OPTIMAL INTERNATIONAL ASSET ALLOCATION WITH TIME-VARYING RISK

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ABSTRACT

This paper examines the optimal allocation each period of an internationally diversified portfolio from the different points of view of a UK and a US investor. We find that investor location affects optimal asset allocation. The presence of exchange rate risk causes the markets to appear not fully integrated and creates a preference for home assets. Domestic equity is the dominant asset in the optimal portfolio for both investors, but the US investor bears less risk than the UK investor, and holds less foreign equity – 20% compared with 25%. Survey evidence indicates actual shares are 6% and 18%, respectively, making the home-bias puzzle more acute for US than UK investors. There would seem to be more potential gains from increased international diversification for the US than the UK investor.

I INTRODUCTION

We examine the potential benefits of holding an internationally diversified portfolio that is re-balanced each period to take account of time variation in the covariance matrix of returns. Flavin and Wickens (1998) show that domestic asset allocation would be greatly improved by re-balancing in this way compared with using an allocation based on a constant covariance matrix. Guidolin and Timmermann (2005) show that ignoring regime shifts in asset markets causes significant welfare costs. Ang and Bekaert (2002) show that this also applies to international asset allocation.

Evidence of large benefits to international portfolio diversification exists from the early literature, e.g., Grubel (1968) and Levy and Sarnat (1970). More recently, Grauer and Hakansson (1987) conclude that a US investor can reap 'remarkably large' gains from including non-US assets in the portfolio of risky assets. Based on a paired *t*-test, they find that realised returns from an internationally diversified portfolio are significantly higher than those generated by a portfolio consisting entirely of domestic stocks. Furthermore, the gains

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increased, as the investor becomes more risk averse. De Santis and Gerard (1997) provide evidence that, even though equity market declines are contagious across countries, US investors may still earn expected gains of 2.1% on average and these have not fallen despite increased financial market integration. Eun and Resnick (1988) and Jorion (1985) both show that hedging foreign exchange risk can potentially increase the gains from international diversification. Many studies concentrate on equity markets, but Levy and Lerman (1988) find that a US investor who diversified across world bond markets could have realised more than twice the mean rate of return on a domestic US bond portfolio with the same risk level.

Our analysis is conducted from both the perspective of a UK and a US investor to see whether investor location affects optimal asset allocation. Although traditional models of finance would suggest otherwise, a recent literature has shown that the location of trade exerts an impact on both the stock abnormal return and the order flow. Both factors obviously influence asset allocation. Focusing on foreign country funds, Bodurtha *et al.* (1995) find evidence of international market segmentation. In particular, the fund price is shown to move more closely with the US market while the underlying stocks have higher correlation with the market upon which they trade. Lau and McInish (2003) also provide evidence of market segmentation by focusing on the Jardine group's decision to change their primary listing from Hong Kong to Singapore. They find evidence of order flow segmentation and a decline in volume traded, implying that certain investor groups are only interested in holding the stock when it trades on a certain market. Clyde *et al.* (1997) show such segmentation may also apply at the domestic level. They report a decline in liquidity for stocks moving from AMEX to Nasdaq but the announcement of the move was associated with positive excess returns.

This result is not only associated with closed-end funds or relatively small markets. Froot and Dabora (1999) find this phenomenon also applies to large individual stocks operating in large stock markets. They examine 'Siamese twin' companies (Unilever, Shell Transport and SmithKline Beecham) that trade on different markets but whose value is determined by a common cash flow. They find significant pricing differentials. Furthermore the differential or relative price is more highly correlated with the market on which the stock is most actively traded. This result proves robust to a number of commonly cited explanations. Bedi *et al.* (2003) find results consistent with those of Froot and Dabora. Werner and Kleidon (1996) focus on British stocks cross-listed on the UK and US markets and find that markets are not perfectly integrated. In particular they provide evidence of order flow segmentation for such stocks. This pattern has also been found in other financial markets. Tse (1999) report that the London and Tokyo markets for futures on Japanese government bonds exhibits segmentation in order flow while Hsieh and Kleidon (1996) present a similar result for the spot $/DM exchange rate trading in London and New York.

Our representative investors are allowed to hold three risky assets: domestic and foreign equity and a domestic long-bond. An optimal portfolio is then obtained based on the excess asset returns over the domestic risk-free rate. Our

tactical allocation strategy is an adaptation of Markowitz's (1959) minimum variance portfolio selection theory in which excess returns are modelled by a multivariate GARCH (M-GARCH) process particularly suited to portfolio analysis. This enables us to determine how the covariance matrix of excess returns, and hence the minimum variance portfolio frontier and the optimal portfolio shares, vary through time. One reason for focussing on minimising the portfolio variance is that, in practice, returns – especially equity returns – are not forecastable (i.e. they are virtually serially independent). We are able to reject the assumption that the covariance matrix of returns is constant.[1] In the absence of transactions costs, this increases the need for the optimal portfolio to be re-balanced each period.[2] This suggests that, like portfolios that comprise only domestic assets, an international tactical asset allocation should aim to exploit the regularities in the covariance structure of excess returns in order to minimise risk.

   Domestic equity dominates the optimal portfolio in each period for both sets of investors. However, the foreign asset is also an important portfolio constituent, and on average has a greater share than the domestic bond. Comparing our results with surveys of actual asset holdings, e.g., French and Poterba (1991) and Cooper and Kaplanis (1994), confirms that the home bias puzzle is still alive[3] and is more acute in the United States than the United Kingdom. Our analysis suggests that the US investor should, on average, apportion 20% of the funds held in risky assets to UK equity, while the optimal portfolio for the UK investor contains on average 25% of its asset holdings in US equity. We find that in the short run shares can differ considerably from these average values, and that most of the re-balancing is between domestic and foreign equity. As our investment opportunity set is restricted, our results may be interpreted as providing a lower bound on optimal foreign equity holdings.

   The paper is structured as follows. Section II describes the econometric model. In Section III the data are described and the estimates are reported. The optimal portfolios are analysed in Section IV. Section V summarises our findings and has some conclusions.

II   ECONOMETRIC MODEL

We aim to identify the optimal portfolio of risky assets in each period for both UK and US investors. The asset allocation strategy used is fully described in Flavin and Wickens (1998). It is assumed that investors use Markowitz's mean-variance portfolio theory to determine the optimal asset allocation. Investors therefore select the weights of the portfolio that corresponds to a tangent from the origin to the portfolio frontier of risky assets. The risk-free asset in each case being the domestic risk-free bond.

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[1] This has also been shown by Clare *et al.* (1998), Cumby *et al.* (1994) among others.

[2] Of course, this is not the only reason to re-balance. In general, the portfolio will need to be re-balanced when expected returns are not equal to realised returns.

[3] Excellent reviews, and potential explanations of the puzzle, are provided by Lewis (1999) and Karolyi and Stulz (2003).

Instead of assuming that the covariance matrix of excess returns is constant through time, we allow it to be time varying. This implies that the portfolio may need re-balancing every period in response to such time variation. We assume that the vector of excess returns is Normally distributed with a time-varying conditional mean and conditional variance generated by a M-GARCH(1,1) process. It is widely recognised that there is a major computational problem in the full information estimation of such models due to the large number of parameters they possess. We therefore adopt the M-GARCH representation set out by Flavin and Wickens (1998).[4] This model is a variant of the Berndt, Engle, Kraft and Kroner (BEKK) representation.[5] If n is the number of risky assets, then compared with the most general formulation of the model, this representation results in the number of parameters increasing at the rate n^2 instead of n^4. When $n = 3$, the number of parameters to be estimated is reduced from 78 using the most general model to 18, a substantial saving. The model may be written as

$$
\begin{aligned}
\mathbf{r}_{t+1} = {}& \mathbf{\upsilon} + \mathbf{\Gamma}\mathbf{r}_t + \mathbf{\Upsilon}d87 + \mathbf{\xi}_{t+1} \\
& \mathbf{\xi}_{t+1}|\mathbf{\Psi}_t \sim N(0, \mathbf{\Omega}_{t+1}), \\
\mathbf{\Omega}_{t+1} = {}& \mathbf{V}'\mathbf{V} + \mathbf{\Phi}'(\mathbf{\Omega}_t - \mathbf{V}'\mathbf{V})\mathbf{\Phi} + \mathbf{\Theta}'(\mathbf{\xi}_t\mathbf{\xi}_t' - \mathbf{V}'\mathbf{V})\mathbf{\Theta},
\end{aligned}
\tag{1}
$$

where $\mathbf{r}_{t+1}$ is a vector of excess returns in period $t+1$ and $d87$ is a dummy variable for the October 1987 stock market crash. $\mathbf{\upsilon}$ and $\mathbf{\Upsilon}$ are $n \times 1$ vectors, $\mathbf{\Gamma}$ is an $n \times n$ matrix and $\mathbf{V}$, $\mathbf{\Phi}$ and $\mathbf{\Theta}$ are $n \times n$ symmetric matrices of parameters.

By making the matrices symmetric rather than unrestricted we are able to economise on parameters as only $3n(n+1)/2$ parameters are required for the covariance matrix. It might seem that an equivalent specification would be to make the matrices triangular, but in fact this has the disadvantage of restricting the dynamic structure of the covariance matrix unnecessarily by introducing an additional lag involving cross-effects. By formulating the conditional covariance structure in this way, we obtain an estimate of both the unconditional (long-run) covariance matrix and the conditional covariance matrix (the short-run dynamics). This is important as it allows us to decide whether or not the short-run dynamics make a sufficiently useful contribution to justify their inclusion, and the time and effort to estimate them. It also allows us to identify which parameters are most significant in determining deviations from the long run. This formulation also guarantees a positive semi-definite covariance matrix.

Initially we choose $n = 3$ and $\mathbf{r} = (ukeq, useq, gvbd)$' where $ukeq$, $useq$ and $gvbd$ refer to the excess return over the domestic risk-free rate of UK equities, US equities and a domestic government bond respectively.[6] From an econometric

[4] This parameterisation is consistent with the covariance stationary model developed in Engle and Kroner (1995).

[5] For a full treatment of multivariate (G)ARCH models, the reader is referred to Bollerslev et al. (1994) or Bollerslev et al. (1992) for a survey of (G)ARCH models in finance.

[6] The main body of the paper is concerned with the case in which the foreign bond is omitted from the opportunity set of the investor. However, at the end of our analysis, the consequences of its inclusion are examined.

point of view, there is a further benefit from working with excess returns, namely that all series are stationary and do not require differencing.[7]

This model can be generalised further by incorporating macroeconomic variables to better predict the conditional moments of the return process as in Flavin and Wickens (2003).

III DATA AND ESTIMATION RESULTS

The data

We use time series data on broad classes of UK and US financial assets. The analysis is conducted from both the perspective of a UK and a US investor. Each investor is assumed to choose an optimal portfolio consisting of three risky assets based on the excess returns of two domestic risky assets and one foreign risky asset over a domestic risk-free asset. The risky assets used in the analysis are UK equities, represented by the Financial Times All Share Index; US Equities represented by the S&P Composite Index, UK government bonds represented by the FT British government stock index; US bonds represented by a Datastream computed government bond index. In each case, the return on the foreign asset is converted into the domestic currency using the end of month exchange rate. The data used in this paper are annualised monthly total returns. The total return data include dividend payments in the case of equities and coupon payments in the case of government bonds. For the UK investor, the rate of return on the UK government 30-day Treasury bill is taken as the risk-free rate of interest, while for the US investor the riskless interest rate is proxied by the Eurodollar rate.[8] These assets are riskless at least in the nominal sense. The data period is from January 1980 to March 1997. All data are sourced from DATASTREAM.

By working with excess returns we prevent volatility in the risk-free rate from incorrectly contributing to the risk of the optimal risky portfolio. As the risk-free rate is perfectly predictable at the start of each period, and is therefore in the investor's information set when the allocation decision is made, its inclusion would tend to over-estimate total portfolio risk.

UK investor: discussion of results

The estimates of the model for the representative UK investor are reported in Tables 1 and 2.[9] In the conditional mean the elements of Γ are generally not significant. This is consistent with the usual finding that total stock and bond returns are serially uncorrelated. The most significant element is Γ_{32} implying

[7] A wide range of Unit root tests were conducted on these series and all results confirm stationarity. Results are available from the authors upon request.

[8] We would prefer to have used the return on a 1-month treasury bill but the series available would result in a shorter time frame. Despite not being perfectly correlated, we argue that the Eurodollar rate is a reasonable proxy.

[9] Maximisation of the likelihood function was achieved using the Berndt, Hall, Hall and Hausmann (BHHH) algorithm in RATS.

Table 1
Conditional mean estimates for UK investor

υ_1	7.23**	Γ_{11}	− 0.06	Y_1	− 403.51
υ_2	5.20	Γ_{12}	0.07	Y_2	− 334.39**
υ_3	− 0.61	Γ_{13}	0.20	Y_3	0
		Γ_{21}	− 0.09		
		Γ_{22}	0.11		
		Γ_{23}	0.07		
		Γ_{31}	0.01		
		Γ_{32}	− 0.06**		
		Γ_{33}	0.10		

Note:
*, **, *** signify that estimate is statistically significant at 10%, 5% and 1% confidence levels, respectively.

Table 2
Conditional covariance estimates for UK investor

V_{11}	57.21***	Θ_{11}	0.27***	Φ_{11}	− 0.80***
V_{12}	26.59***	Θ_{12}	− 0.13**	Φ_{12}	− 0.19**
V_{13}	15.50***	Θ_{13}	0.15***	Φ_{13}	− 0.15
V_{22}	56.98***	Θ_{22}	0.28***	Φ_{22}	− 0.73***
V_{23}	− 4.94**	Θ_{23}	0.02	Φ_{23}	− 0.01
V_{33}	24.06***	Θ_{33}	0.43***	Φ_{24}	0.39***

Note:
*, **, *** signify that estimate is statistically significant at 10%, 5% and 1% confidence levels, respectively.

that the lagged excess return on the US equity has some explanatory power for the excess return on UK bonds, but it is difficult to think of a good reason why this should be. Consequently, we employ a constant vector of expected asset returns to generate the portfolio shares. This has the added advantage that all of the variation in the estimated frontiers, and hence the portfolio shares, can be attributed to variation in the conditional covariance matrix of excess returns. This is also the assumption made by Cumby *et al.* (1994) who use the historical mean of each asset as its expected value. Jobson and Korkie (1981) advocate the use of global shrinkage based on Stein estimators whereby all assets of the same class have the same expected excess return. This is an extreme case of Stein estimation with the individual asset being assigned a weight of zero and the global mean having a weight of one. They show that this approach significantly improves the practical application of the mean-variance framework. As we are working with financial asset indices as opposed to individual securities, these approaches reduce to the same thing. Furthermore Ang and Bekaert (2002) fail to reject the hypothesis that means are equal across high and low volatility regimes. Another reason for making this assumption is that the sensitivity of the portfolio shares to small variations in the mean is far greater than that to variations in the covariance matrix, Kallberg and Ziemba (1984). Best and Grauer (1991) show that even small changes in the mean vector can result in

dramatic variation in the composition of the estimated optimal portfolio of risky assets.

Continuous re-balancing of the portfolio to changes in the predicted excess return would not only be expensive due to transaction costs, it would also be counter-productive because of the lack of persistence of the deviations of excess returns from their unconditional means. This is not true of re-balancing due to changes in the conditional variance because of their much higher degree of persistence and their lower volatility.

In contrast, the elements of the matrices determining the conditional covariance matrix are generally significant. All the elements of $\mathbf{V}$, which determines the unconditional or long-run matrix, and most of the elements of the matrices that determine the short run, $\mathbf{\Theta}$ and $\mathbf{\Phi}$, are highly significant. The significance of the diagonal elements of $\mathbf{\Theta}$ and $\mathbf{\Phi}$ indicates that the conditional variances differ considerably from the unconditional variances, showing considerable volatility clustering even at monthly intervals. The only off-diagonal elements not significant are $\mathbf{\Theta}_{32}$, $\mathbf{\Phi}_{31}$ and $\mathbf{\Phi}_{32}$. The significance of the {3,1} elements suggests that the allocation between UK equity and UK bonds will need to be re-balanced in the short-run to achieve optimality.

The {2,1} elements of $\mathbf{\Theta}$ and $\mathbf{\Phi}$ show the volatility contagion between the UK and the US stock markets. This, together with the {2,1} element of the long-run covariance matrix, is the reason why investors may want to hold an internationally diversified portfolio in order to reduce risk. For example, the long-run covariance matrix is

$$H = \begin{bmatrix} 3273 & & \\ 1521 & 3953 & \\ 887 & 131 & 843 \end{bmatrix},$$

implying a correlation between the excess returns over the UK risk-free rate of UK and US equity returns of 0.42. This also implies that to achieve an optimal portfolio re-balancing between UK and US equity will be required.

US investor: discussion of results

Results for the US investor are presented in Tables 3 and 4. The results are similar to those for the United Kingdom. Again, $\mathbf{\Gamma}$ is almost insignificant, though here there does seem to be some significant persistence in the excess return on US bonds. The elements of $\mathbf{V}$ are all statistically significant. The estimates of the diagonal elements of $\mathbf{\Theta}$ are all significant showing considerable ARCH effects and hence differences between the long-run and short-run covariance matrices. The significance of the {3,2} elements of $\mathbf{\Theta}$ and $\mathbf{\Phi}$ indicates that there will need to be a re-balancing between US equity and US bonds in the short run to achieve optimality.

The main difference is that there are no significant contagion effects between the US and UK stock markets. Taking together the UK and US results, this seems to indicate that causality runs from the US to the UK stock market. It would also suggest that the gains to the US investor from re-balancing the

Table 3
Conditional mean estimates for US investor

v_1	10.86**	Γ_{11}	-0.05	Y_1	-350.93
v_2	9.40***	Γ_{12}	0.14	Y_2	$-327.58*$
v_3	2.11	Γ_{13}	-0.002	Y_3	0
		Γ_{21}	0.02		
		Γ_{22}	-0.01		
		Γ_{23}	0.20		
		Γ_{31}	0.02		
		Γ_{32}	-0.07		
		Γ_{33}	$0.20**$		

Note:
*, **, *** signify that estimate is statistically significant at 10%, 5% and 1% confidence levels, respectively.

Table 4
Conditional covariance estimates for US investor

V_{11}	64.60***	Θ_{11}	$-0.32*$	Φ_{11}	-0.38
V_{12}	22.49***	Θ_{12}	0.10	Φ_{12}	-0.05
V_{13}	5.50***	Θ_{13}	-0.02	Φ_{13}	$0.39*$
V_{22}	39.09***	Θ_{22}	$-0.26**$	Φ_{22}	0.24
V_{23}	6.78**	Θ_{23}	$-0.21***$	Φ_{23}	$-0.41**$
V_{33}	18.50***	Θ_{33}	$0.22***$	Φ_{24}	$0.47**$

Note:
*, **, *** signify that estimate is statistically significant at 10%, 5% and 1% confidence levels, respectively.

portfolio in the short run between US and UK assets are likely to be small, though they may still be positive. The long-run covariance matrix is

$$H = \begin{bmatrix} 4173 & & \\ 1453 & 2034 & \\ 355 & 389 & 419 \end{bmatrix},$$

giving a correlation between the excess returns over the US risk-free rate on US and UK equity of 0.50.

IV OPTIMAL ASSET ALLOCATION

Frontier movements

Variations in the optimal portfolio weights when short sales are permitted are due entirely to movements in the portfolio frontier. These are brought about by new information on next period's conditional covariance matrix that causes it to vary over time. Estimates of the unconditional and conditional variances facing UK and US investors are plotted in Figures 1 and 2. For each country fluctuations in the exchange rate make foreign equity the most volatile asset. Nonetheless, since 1993, there has been a noticeable decline in volatility for all assets, and especially for equity returns expressed in sterling. This reflects the
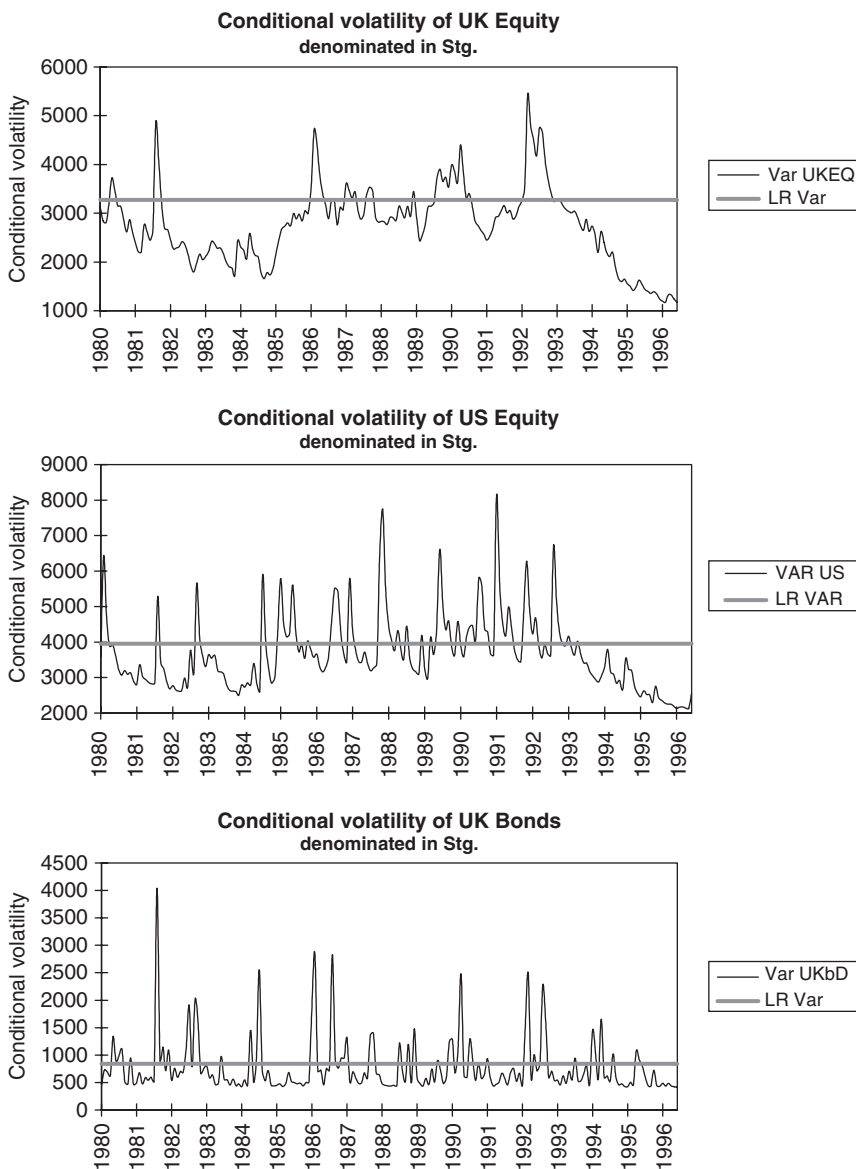
Conditional volatility of UK Equity
denominated in Stg.

Conditional volatility of US Equity
denominated in Stg.

Conditional volatility of UK Bonds
denominated in Stg.

Figure 1. Conditional asset return volatilities from perspective of UK investor.

relative stability of the £/$ exchange rate over this period. The graphs also show that short-run deviations of the conditional variances can be quite large, and are therefore likely to have a significant impact on the portfolio frontiers and hence on asset allocation in the short run.

Some idea of the fluctuations in the portfolio frontiers facing UK and US investors can be obtained from Figures 3 and 4. Figure 3 shows, for each

Conditional volatility of UK Equity
denominated in US$

Conditional volatility of US Equity
denominated in US$

Conditional volatility of US Bond
denominated in US$
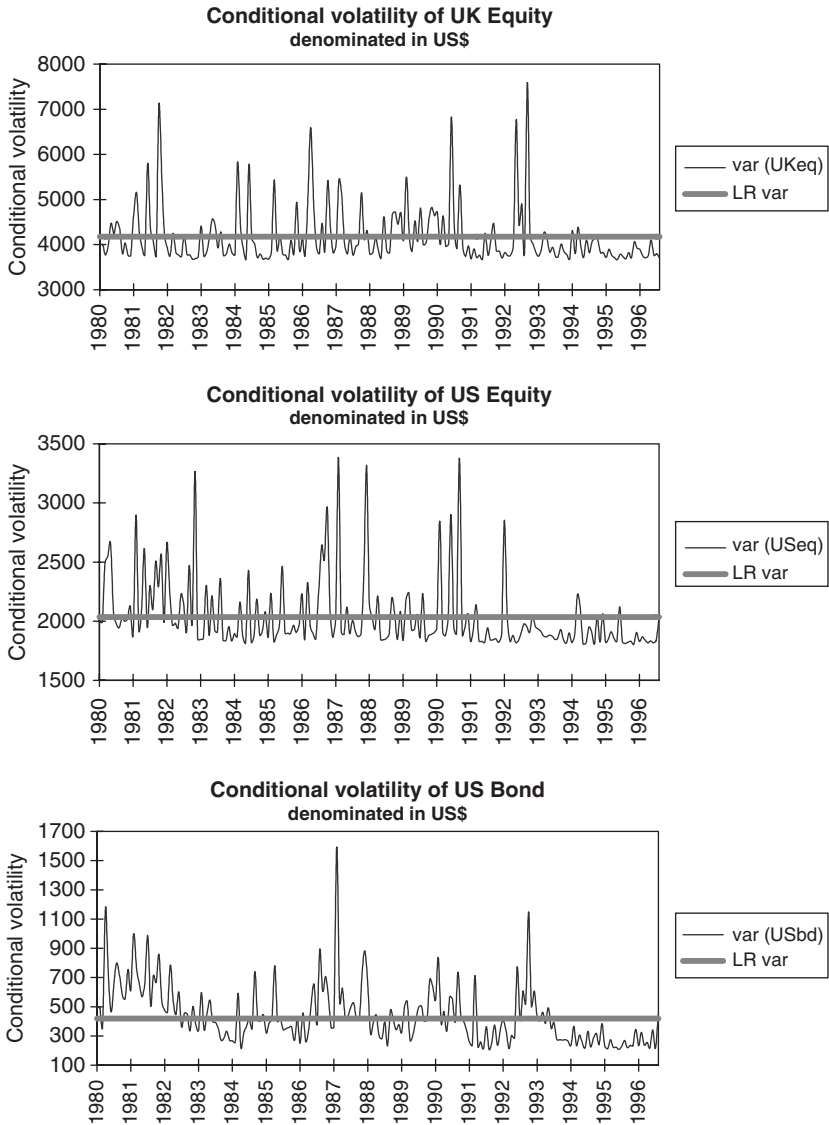
Figure 2. Conditional asset return volatilities from perspective of US investor.

country, the frontier based on the long-run covariance matrix and the mean frontier in the short run. The position of the frontiers reflect the minimum portfolio standard deviation for a given portfolio return, hence this is just another way of comparing portfolio standard deviations. For each country the mean frontier lies to the left of the long-run frontier. It is therefore possible for investors to reduce their portfolio risk by re-balancing their portfolios each period. Another implication is that the actual portfolio risk borne by investors

Portfolio frontiers
Conditional vs Unconditional
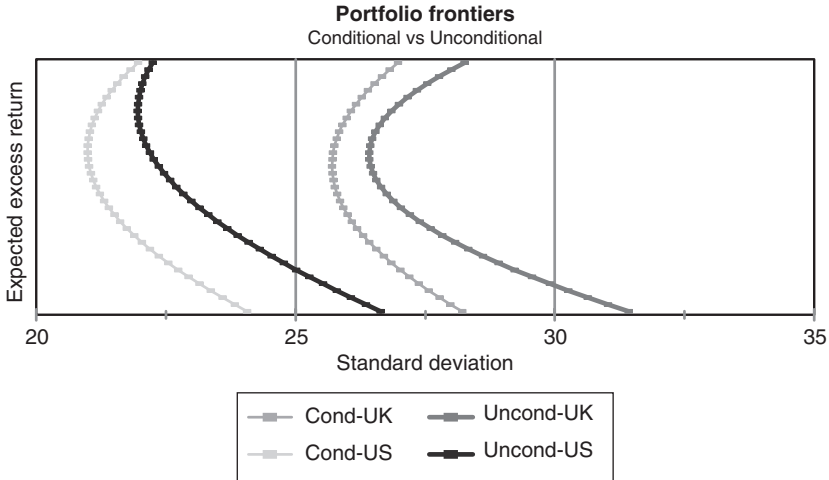
Figure 3. Conditional and unconditional portfolio frontiers of the UK and US investor.

Distribution of Portfolio Frontiers
UK and US Investors
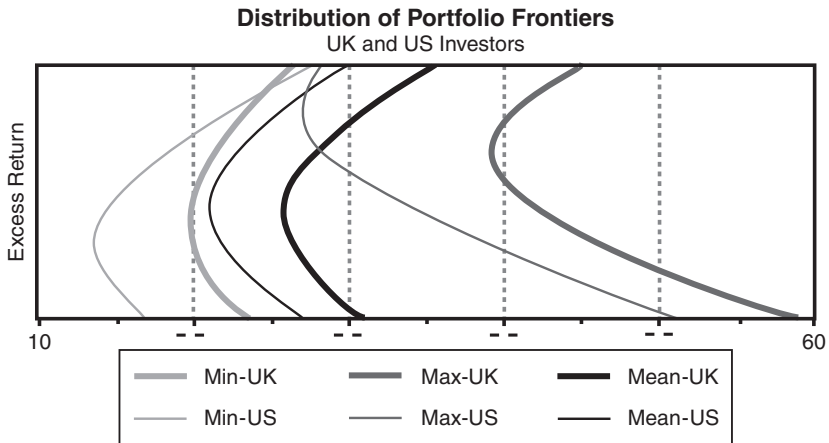
Figure 4. Portfolio frontier distribution for UK and US investor.

who use the long-run covariance matrix will be different from that shown by the long-run frontier. The frontier for the US investor lies to the left of that for the UK investor. This implies that the US investor bears less risk than the UK investor to achieve the same return. The optimal portfolio of risky assets available to the US investor should therefore deliver a higher Sharpe Performance Index than that of the UK investor.

Figure 4 provides more information on the distribution of frontier movements. It displays the maximum, minimum and mean frontiers for each

country. This shows that although the conditional distribution of frontiers for the US investor is shifted to the left of that for the UK investor, there is considerable overlap in the distributions. The graph also indicates that the conditional distributions are positively skewed, with a few periods when portfolio risk is much higher than the mean.

Optimal portfolios

The optimal portfolio of risky assets when there are no restrictions on short sales is obtained from the point of tangency between the portfolio frontier and the Capital Market line, which goes through the origin. The resulting portfolio weights vary each period. Figures 5 and 6 show the time variation in the excess return and the standard deviation of the optimal portfolios of UK and US investors, and Figure 7 shows the Sharpe Performance Index (ratio of excess
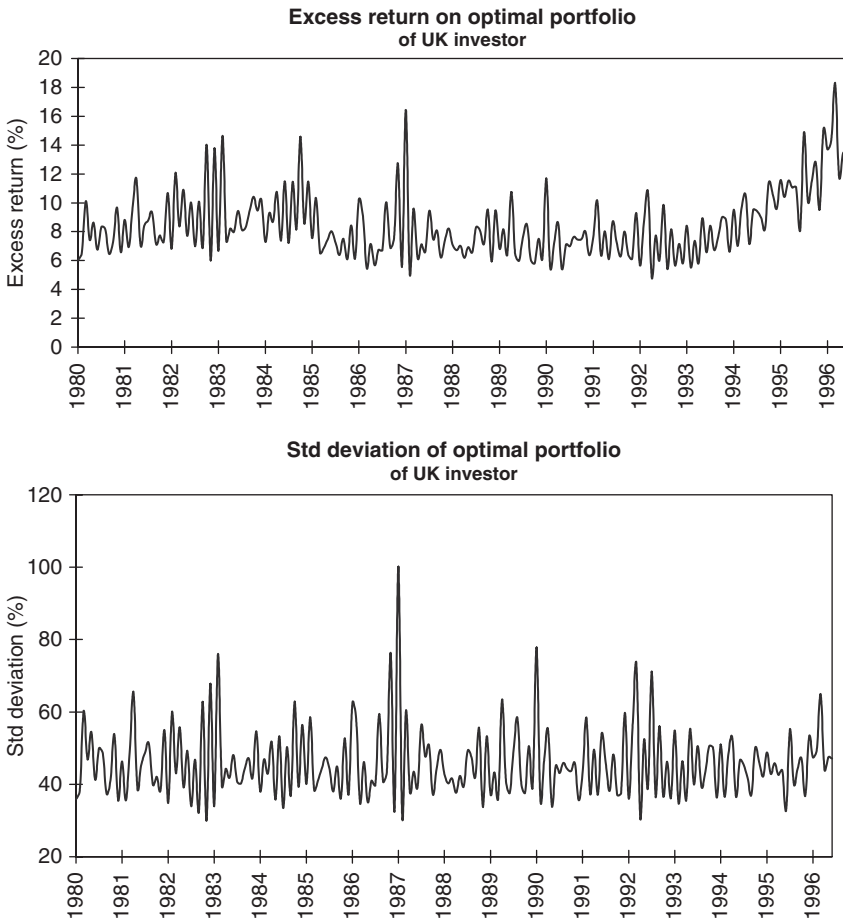
Excess return on optimal portfolio
of UK investor

Std deviation of optimal portfolio
of UK investor

Figure 5. Portfolio characteristics of UK investor's optimal portfolio.

Return on optimal portfolio
of US investor

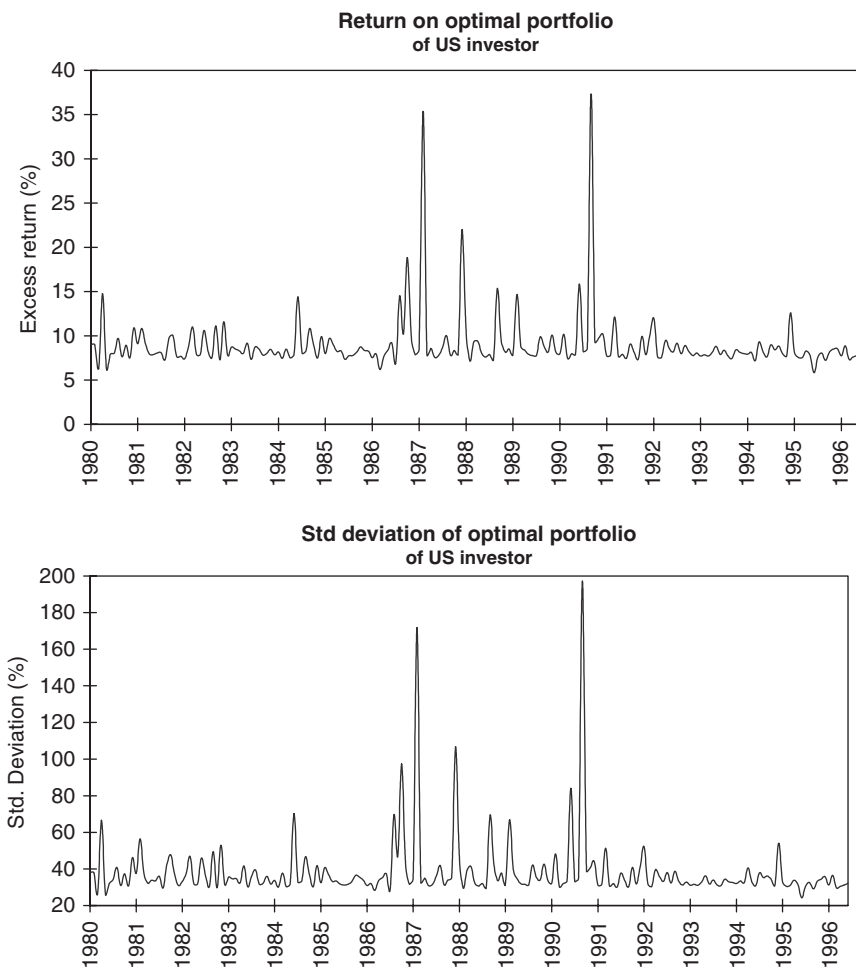**Std deviation of optimal portfolio**
of US investor

Figure 6. Portfolio characteristics of US investor's optimal portfolio.

return to risk). The US portfolio has a higher mean excess return (9.1% vs. 8%) and achieves a higher (0.24 vs. 0.18) and more stable SPI than the UK portfolio. For the United Kingdom, however, there has been a remarkable improvement in the SPI since 1992 due to the strong and persistent growth of equity prices, with the result that the SPI for the United Kingdom has exceeded that for the United States since the end of 1994.

The optimal portfolios of our representative investors differ indicating that location influences the asset allocation decision. The optimal portfolio in each period is weighted towards domestic assets, though not to the extent reported in practice. Given the sensitivity of asset holdings to changes in means, variances and covariances, it appears that the presence of exchange rate risk is sufficient to make these two large markets appear segmented. The following sub-sections examine each investor's decision in turn.
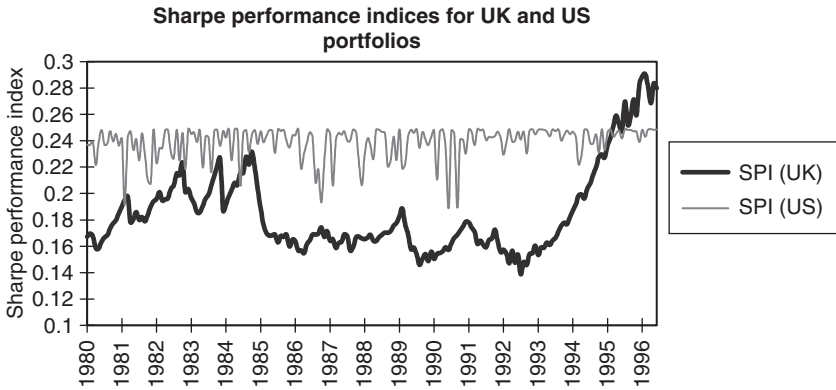
Figure 7. Sharpe Performance Indices for UK and US portfolios.

Table 5
Constant weight vector for UK investor

	UK equity	US equity	UK bond
Asset holding (%)	64	26	10

UK Investor

We consider the UK investor who is able to take unlimited short positions. We first look at the prescribed weights assuming a constant covariance matrix. This matrix is taken to be the long-run matrix, $\mathbf{V'V}$, estimated above. It fails to recognise the benefits of portfolio re-balancing in response to changes in the financial environment, which has been shown to be welfare reducing by Guidolin and Timmermann (2005). Table 5 shows the weight vector. In comparison with our time-varying allocation, this method of portfolio selection tends to overstate the importance of the UK bond, but even here US equity accounts for a large portion of the portfolio.

The time series of shares in the optimal risky portfolio of the UK investor is shown in Figure 8 and summary statistics are reported in Table 6. UK equity dominates the portfolio, accounting on average for 77% of the investment. Although the optimal proportion of UK equity fluctuates a great deal, it is never held short. In many periods it is optimal to hold over 100% of total wealth in UK equity. The optimal proportion of US equity has a mean of 27% and is relatively stable. For only one period in the entire 200 period sample is it optimal to hold US equity short. In contrast to domestic and foreign equity, it is frequently optimal to go short in UK bonds. The mean optimal proportion of UK bonds is -4%, and varies between a maximum of 55% and a minimum of -166%. The volatility of this proportion is similar to that of UK equity. The usual reason for going short in UK bonds, therefore, is to buy UK equity. This is indicated very clearly by the optimal allocations after 1992, when the rise in
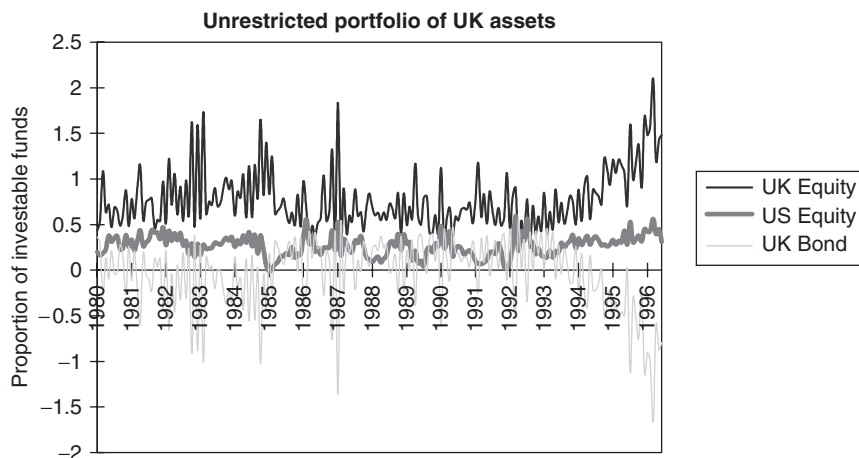
Figure 8. Unrestricted asset holdings in UK investor's optimal portfolio.

Table 6
Summary of unrestricted holdings for a UK investor

	Mean (%)	Minimum (%)	Maximum (%)
UK equity	77	24	210
US equity	27	− 2	60
UK bond	− 4	− 166	55

the share of UK equity is matched by a corresponding fall in share of UK bonds. This also explains the improvement in the SPI for UK investors after 1992.

It is unlikely that investors would follow this investment strategy for two reasons. First, the transactions costs of continuously rebalancing the portfolio in this way may be too high, although the use of indexed trackers or futures would help make it more feasible. Second, and in practice probably more important, UK mutual fund managers, the major holders of assets by far, are prohibited by law from going short. We therefore construct optimal portfolios subject to the constraint that asset shares must be non-negative. For the unconstrained portfolio it was possible to obtain a closed-form expression for the portfolio shares and hence find the return on the portfolio in each period. For the constrained portfolio we use quadratic programming to minimise the variance of the portfolio subject to a target rate of return that is chosen as the mean return on the unconstrained optimal portfolio. This implies that, in terms of the mean portfolio return, the investor is not penalised by the restriction and it aids comparison with the unrestricted case.

Figure 9 shows how the restricted shares vary over time, and Table 7 provides summary statistics. UK equity still dominates the portfolio with a mean of 71%, but its range of variation is reduced by a factor of about 9, having a maximum of 89% and a minimum of 62%. The mean share of US equity is similar and its range of variation is halved. The mean share of UK bonds is 4%, and its range
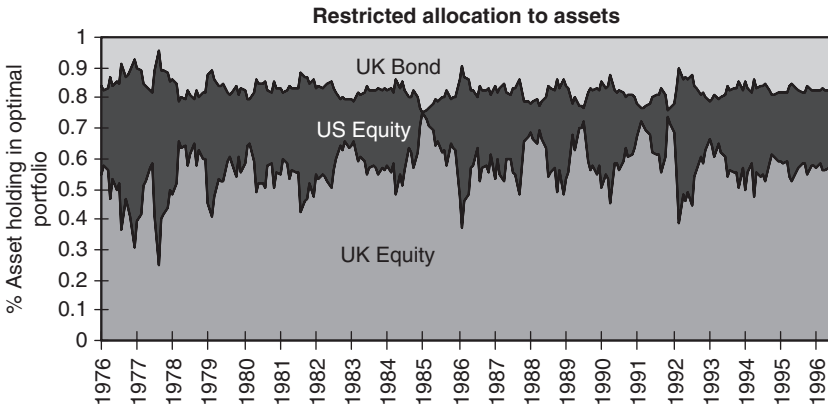
Figure 9. Restricted asset holdings in UK investor's portfolio.

Table 7
Summary of restricted holdings for a UK investor

	Mean (%)	Minimum (%)	Maximum (%)
UK equity	71	62	89
US equity	25	0	38
UK bond	4	0	11

of variation is reduced by a factor of about 20. Now that borrowing by selling domestic bonds is prohibited, portfolio re-balancing takes place mainly between domestic and foreign equity. This results in a considerable reduction in the degree of re-balancing. Survey evidence shows that UK investors hold up to 18% of their wealth in foreign assets. Our estimate is that a UK investor faced with the opportunity to form an optimal portfolio from these three risky assets should hold about 25% of wealth in US equity. The difference between the two is a measure of the extent of home bias by UK investors.

US investor

Once more, we begin our analysis by computing the vector of weights implied by the constant covariance matrix. Table 8 presents these asset holdings. This approach places too much weight on the domestic bond to the detriment of home equity. It also fails to recognise the welfare benefits of portfolio re-balancing.

The time-varying asset shares for the US investor are shown in Figure 10 with summary statistics in Table 9. The results are similar to those for the UK investor. Domestic equity has the largest share and the greatest variation, and whenever in excess of 100% of wealth is invested in US equity, it is always funded by adopting a short position in the domestic bond. The mean shares are: domestic equity 64% (compared with 77% for the United Kingdom), foreign

Table 8
Constant weight vector for US investor

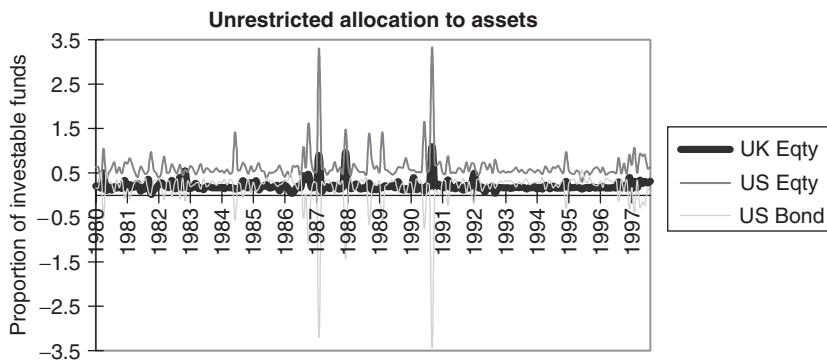|	UK equity	US equity	US bond
Asset holding (%)	20	62	18

Figure 10. Unrestricted asset holdings in US investor's optimal portfolio.

Table 9
Summary of unrestricted holdings for a US investor

	Mean (%)	Minimum (%)	Maximum (%)
UK equity	20	2	109
US equity	64	30	332
US bond	16	− 342	57

equity 20% (25%) and domestic bonds 16% (− 4%). US equity fluctuates wildly, moving between a range of 30% and 332%. Thus again this investment strategy looks excessively volatile.

Restricting the US investor to holding only non-negative positions provides Figure 11 and Table 10. Again the mean shares are hardly altered but the variation in the shares is greatly reduced compared with the unrestricted portfolio. The range of the share of US equity is from 38% to 86% and the share of the domestic bond is remarkably stable moving only between 14% and 20% of the portfolio. Even more than for the United Kingdom, re-balancing the portfolio is achieved mainly by substituting domestic for foreign equity, leaving the share of the US domestic bond relatively unchanged. Even so, UK equity still has a mean share of 20%. Survey evidence shows that US investors hold as little as 6% of their wealth in foreign assets. The home bias problem therefore seems to be much more a feature of US than UK investment. It may also be a consequence of the growth in the 1990s of the number of non-US companies listing in the US via ADRs and direct listings. US markets are potentially more
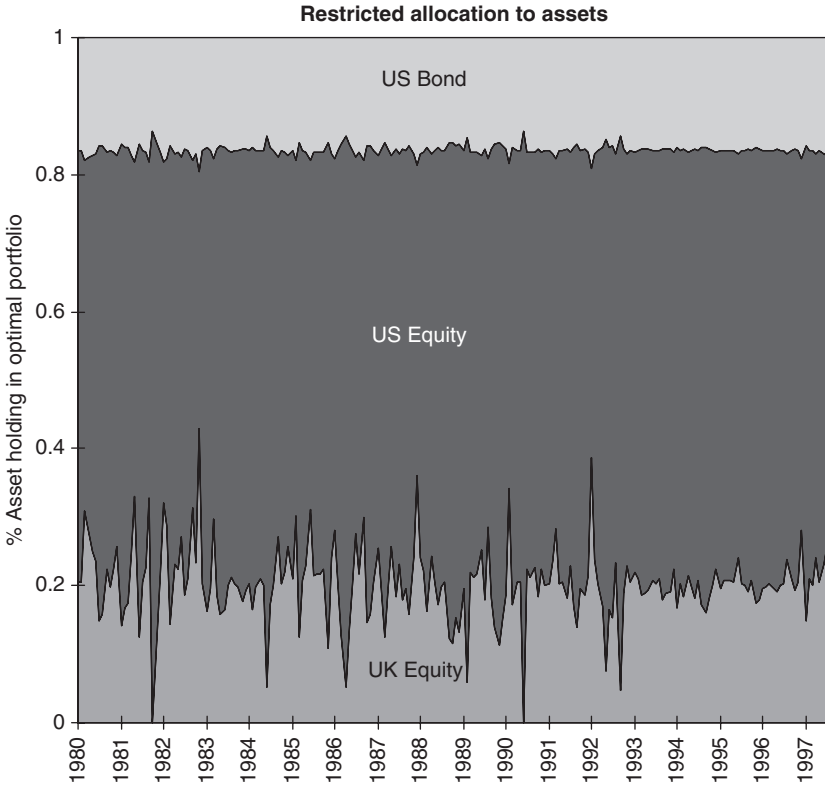
Figure 11. Restricted asset holdings in US investor's portfolio.

Table 10
Summary of restricted holdings for a US investor

	Mean (%)	Minimum (%)	Maximum (%)
UK equity	20	0	43
US equity	63	38	86
US bond	17	14	20

diversified than those of the United Kingdom. However the aforementioned studies of Froot and Dabora (1999) and Lau and McInish (2003) suggest that location factors may influence the price movement of such stocks and thus reduce the potential diversification benefits of these dual-listed stocks.

The foreign bond

In the preceding analysis, the opportunity set of the investor was limited by excluding the foreign bond. Here we examine the consequences of its inclusion.

Table 11
Unconstrained asset holdings including the foreign bond

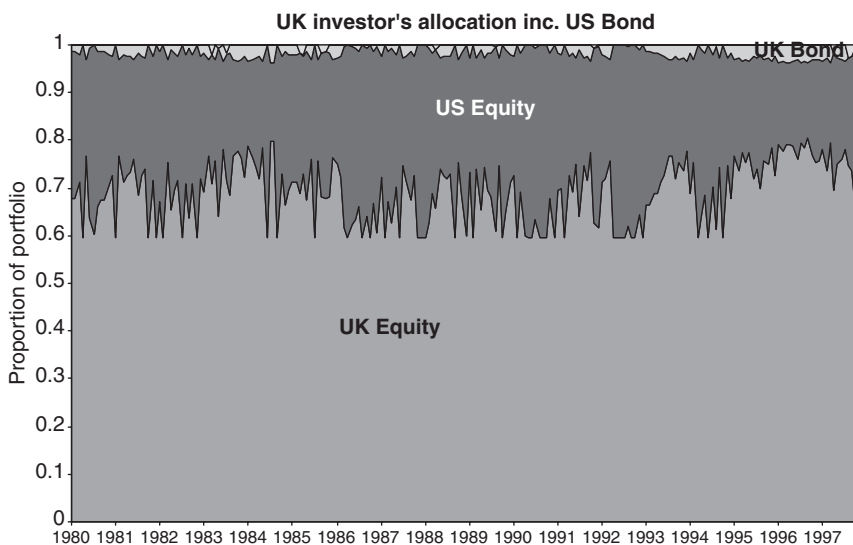|	UK equity	US equity	UK bond	US bond
UK investor	73	54	11	− 38
US investor	69	82	− 40	− 11

Figure 12. UK investor's asset allocation including foreign bond.

Table 12
Constrained asset holdings including the foreign bond

	UK equity	US equity	UK bond	US bond
UK investor	69	29	2	0
US investor	30	67	0	2

For both investors, we find that the unconstrained share held in the foreign bond is always negative and quite large. Taking the UK investor to illustrate, Table 11 reports the average unconstrained percentage holding. Our main interest, however, is in the constrained portfolio that prohibits short sales. In this case the optimal share of the foreign bond is negligible. Consequently, we pay more attention to the portfolios excluding the foreign bond. Table 12 summarises the mean holdings in the constrained allocation, while Figures 12 and 13 plot the restricted time-varying weights.
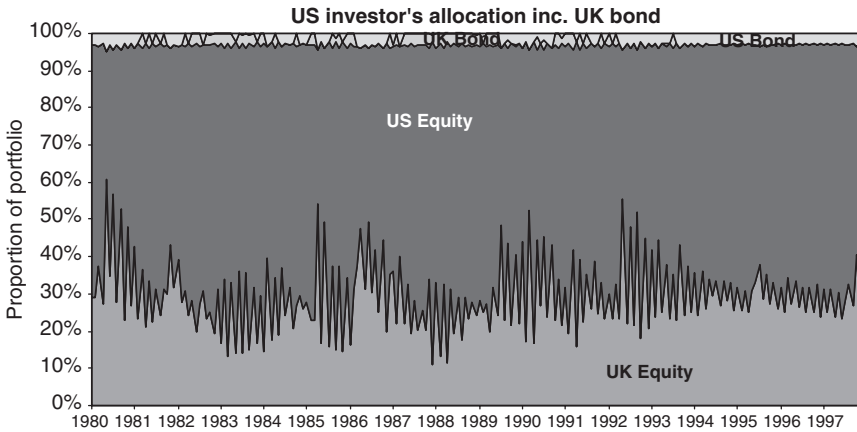
Figure 13. US investor's asset allocation including foreign bond.

V CONCLUSIONS

The aim of this paper has been to re-examine the issue of the optimal allocation each period of an internationally diversified portfolio. These results are then used to provide new ways of determining whether the location of an investor ought to affect portfolio selection. The example used in the analysis is the optimal mix of domestic and foreign equity, and domestic bonds that should be held by UK and US investors, two countries that have not had significant barriers to investing abroad for some time. Two tactical investment strategies are compared. Both are versions of Markowitz's mean-variance portfolio theory in which investors use the joint conditional distribution of excess returns, which is time varying, to re-balance their portfolios each period. One allows investors to hold unlimited short positions; the other assumes that investors are constrained from going short, the situation faced by most fund managers. The conditional covariance matrix is estimated from a form of the M-GARCH model particularly suited to portfolio analysis.

Investor location influences portfolio selection. Mean-variance weights are known to be sensitive to changes in asset characteristics and it appears that currency fluctuations is sufficient to drive a wedge between the two markets and make them appear segmented. For both investors, although domestic equity is the dominant asset, it is optimal to hold between 20% and 27% of wealth in foreign equity. This compares with survey evidence, which indicates that in practice UK investors hold around 18% in foreign assets, while US investors hold only about 6%. The home-bias puzzle seems therefore to be more acute for US than UK investors. Put another way, there seems to be more potential gains from increased international diversification for the US than the UK investor. It may be argued that the greater presence of cross-listed equities on the US market alleviates this problem as the number of ADRs grew rapidly during the 1990s but given that other derivatives respond to US factors, this source of

diversification is probably not as strong as one might expect. We stress that since our investment opportunity set is small, our figures represent a lower bound on the optimal holdings of foreign assets.

We also find that the location of the investor is important in determining the investment performance of the portfolio. The portfolio frontiers facing the US investor lie nearer the origin than for the UK investor, implying that US investors can achieve the same return as UK investors but with less risk. This 'risk-return' advantage is also shown in the higher average Sharpe Performance Index for the US – even though since 1993 the SPI for the United Kingdom has steadily improved and now lies above that for the United States.

Finally, our results confirm the findings of Flavin and Wickens (1998) that using a constant covariance matrix to construct the assets shares produces a misallocation of resources and a false estimate of one-period risk, that it is optimal to re-balance the portfolio each period, and that prohibiting short positions greatly reduces the amount of portfolio re-balancing required.

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