive me, that on bliss so high
> Lingers thrilling phantasie:
> That the one Image, dear and bright,
> Feeds thought by day, and dreams by night:
> That Hope presumes to mingle thee,
> With visions of my destiny![40]

Hamilton also combined, in the same ode, his aspiration to greatness with his love for Catherine:

> Perchance it may be mine to soar
> Higher than mortal ever before:
> Climb the meridian steeps of fame,
> And leave an everlasting name.
> If such my lot .... O then how sweet,
> To lay my triumphs at thy feet.

Shortly after sending this ode, Hamilton received news that Catherine was to marry William Barlow, who was fifteen years her senior. This apparently was an arranged marriage, because later letters show that she had always loved Hamilton. With news of the marriage, Hamilton was driven close to suicide. He contemplated throwing himself into the Royal Canal as he walked to Dunsink Observatory for a breakfast meeting with Brinkley.

He refers to this event in a letter to his colleague, P.G. Tait, thirty-two years later:

> I wish I could add that it was religion, or even generally my belief in the Bible, which protected me. My recollection has always been that it was simply a feeling of personal courage, which revolted against the imagined act, as one of cowardice. I would not leave my post; I felt that I had something to do.[41]

Hamilton never forgot Catherine. In 1848 he wrote to his friend, the poet, Aubrey de Vere:

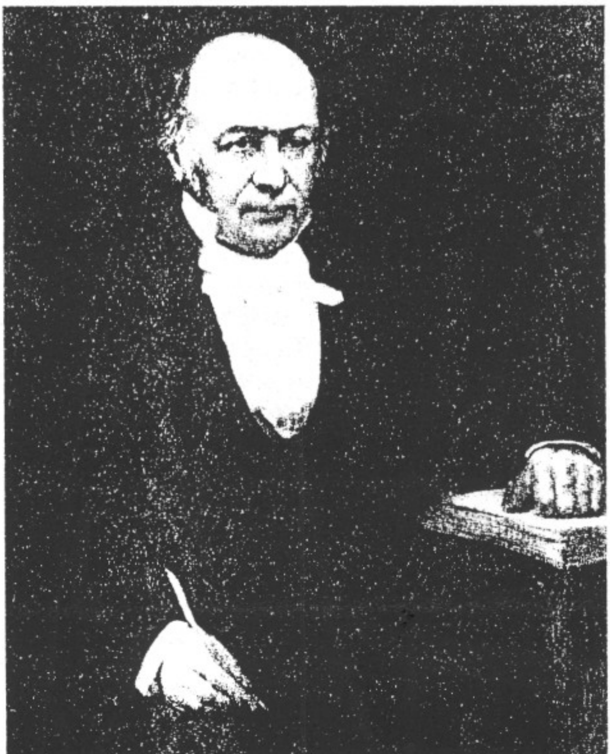
> The same remembrance has run like a river through my life, hidden seemingly for intervals, but breaking forth again with an occasional power which terrifies me – a really frightful degree of force and vividness.[42]

Life at Dunsink Observatory

Dunsink Observatory was built in the 1780s and Henry Ussher was the first Professor there. Ussher was a descendant of the renowned Archbishop Ussher, who in the seventeenth century had made the famous scientific pronouncement that the world was created in October 4004 BC![43] Brinkley succeeded Ussher in 1792, and the title "Astronomer Royal for Ireland" was then added to the designation of the Dunsink post. Dunsink was one of four important observatories in Ireland during Hamilton's lifetime. The other three were at Birr, Armagh and Markree, Co. Sligo.[44] At Birr, William Parsons possessed the world's greatest telescope in 1845. This famous telescope has been recently restored and is located in Birr Castle. The mirror on the restored telescope is a recent addition, while the original mirror has been retained in London. Romney Robinson conducted the observatory in Armagh, and Edward Cooper owned the world's largest refractor lens at Markree in 1831.

The prospective salary in Dunsink was less than half that of a Senior Fellow in Trinity but the Dunsink post came with a free mansion and eighteen acres of land.[45] The position of Astronomer Royal for Ireland became vacant in 1827 because Brinkley had just been appointed Bishop of Cloyne. Hamilton was strongly encouraged to apply for the post, even though he was still an undergraduate in Trinity. There were many eminent people interested in the post, but Hamilton was offered the position one week after his application was received. Hamilton accepted and thus became a Professor while still an undergraduate (he graduated with a B.A. shortly afterwards)! There were two important considerations for Hamilton in deciding to accept the post. Firstly, it provided a perfect residence for him, and his sisters, whom he had vowed to support when their parents died. It would take a long time before he could afford such a house if he had stayed in Trinity. Secondly, the post involved very little teaching, with only one annual University course of 12 lectures on astronomy. Thus, Hamilton could concentrate on his mathematical research.

In 1827 Alexander Nimmo, the engineer for the Western District of Ireland, took Hamilton on a two-month tour of various engineering feats around Ireland and England. It was while staying in the Lake District that Hamilton first met William



William Rowan Hamilton

Wordsworth, who was then fifty-seven. They found each other's company quite stimulating, as can be seen in a letter from Hamilton to Eliza:

> He (Wordsworth) walked back with our party as far as their lodge; and then, on our bidding Mrs. Harrison goodnight, I offered to walk back with him, while my party proceeded to the hotel. This offer he accepted and our conversation had become so interesting that when we arrived at his house, a distance of about a mile, he proposed to walk back with me on my way, a proposal which you may be sure that I did not reject; so far from it that when he came to turn once more towards his home, I also turned once more along with him. It was very late when I reached the hotel after all this walking ...[46]

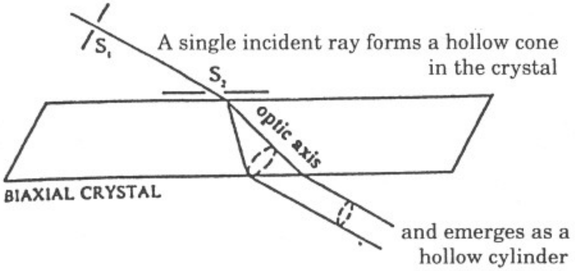
Wordsworth wrote to Hamilton:

> Seldom have I parted – never, I was going to say – with one whom, after so short an acquaintance, I lost sight of with more regret. I trust we shall meet again.[47]

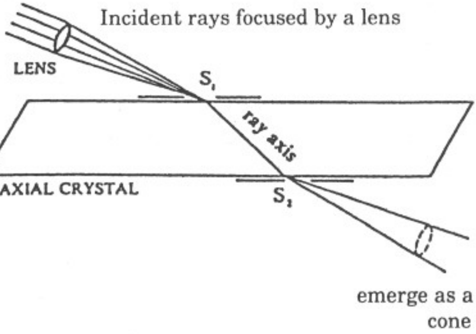
Two years later, at Hamilton's suggestion, Wordsworth made his first visit to Ireland. He stayed for several days at Dunsink where they had many spirited discussions about poetry, science and philosophy. They kept in regular correspondence until Wordsworth's death in 1850. Hamilton and Eliza stayed with Wordsworth in the Lake District for a month in 1830. Hamilton's assertion, that science was an aesthetic creation with beauty and fascination, was "to Wordsworth an entirely new revelation and had the effect of raising his conception, which had before been unduly depreciatory, of the dignity both of science itself and of its most eminent votaries".[48] Wordsworth once said that Hamilton and Coleridge were the two most wonderful men, taking all their endowments together, that he had ever met.[49] Hamilton was the godfather of Wordsworth's son, William.

Prediction of conical refraction

The international scientific community was stunned by Hamilton's prediction of conical refraction, in 1832. Refraction of light is the process by which a ray of light bends as it passes from one medium to another. This explains why a stick appears bent when immersed in water: it is because light is bent as it passes from the air into the water. Hamilton's purely mathematical theory predicted that a ray of light, entering certain biaxial crystals under certain conditions, would emerge as a hollow cone light beam or a hollow cylinder light beam.[50] This phenomenon of conical refraction was totally unexpected: nobody had ever imagined it. Hamilton asked Humphrey Lloyd, a scientist in Trinity, to verify his theory by experiment. The quantitative and qualitative verification by Lloyd's experiment astounded the scientific community. Figure 2 shows how conical refraction occurs.[51]



Internal conical refraction. S_1 and S_2 are movable pinholes.



External conical refraction. S_1 and S_2 are movable pinholes.

Figure 2.

Here was a purely mathematical prediction of a totally unexpected natural phenomenon being experimentally verified quantitatively and qualitatively. Mathematics had always been fundamental in understanding how nature works, but this apparently was the first time that a mathematical prediction of an unimagined physical phenomenon had been verified so precisely.[52] George Airy, the Astronomer Royal for England, said: "Perhaps the most remarkable prediction that has ever been made is that lately by Professor Hamilton".[53] William Whewell, the mineralogist in Cambridge who coined the word "Scientist"[54] in 1840, said: "Few things have been more remarkable".[55] Hamilton won the Royal Medal of the Royal Society of London in 1835 for his work on conical refraction.

There have been more famous mathematical predictions of unforeseen physical phenomena since Hamilton's one. In 1845 John Adams and Urbain Leverrier were studying aberrations in the orbit of Uranus and their mathematical results led them to predict the existence of a new planet. They told astronomers where and when to look, and this culminated in the discovery of the planet Neptune, on the night of 23 September 1846, at Berlin Observatory.[56] Perhaps the most important mathematical prediction of an unexpected physical phenomenon was the prediction of electromagnetic waves[57] by James Clerk Maxwell in 1864.[58] Maxwell's mathematics led him to predict the existence of this new phenomenon which nobody had ever imagined. Not many people believed him and it took twenty-two years before the German physicist, Heinrich Hertz, verified his theory by detecting radio waves experimentally. Radio waves are just one class of electromagnetic waves. Maxwell was dead by the time Hertz verified his prediction but Hertz followed Maxwell's advice on how to detect the radio waves. Nine years later, in 1895, Marconi sent the first wireless telegraph, and then in 1901, he conducted the first transatlantic wireless communication by transmitting and receiving radio signals. Radio, television, radar and telecommunications all followed from this. So much of our society today depends on these electromagnetic waves, the discovery of which can be traced back to the mathematics of Maxwell. Hamilton deserves a mention here too because his quaternions, which we will meet later on, were used in a fundamental way in Maxwell's original formulation of electromagnetism.

Marriage

Hamilton first met Helen Bayly at local social events in 1832. Helen was from Nenagh, Co. Tipperary, but she used to stay with her two married sisters at Scripplestown and Dunsinea, which were both near Dunsink. Hamilton married Helen a year later. Their first child, William Edwin, was born in 1834. They had three children in all: another son, Archibald Henry and a daughter, Helen Eliza. William Edwin has never been traced fully. He became a civil engineer and was last heard of in 1891 when he was working for the *Planet* newspaper in Chatham, Ontario, in Canada.[59] It is not known whether he ever had any children and so there may possibly be some unknown descendants of Hamilton. Archibald Henry was a curate in Clogher when he died in 1914. Helen Eliza married John O'Regan in 1869 but tragically died a year later, following the birth of her son, John. Hamilton's only known descendants come from this son, John. The O'Regans now live in Marlborough in England.

General Method in Dynamics

Following on from ideas in his "Theory of System of Rays", Hamilton was able to show that the laws of optics (i.e. light) and the laws for the motion of objects (i.e. dynamics) could be expressed in the same mathematical form. Here were two completely different areas in practice, optics and dynamics, but Hamilton constructed a beautiful mathematical theory that united the two at a highly abstract level. His new theory appeared in two papers called "General Method in Dynamics". These papers were to be Hamilton's most important contribution to science. It is essential to have a theory of dynamics if we are to understand how objects (e.g. planets, satellites, cars, aeroplanes, etc.) move, and this understanding of motion is crucial to our understanding and operation of the physical world around us. The papers were conveyed to the Royal Society of London by the scientist Francis Beaufort, and published in 1834. Beaufort was born in Navan in 1774 and had earlier studied the basics of science at Dunsink under Ussher. He is famous for his "Beaufort Scale", which is the standard system for measuring wind speed. For example, 6 on the Beaufort Scale means a strong breeze with speed 25-31 m.p.h., whereas 8 corresponds to a gale with speed 39-46 m.p.h.

Hamilton searched for beauty in mathematics and was not necessarily interested in the practical uses. He elucidates this point in the introductory remarks to his papers on dynamics, explaining that he finds it "intellectually pleasing" that he has reduced the study of forces and motion of an object from many equations down to just two equations:

> The difficulty is therefore at least transferred from the integration of many equations of one class to the integration of two of another; and even if it should be thought that no practical facility is gained, yet an intellectual pleasure may result from the reduction of the most complex and, probably, of all researches respecting the forces and motions of body, to the study of one characteristic function, the unfolding of one central relation ...[60]

The opinion among the majority of scientists was that Hamilton had developed a very elegant mathematical theory, but they had no real practical use for it since they already had previous simpler techniques which worked equally well in most situations.

The renowned astronomer Herschel was viewing the southern heavens from the Cape of Good Hope, when he received a detailed account from Hamilton that described the new work on dynamics. Herschel replied:

> Alas! I grieve to say that it is only the general scope of the method which stands dimly shadowed out to my mind amid the gleaming and dazzling lustre of the symbolic expressions in which it is conveyed ... I could almost regret that you had taken so much trouble for one who can now only look on as a bystander, and mix his plaudits with the smoking of your chariot wheels, and the dust of your triumph.[61]

It was also during Herschel's time in the Cape of Good Hope that the famous "Moon Hoax" occurred. A New York newspaper reported that Herschel had seen winged creatures, resembling human beings, on the moon. Communications between New York and the Cape of Good Hope were slow and so the hoax continued for several weeks![62]

Hamilton's new method of dynamics was far ahead of its time. It took over eighty years before Hamilton's profound mathematics in his "General Method in Dynamics" was to play a crucial role in a major breakthrough for our understanding of the physical world. In the early part of this century, physicists were beginning to realise that Newton's laws of motion did not work well at the atomic level where very small particles like electrons and atoms are in motion. Newton's laws had worked very well for larger objects. An understanding at the atomic level is fundamental to an understanding of the physical world, and so physicists were attempting to develop a new theory for the atomic level. This theory was called "Quantum Mechanics (or Wave Mechanics)", and Hamilton's method of dynamics was indispensable for it. The Austrian physicist, Erwin Schrödinger, who won the Nobel Prize in 1933, was one of the principal scientists working on "Wave Mechanics" in the 1920s. He said:

> I daresay not a day passes – and seldom an hour – without somebody, somewhere on the globe, pronouncing or reading or writing Hamilton's name. That is due to his fundamental discoveries in general dynamics. The Hamiltonian Principle has become the cornerstone of modern physics, the thing with which a physicist expects every physical phenomenon to be in conformity ... The modern development of physics is continually enhancing Hamilton's name. His famous analogy between dynamics and optics virtually anticipated wave-mechanics, which did not have to add much to his ideas ... The central conception of all modern theory in physics is "the Hamiltonian". If you wish to apply modern theory to any physical problem, you must start with putting the problem "in Hamiltonian form". Thus Hamilton is one of the greatest men of science the world has produced.[63]

Hamilton's aesthetically pleasing and highly abstract mathematics from 1834 became the cornerstone of the modern understanding of the physical world in the twentieth century.

Schrödinger fled Nazi-controlled Austria during World War II and came to Ireland at Eamon de Valera's invitation. De Valera had established the Dublin Institute for Advanced Studies (DIAS) in 1940 and Schrödinger worked there as Director for many years. De Valera had a keen interest in mathematics. He lectured in mathematics at Maynooth, where his close friend, Pádraig de Brún, was Professor of mathematics.[64] The DIAS was de Valera's idea of an international centre of excellence in mathematics and Celtic Studies. The Institute now has three schools: The School of Celtic Studies, the School of Theoretical Physics[65] and the School of Cosmic Physics.

De Valera greatly admired Hamilton. He declared in Dáil Éireann: "This is the country of Hamilton, a country of great mathematicians".[66]

Our current Taoiseach is also familiar with Hamilton, as he observes his image almost daily. There is a statue of Hamilton on the steps of Government Buildings in Merrion Street, dating back to the time when the buildings housed the College of Science.

President of the Royal Irish Academy

In 1837 Hamilton was elected president of the Royal Irish Academy. The RIA had been set up in 1795 from the amalgamation of two college societies known as the Palaeosophers and the Neosophers.[67] Its birth therefore came from a fusion of interests in the old and the new. Hamilton's broad interests still included the antiquities and he worked hard to ensure that important historical treasures were taken care of. For example, he was instrumental in obtaining the necessary funds so that the *Annals of the Four Masters* could be printed.[68] Hamilton proved a dedicated and competent president until his term in office ended in 1846, when he was succeeded by Humphrey Lloyd.

Hamilton was an eloquent speaker and attracted large audiences to the introductory lecture of his annual course on astronomy. This lecture was considered to be lively intellectual entertainment, and many non-academics attended. In the lecture, Hamilton "gave free scope to his views on the philosophy of science, to his admiration for the kings of thought, and to his eloquent assertion of the kinship between science and poetry".[69]

His rhetoric was also relished by the scientific community and he was sometimes called upon to make the "farewell speech" at conferences. During such conferences, which were usually held in England, Hamilton would proclaim himself as an Irishman in his address. His love for Ireland is reflected in the following poem, which he penned as he approached the Irish coast:

> My native land, appear! these eyes await
> Impatiently thy rising over the bare
> Expanse of waters; fondly searching where
> Thy fair but hidden form lingers so late …
> In thee my homeward thoughts still claim their share,
> My heart, my life, to thee are dedicate.[70]

Quaternions

The quaternions are Hamilton's most celebrated contribution to mathematics. "Number couples" had been important in mathematics and science, and Hamilton was attempting to extend his theory of number couples to a theory of "number triples (or triplets)".[71] He hoped these triplets would provide a natural mathematical structure and a new way for describing our three-dimensional world, in the same way that the number couples played a significant role in two-dimensional geometry. He was having a difficult time in his quest for a suitable theory of triplets.[72] Then, on Monday 16 October 1843, Hamilton's mind gave birth to quaternions, as he walked on the banks of the Royal Canal. Hamilton recollects the event, in a letter to his son Archibald, one month before his death in 1865:

> Although your mother talked with me now and then, yet an undercurrent of thought was going on in my mind, which gave at last a result, whereof it is not too much to say that I felt at once an importance. An electric current seemed to close; and a spark flashed forth, the herald (as I foresaw, immediately) of many long years to come of definitely directed thought and work. ... Nor could I resist the impulse – unphilosophical as it may have been – to cut with a knife on a stone of Brougham Bridge as we passed it, the fundamental formula ...[73]

Hamilton realised that if he worked with "number quadruples" and an unusual multiplication operation, then he would get everything he desired. He named his new system of numbers "Quaternions" because each number quadruple had four components. He had created a totally new structure in mathematics. The mathematical world was shocked by his audacity in creating a system of "numbers" that did not satisfy the usual "commutative rule for multiplication" (this is the rule which says that it does not matter in which order you multiply two ordinary numbers, i.e. two times three equals three times two. The quaternions did not satisfy this rule). Again, we see Hamilton's highly original and creative mind at work. He did not care if, in his new number system, he had to discard the usual rules of multiplication for ordinary numbers. His strong philosophy provided him with an open mind for conjuring up any abstract mathematical system that he found interesting and aesthetically pleasing. Two weeks later, the prominent mathematician, John Graves, wrote:

> There is still something in the system [of quaternions] which gravels me. I have not yet any clear view as to the extent to which we are at liberty arbitrarily to create imaginaries, and to endow them with supernatural properties. You are certainly justified by the event ... [but] what right have you to such luck, getting at your system by such an inventive mode as yours?[74]

Hamilton's abstract mathematical creation played a crucial role in some major scientific breakthroughs. As mentioned earlier, quaternions played an important role in Maxwell's theory of electromagnetic waves. Thus, the inventions of radio, television, radar, X-rays and countless other significant products of our time, are directly connected to Hamilton's quaternions. According to Maxwell, "Quaternions allow one to express the physics of electricity and magnetism much more directly than is possible with coordinates, so that the mathematics reveals more clearly the nature of the phenomena".[75] Also, the theory of "vector analysis", which is now indispensable in physics and its applications, is an offspring of quaternions.[76] Hamilton subsequently published many papers and several books on quaternions.

Hamilton has been called the "Liberator of Algebra", because his quaternions smashed the previously accepted notion that a useful algebraic number system should satisfy the rules of ordinary numbers in arithmetic. His quaternions opened up a whole new mathematical world, in which mathematicians were now free to conceive new algebraic number systems that were not shackled by the rules of ordinary numbers. Hence, we may assert that modern algebra was born on 16 October 1843, on the banks of the Royal Canal in Dublin. The event is now commemorated by a plaque (Figure 3), which was erected by Eamon de Valera.

The plaque is on the west side of the bridge, at Broombridge train station, in Cabra. De Valera also paid homage to quaternions with a little graffiti of his own by scratching the fundamental formula on the wall of his prison cell in Kilmainham.

Here as he walked by
on the 16th of October 1843
Sir William Rowan Hamilton
in a flash of genius discovered
the fundamental formula for
quaternion multiplication

$$i^2 = j^2 = k^2 = ijk = -1$$

& cut it on a stone of this bridge

Figure 3.

Of course, Brendan Behan also created a famous link between the Royal Canal and a mathematical concept!![77]

In 1990 Professor Anthony O'Farrell, of N.U.I. Maynooth, initiated an annual walk to celebrate Hamilton's creation of the quaternions. The walk takes place on 16 October and participants retrace Hamilton's steps by commencing at Dunsink Observatory and then strolling down to the canal at Ashtown train station, before following the canal to Broombridge.[78]

Hamilton had a strong interest in metaphysics and this played a role in his mathematical research. He was influenced by Kant and Coleridge. He wrote:

> The quaternion [was] born, as a curious offspring of a quaternion of parents, say of geometry, algebra, metaphysics and poetry ... I have never been able to give a clearer statement of their nature and their aim than I have done in two lines of a sonnet addressed to Sir John Herschel:
>
> "And how the one of Time, of Space the Three,
> Might in the Chain of Symbols girdled be."[79]

Hamilton is referring here to an interpretation of his (four-dimensional) quaternions as being a mingling of one-dimensional time and three-dimensional space. He was close to the four-dimensional world of Einstein's relativity. The scientist, Cornelius Lanczos, wrote: "It is astonishing to see how the quaternions of Hamilton foreshadowed our four-dimensional world, in which space and time are united into a single entity, the space-time world of Einstein's Relativity".[80]

Later years

Hamilton had never forgotten Catherine Disney and she would later reappear in his life. In the late 1840s Hamilton started coaching her son James, who was then competing for a fellowship in Trinity. In 1848 Catherine wrote to Hamilton, thanking him for his support. Many letters flowed back and forth between them over a period of six weeks, and it was made clear for the first time that she had always loved him.[81] Catherine was now in severe emotional distress at home and made an unsuccessful attempt at suicide by taking an overdose of laudanum. She then left her husband and returned to Dublin to stay with her mother and brothers. Hamilton and Catherine remained friends. In October 1853, Hamilton received a pencil case from Catherine with the inscription "From one whom you must never forget, nor think unkindly of, and who would have died more contented if we had once more met".[82] Hamilton rushed to see her at her brother's house in Donnybrook. She died two weeks later. Catherine's son, James Barlow, became Professor of Modern History in Trinity in 1860 and later Vice-Provost in 1899. He is remembered with a plaque in the college chapel.

In 1855 Hamilton received an unusual request:

> A very odd and original lady has also lately had a baby; such things, as you know, will happen ... (Recently) when I met her for the first time in my life, she told me of this young Pagan as she called him. And she asked me to be a godfather, perhaps because she is an admirer of Wordsworth. However I declined.[83]

The "young pagan" was Oscar Wilde. Lady Wilde soon became a close friend of Hamilton. On a recent visit to the Dublin Writers' Museum, I noticed that Oscar Wilde was born on 16 October 1854. Who knows what Hamilton's reply might have been if he had known this! October 16 was, of course, the date of his momentous insight by the banks of the Royal Canal, eleven years earlier.

Hamilton died on 2 September 1865. He had known for several days that he was dying. The causes of death were gout and bronchitis. His close friend, Robert Graves, had been with him just before he died. Graves wrote:

> ... he breathed his last, having first, as I learned the following day, solemnly stretched himself at full length upon his bed, and symmetrically disposed his arms and hands, thus calmly to await his death.[84]

Hamilton is buried in Mount Jerome Cemetery in Dublin. He once wrote: "I have very long admired Ptolemy's description of his great astronomical master, Hipparchus, as a labour-loving and truth-loving man. Be such my epitaph".[85]

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1. H, xv.
2. Nicolson, Harold, *The Desire to Please: A story of Hamilton Rowan and the United Irishmen* (1943), 112.
3. Quane, Michael, "Meath Diocesan School", *Ríocht na Midhe*, V, No. 1 (1971), 51.
4. *Ibid.*, 52.

5. H, 26.
6. G, Vol. 1, 31.
7. *Ibid.*, 33.
8. *Ibid.*, 40.
9. *Ibid.*, 55.
10. H, 27.
11. G, Vol. 1, 75.
12. *Ibid.*, 120.
13. *Ibid.*, 129.
14. *Ibid.*, 111.
15. H, 15.
16. G, Vol. 1, 88.
17. Newton's *Principia*, published in 1687, contained his famous mathematical laws of motion and gravitation, which revolutionised science.
18. G, Vol. 1, 110.
19. Polya, George, *Mathematical Methods in Science* (1977).
20. Kline, Morris, *Mathematics in Western Culture* (1990).
21. Guillen, Michael, *Five Equations that changed the World* (1995).
22. Stewart, Ian, *Nature's Numbers* (1995).
23. H, 21.
24. H, 387.
25. H, 21.
26. H, 105.
27. G, Vol. 1, 284.
28. *Ibid.*, 193.
29. *Ibid.*, 114.
30. H, 43.
31. O'D, front flap.
32. Descartes conceived the system whereby the position of any point in three-dimensional space is represented by three coordinates. A geometrical figure, like a circle or a sphere in space, can then be considered as the set of points whose coordinates satisfy some equation. This will be an algebraic equation and so algebraic concepts and techniques can be employed to solve a geometrical problem. The etymology of the word geometry gives a taste of the history of mathematics. The word comes from the Greek geo (meaning earth) and metron (meaning measure). In ancient times, as well as now, geometry proved indispensable for measurement purposes. For example, in the fourteenth century BC, in Egypt, it was used by the Pharaoh to measure how much land a farmer lost from the annual flooding of the Nile. This would determine how much compensation the farmer received.
33. H, 61.
34. G, Vol. 1, 592.
35. *Ibid.*, 156-7.
36. *Ibid.*, 161.
37. *Ibid.*, 160.
38. H, 404.
39. *Ibid.*, 38.
40. G, Vol. 1, 174.
41. O'D, 54.
42. H, 40.
43. Moore, Patrick, *Armagh Observatory*, 5.
44. McKenna, Susan, *Vistas in Astronomy* (1968), Vol. 9, 283-96.
45. O'D, 66.
46. G, Vol. 1, 264.
47. *Ibid.*, 268.
48. William Rowan Hamilton, *Mathematical Papers*, Vol. 1, xiv.
49. G, Vol. 1, 269.
50. A biaxial crystal is one with two axes of symmetry. An example is aragonite. The emergence of a hollow cone light beam is called external conical refraction because the

cone of light appears outside the crystal. The emergence of a hollow cylinder light beam is called internal conical refraction because the cone of light appears inside the crystal.

51. H, 89-90.
52. O'D, 106.
53. G, Vol. 1, 637.
54. H, 387.
55. O'D, 105.
56. H, 326.
57. Electromagnetic waves can be considered as oscillating electric and magnetic fields travelling together through space at the speed of light. The range of possible wavelengths of the electromagnetic waves embraces radio waves, infra-red radiation, visible light, ultra-violet radiation, X-rays and gamma radiation. This range is called the electromagnetic spectrum.
58. Kline, *op. cit.*, 351.
59. O'D, 186.
60. William Rowan Hamilton, *Mathematical Papers*, Vol. 3, 105.
61. G, Vol. 2, 127.
62. O'D, 133.
63. Schrödinger, Erwin, *Scripta Mathematica*, Vol. 2 (1945), 82.
64. *St. Patrick's College, Maynooth*, The Irish Heritage Series, No. 47 (1984). De Valera lectured in Maynooth in 1913-4. As well as being a mathematician, De Brún translated the New Testament, and parts of Homer and Dante, into Irish. He later became President of U.C.G.
65. Theoretical Physics is also called Mathematical Physics. It is essentially the application of mathematical theory to physics.
66. Dublin Institute for Advanced Studies, *Golden Jubilee: 1940-1990*, 2.
67. O'D, 116.
68. O'D, 120.
69. G, Vol. 1, 639.
70. G, Vol. 2, 528.
71. A number couple is a pair of numbers. A point in a two-dimensional region can be represented by two coordinates, and so number couples can represent points there. There was a theory of number couples which showed how to add and multiply them. A triplet is a triple of numbers and so can represent points in our three-dimensional world. In Hamilton's time, there was no theory of triplets which had a satisfactory multiplication defined for it.
72. There was a good reason why Hamilton was having a difficult time in this quest. We now know that the kind of "suitable" theory of triplets, that he was seeking, does not exist.
73. G, Vol. 2, 434.
74. *Ibid.*, 443.
75. H, 317.
76. H, 318.
77. "The oul' triangle
    Goes jingle jangle
    Along the banks of the Royal Canal".

    These famous lines were penned by Behan in his play *The Quare Fellow*. The sound of the triangle indicated that it was unlocking time in Mountjoy Prison, where Behan was once an inmate.
78. All are welcome to participate in the walk. Contact the Department of Mathematics in N.U.I. Maynooth for details.
79. H, 247.
80. Lanczos, C., *University Review of N.U.I.*, IV, 2 (1967), 160.
81. O'D, 158.
82. H, 351.
83. G, Vol. 1, 269.
84. G, Vol. 3, 211.
85. Bell, E.T., *Men of Mathematics* (1965), 361.