Public Policy Towards R&D in Oligopolistic Industries

By DERMOT LEAHY AND J. PETER NEARY *

We consider the free-market and socially optimal outcomes in a general oligopoly model with many firms which first engage in R&D and then compete in either output or price. Strategic behavior by firms tends to reduce output, R&D, and welfare and so justifies higher subsidies except when R&D spillovers are low and firms' actions are strategic substitutes. It also reduces the benefits of R&D cooperation. Moreover, policies to encourage cooperation are likely to be redundant (since it is always privately profitable) and simulations suggest that the welfare cost of lax competition policy is high. (*JEL* D43, L13, O32)

The importance of determining optimal policy towards R&D cannot be exaggerated, given the worldwide interest in fostering R&D and given the significant differences between European and U.S. policies towards interfirm cooperation on R&D.[1] Moreover, the problem is an inherently difficult one because of the complex nature of the R&D process. Since R&D is a component of fixed costs, industries where it is important tend to be concentrated. Hence R&D policy must go hand in hand with competition policy. At the same time, R&D is like any form of investment in that it precedes the production stage. Hence, issues of time consistency and strategic commitment inevitably arise in considering the choice of R&D policy. Finally, R&D by one firm typically leads to spillovers which benefit other firms, so that R&D exhibits many of the characteristics of a public good, albeit one that is mostly privately produced. The degree to which such spillovers occur and can be internalized is another crucial influence on the desirable pattern of intervention.

All these aspects of R&D generate incentives for firms to behave strategically, but, as previous writers have shown, the effects of such incentives are ambiguous. James A. Brander and Barbara J. Spencer (1983) showed that oligopolistic firms which invested strategically in R&D, with a view to improving their future competitive position vis-à-vis their rivals, would normally carry out *more* R&D than the cost-minimizing level. In this respect, the strategic incentives to which R&D give rise are identical to those arising from investment in physical capital, as considered for example by A. Michael Spence (1977), Avinash Dixit (1980), and Drew Fudenberg and Jean Tirole (1984). However, Brander and Spencer did not allow for any R&D spillovers between firms. Spence (1984) focused on this issue and noted that such spillovers dilute the strategic incentive for firms to engage in R&D (because each firm is adversely affected by the positive benefits which its own R&D confers on its rivals). Spence suggested that cooperation on R&D might internalize this negative externality, though he did not present a complete analysis. It was left to Claude d'Aspremont and Jacquemin (1988) to formalize this argument, and to show that, with sufficiently large spillovers, cooperation on R&D (though with subsequent competition at the output stage) indeed leads to more output, R&D, and welfare.

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[1] See Alexis Jacquemin (1988) and Stephen Martin (1996).

These papers and the literature they have inspired have thrown considerable light on aspects of R&D in oligopolistic markets.[2] Nevertheless, there remain a number of issues which require further exploration and which form the subject of this paper. The first of these is the need to disentangle the separate influences of strategic behavior on the one hand and R&D cooperation on the other. Whereas Brander-Spencer and Spence compare strategic and nonstrategic behavior in the absence of cooperation, d'Aspremont-Jacquemin and subsequent writers take strategic behavior for granted and concentrate on comparing the outcomes with and without R&D cooperation. Each of these approaches is incomplete and, as we shall see, potentially misleading. The first objective of this paper, therefore, is to present a comprehensive analysis of these issues in a generalization of the d'Aspremont and Jacquemin model of two-stage duopoly which allows for nonlinear demands, price as well as quantity competition, and many firms. By focusing on the incentives to engage in R&D, by invoking stability conditions in a natural way, and by making use of a new geometric technique, we are able to give a more comprehensive ranking of output, R&D, and welfare in the different cases.

A ranking of welfare levels is essential if they are to be evaluated from a public policy perspective. However, it is not sufficient as a guide to whether intervention is desirable or not. To determine this, it is also necessary to examine which of the equilibria will be chosen in the absence of intervention. In this context, the second objective of this paper is to compare the levels of industry profits in the different equilibria. This allows in particular an investigation of the market incentives for firms to engage in R&D cooperation without any intervention.

Finally, the third objective of the paper is to consider explicitly the nature of optimal intervention in each equilibrium. While most previous papers have had a public policy focus, they have not attempted to characterize explicitly the optimal package. We do this both for the first-best case, when both output and R&D subsidies are chosen optimally, and for the second-best case, when only R&D subsidies are available. We also address the problem of dynamic consistency which arises in the first-best case. If output subsidies are positively influenced by investment in R&D, and if the government cannot commit in advance to a subsidy rate, then firms have a further incentive to invest in R&D. Anticipating this strategic behavior, the government, in turn, has an incentive to offer a lower subsidy, just as strategic behavior by firms enjoying learning by doing was shown in Leahy and Neary (1994) to justify lower, rather than higher, subsidies.

The plan of the paper is as follows. Section I introduces the model and Section II considers the free-market outcome, isolating the separate influences of strategic behavior and R&D cooperation. Section III then shows how the first-best outcome can be attained by appropriate R&D and output subsidies, under different assumptions about the extent of commitment and cooperation. Section IV turns to the case where the government can only use R&D subsidies and considers the optimal second-best subsidies in this case. Section V looks at explicit solutions of the model for particular functional forms and considers the robustness of the conclusions to the relaxation of a key assumption. Finally, Section VI concludes with a summary of results.

I. The Model

We consider an industry of n identical firms, which compete over two periods. In the first, preproduction, period, each firm chooses its level of R&D, x_i; while in the second period, it chooses the level of an "action" a_i, which may be either output, q_i, or price p_i. This general specification encompasses both output

[2] Irene Henriques (1990) considers the stability of the d'Aspremont-Jacquemin model and Raymond De Bondt et al. (1992) and Morton I. Kamien et al. (1992) extend it to allow for differentiated products and price as well as quantity competition. Kotaro Suzumura (1992) allows for many firms and general demands in Cournot competition, as does Steffen Ziss (1994), who independently derives a diagram similar to ours. Relative to these papers, we make a number of contributions: we allow for general functional forms in both output and price competition; we calculate explicit expressions for the marginal return to R&D in each equilibrium; we disentangle the separate influences of R&D cooperation and strategic behavior; we consider industry profits; and we derive explicit expressions for the optimal subsidies.

(Cournot) and price (Bertrand) competition. Since we consider only symmetric equilibria in pure strategies, we assume that in Bertrand competition products are symmetrically differentiated across firms.

We assume that marginal production costs are independent of output but decreasing in R&D, both that of the firm itself and (through spillover effects) of its rivals:

$$(1) \qquad c_i = c_i(x_i, X_{-i}),$$

where X_{-i} is the total R&D carried out by all $n - 1$ firms other than firm i. We define:

$$(2) \qquad \theta \equiv -\partial c_i / \partial x_i > 0,$$

as the direct cost-reducing effect of R&D per unit output; and

$$(3) \qquad \beta \equiv \{\partial c_i / \partial X_{-i}\} / \{\partial c_i / \partial x_i\}, \quad 0 \leq \beta \leq 1,$$

as the spillover coefficient, measuring (as a fraction of θ) the extent to which firm i benefits from R&D carried out by any other firm. The terms θ and β need not be constant, but we will focus on symmetric equilibria in which they are common across all firms.[3] Finally, we define:

$$(4) \qquad \xi \equiv \{1 + (n - 1)\beta\}\theta > 0.$$

This shows the effect on each firm's marginal cost of a unit increase in R&D by all firms. In symmetric equilibria, this can be interpreted as the marginal social return to R&D per unit output.

Each firm's profits may be written as follows:

$$(5) \qquad \pi^i = R^i[c_i(x_i, X_{-i}), a_i, \mathbf{A}_{-i}] - \Gamma_i(x_i) + \sigma x_i + S^i(a_i, \mathbf{A}_{-i}, s).$$

[3] De Bondt and Irene Henriques (1995) consider asymmetric R&D spillovers. Stephen W. Salant and Greg Shaffer (1996) show that even *ex ante* identical firms may have incentives to make unequal investments in R&D. We assume a sufficiently low θ that this possibility may be ignored.

R^i denotes the firm's net revenue from production and sales, which depends on its unit production costs and on its own and other firms' actions in the second period. ($\mathbf{A}_{-i}$ denotes the vector of actions by all firms other than firm i.) In addition, the firm incurs R&D costs $\Gamma_i(x_i)$, and it receives subsidies. We denote by σ and s the per unit subsidies to R&D and output, respectively. Subsidy revenue in the second period is denoted by S^i which equals sa_i in Cournot competition and $sq_i(a_i, \mathbf{A}_{-i})$ in Bertrand competition.

In specifying firm behavior, there are two further issues to be considered, the *degree of cooperation* and the *order of moves*. Concerning the former, we are mainly interested in the implications of R&D cooperation, by which we mean that each firm chooses its R&D in order to maximize *industry* profits. We therefore contrast the cases where firms either do or do not cooperate on their levels of R&D, assuming that they choose their second-period actions in a noncooperative fashion. For reference we also consider the cartel case, where both R&D and output levels are chosen cooperatively.

As for move order, the decisions on R&D and on output or price have a natural temporal sequence. However, firms may or may not be able to commit to their second-period actions at the same time as they choose their R&D; and the government may or may not be able to commit to both subsidies in advance of firms' decisions. We assume that the government can always commit *intratemporally:* it can set the level of each period's subsidy (to R&D in period 1 and to output in period 2) before firms choose the corresponding variable. As in Leahy and Neary (1994), this leaves three alternative assumptions about the degree of *intertemporal* commitment, each implying a different order of moves:

1. *Full Commitment Equilibrium (FCE)*. In this case, the game has two stages. In the first stage the government chooses both subsidies and in the second stage the firms choose simultaneously their R&D levels and their second-period actions.
2. *Government-Only Commitment Equilibrium (GCE)*. This game has three stages. As in FCE, the government first chooses

both subsidies. Firms then choose their R&D levels at the second stage and their second-period actions at the third stage.

3. *Sequence Equilibrium (SE)*. In this four-stage game, no intertemporal commitment is possible. The government chooses its R&D subsidy σ; each firm then chooses its R&D level; next the government chooses its output subsidy s; and finally each firm chooses its second-period action.

We assume subgame perfection throughout, so at each stage each agent anticipates how its actions will influence the actions of all other agents at every future stage. Hence, the differences between the three assumptions about move order reflect differences in the constraints, institutional or other, on agents' ability to commit to future actions. Since under each assumption firms may or may not cooperate on R&D, we have a total of six combinations to be considered. Following Fudenberg and Tirole (1984), we use the term "strategic behavior" to refer to investments in R&D carried out by firms in GCE and SE with a view to affecting the environment in which the second-period game is played. (Of course, the firms are always Nash players so, strictly speaking, strategic considerations arise even in FCE.) In the next section, we examine the free-market outcome, where the government commits to zero subsidies in both periods (so the FCE and GCE cases effectively reduce to one- and two-stage games, respectively.)

II. Equilibria Without Government Intervention

A. *Second-Period Competition*

In oligopoly equilibria, each firm's first-order condition in period 2 is independent of the degree of R&D cooperation and the order of moves, and sets equal to zero the partial derivative of its profit function with respect to its own action: $\pi_i^i = 0$.[4] The second-order condition requires that $\pi_{ii}^i < 0$; while the cross derivative π_{ij}^i (which measures the effect of a unit increase in firm j's action on the marginal profitability of firm i) is negative if actions are *strategic substitutes* and positive if they are *strategic complements*. We will use the term "normal" Cournot competition for the case where outputs are strategic substitutes and "normal" Bertrand competition for that where prices are strategic complements. And we assume that the own effect dominates the cross effect:

ASSUMPTION 1: $\pi_{ii}^i - \pi_{ij}^i < 0$.

Assumption 1 must hold in Cournot competition with homogeneous products (since $\pi_{ii}^i - \pi_{ij}^i = p' < 0$, where p' is the slope of the inverse demand function) and whenever actions are strategic complements, including normal Bertrand competition (since $\pi_{ij}^i > 0$).

Additional restrictions are implied by stability. We show in the Appendix that:

LEMMA 1: *The symmetric game in second-period actions is stable if, and only if:*

$$(6) \quad \Delta \equiv -\{\pi_{ii}^i + (n-1)\pi_{ij}^i\} > 0.$$

This result can be used to illustrate the symmetric equilibrium graphically, in a space whose coordinates are the period-2 action and the level of R&D of the typical firm. Totally differentiating the first-order condition, its slope is shown in the Appendix to be:

$$(7) \qquad \frac{da}{dx} = \frac{\xi}{\Delta} q_a,$$

where $q_a = \partial q_i / \partial a_i$ is positive (equal to one) in Cournot competition and negative (equal to $\partial q_i / \partial p_i$) in Bertrand competition. So, to sign (7), we need only make the following assumption:

ASSUMPTION 2: *The stability condition given in Lemma 1, $\Delta > 0$, holds at every point*

[4] In the absence of subsidies, $\pi_i^i = R_i^i$, so profit-function derivatives could be replaced by revenue-function derivatives throughout this section. However, it is more convenient to work with the former since they also apply in later sections when subsidies are in force. This distinction is particularly important in the case of Bertrand competition, since output subsidies imply that $\pi_i^i = R_i^i + S_i^i = R_i^i + s\partial q_i / \partial p_i$.

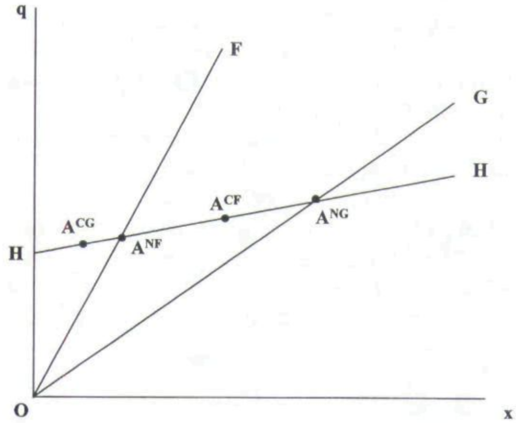

FIGURE 1. COURNOT COMPETITION WITH LOW R&D SPILLOVERS

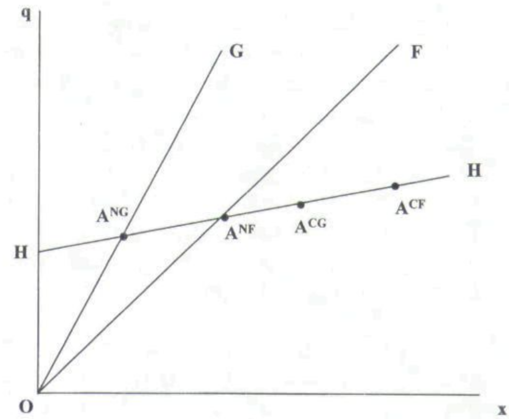

FIGURE 2. COURNOT COMPETITION WITH HIGH R&D SPILLOVERS

along HH, the locus representing the period-2 equilibrium condition, $\pi_i^i = 0$.

Given Assumption 2, the locus of (a, x) combinations satisfying the period-2 first-order condition is upward sloping in Cournot competition and downward sloping in Bertrand competition, as illustrated by the HH schedules in Figures 1 to 3. In all cases, higher R&D by the representative firm is associated with higher output.

B. *Effects of Strategic Behavior Without R&D Cooperation*

Consider next the firms' choice of R&D. The simplest case is that of "No-Cooperation FCE": firms neither cooperate on R&D nor behave strategically. The level of R&D is then chosen by setting its marginal private return equal to its marginal cost:

$$(8)\qquad \frac{\partial \pi^i}{\partial x_i} = \frac{\partial R^i}{\partial c_i}\frac{\partial c_i}{\partial x_i} - \Gamma_i' = \mu_F^N q - \Gamma' = 0, \qquad \forall i; \qquad \mu_F^N \equiv \theta.$$

(Firm subscripts can be omitted in symmetric equilibria.) In this case, the marginal private return to R&D per unit output, which we write as μ_F^N (for "No-Cooperation FCE"), is simply the reduction in the firm's own unit costs, θ.

By contrast, if firms behave strategically (so equilibrium is GCE rather than FCE), they also take account of how their R&D affects the period-2 choices of other firms:

$$(9)\qquad \frac{d\pi^i}{dx_i} = \frac{\partial \pi^i}{\partial x_i} + \sum_{j\neq i}\frac{\partial \pi^i}{\partial a_j}\frac{da_j}{dx_i} = \frac{\partial \pi^i}{\partial x_i} + (n-1)\pi_j^i\frac{da_j}{dx_i} = 0, \quad \forall i,$$

where $\pi_j^i = \partial\pi^i/\partial a_j$. As we show in the Appendix, the strategic effect on profits is:

$$(10)\qquad \pi_j^i\frac{da_j}{dx_i} = \alpha(\bar{\beta} - \beta)\theta q, \qquad i \neq j,$$

$$(11)\qquad \text{where: } \alpha \equiv -\frac{\pi_j^i q_a}{q}\frac{\pi_{ii}}{(\pi_{ii} - \pi_{ij})\Delta} > 0 \quad \text{and } \bar{\beta} \equiv \frac{\pi_{ij}}{\pi_{ii}} < 1.$$

Since α is positive in both output and price competition, the sign of (10) depends only on whether β is greater or less than the threshold value $\bar{\beta}$. Summarizing:

LEMMA 2: *In a symmetric equilibrium, the strategic effect of an increase in the R&D of one firm on its own profits is positive if, and only if, the spillover coefficient* β *is less than*

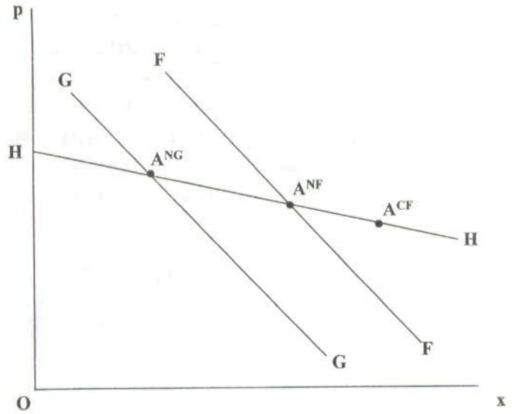

FIGURE 3. BERTRAND COMPETITION
[A^{CG} (NOT DRAWN) LIES TO THE LEFT OF A^{NG} FOR LOW SPILLOVERS AND TO ITS RIGHT OTHERWISE]

$\bar{\beta}$. This threshold value is strictly less than one and it is positive if, and only if, second-period actions are strategic substitutes.

Corollaries of Lemma 2 are that the strategic effect of higher R&D by one firm always lowers its own profits when spillovers are at their maximum ($\beta = 1$); while in the absence of spillovers ($\beta = 0$) it raises them in normal Cournot competition and lowers them in normal Bertrand competition.

The first-order condition for R&D in a symmetric GCE may now be written as:

$$\text{(12)} \qquad \frac{d\pi^i}{dx_i} = \mu_G^N q - \Gamma' = 0, \quad \forall i;$$

$$\mu_G^N \equiv [1 + (n-1)\alpha(\bar{\beta} - \beta)]\theta.$$

Here, μ_G^N (for "No-Cooperation GCE") denotes the marginal private return to R&D per unit output when firms behave strategically.[5] We now wish to compare the FCE and GCE equilibria. Clearly, at any given point μ_G^N is greater than θ if, and only if, R&D spillovers are sufficiently low; i.e., if, and only if, β is less than the threshold value $\bar{\beta}$. But this result is only a local one, which cannot directly be extended to a comparison between two distinct equilibria, since μ_G^N and θ need not be constant. Fortunately, a global comparison may be made rigorously with some additional assumptions:

ASSUMPTION 3: *Equilibrium of each type is unique.*

ASSUMPTION 4: *The profit functions in either one of the two regimes to be compared exhibit the Seade stability condition with respect to R&D levels at all points along HH between the two equilibria.*

The Seade stability condition (Jesus Seade, 1980) requires that the first-order condition for optimal choice of R&D by a single firm be decreasing in a uniform increase in R&D by *all* firms: $\partial^2\pi/\partial x_i \partial x < 0$. We can now rank the levels of output and R&D in the two equilibria:

PROPOSITION 1: *With no R&D cooperation and given Assumptions 2, 3, and 4, output and R&D are higher with strategic behavior than without (i.e., in GCE than in FCE) if, and only if, the spillover parameter is less than the threshold level $\bar{\beta}$ (where both β and $\bar{\beta}$ are evaluated at the equilibrium corresponding to the regime opposite to that specified in Assumption 4).*

This result generalizes the findings of Brander and Spencer (1983) to allow for R&D spillovers: they showed that, in normal Cournot competition with no spillovers, output and R&D are higher when firms behave strategically. By contrast, our result shows that this is only true for $\beta < \bar{\beta}$. A corollary is that it is never true in normal Bertrand competition.

Proposition 1 is illustrated in Figures 1 to 3. In each figure the F and G schedules reflect the first-order conditions for R&D in FCE and GCE, respectively, and their intersection points with HH (A^{NF} and A^{NG}) represent the FCE and GCE equilibria, respectively. Stability requires that F and G cut HH from below

[5] Algebraically, μ_G^N could be negative if β is high and α is large. (In homogeneous-product Cournot competition, this requires extremely convex demand.) Henceforward we rule out such cases and consider only interior equilibria in which R&D is positive.

TABLE 1—MARGINAL PRIVATE RETURN TO R&D PER UNIT OUTPUT IN DIFFERENT EQUILIBRIA

	No cooperation on R&D	Cooperation on R&D
FCE	θ	ξ
GCE	$[1 + (n-1)\alpha(\bar{\beta} - \beta)]\theta$	$\phi\xi$

Notes: $\xi \equiv [1 + (n-1)\beta]\theta$;
$\Delta \equiv -\{\pi_{ii} + (n-1)\pi_{ij}\} > 0$;
$\phi \equiv 1 - (n-1)h < 1$;
$\alpha \equiv h \dfrac{\pi_{ii}}{\pi_{ii} - \pi_{ij}} > 0$;
$\bar{\beta} \equiv \dfrac{\pi_{ij}}{\pi_{ii}} < 1$;
$h \equiv -\dfrac{\pi^i_j q_a}{q\Delta} > 0.$

in Cournot competition and from above in Bertrand competition, as shown. Figures 2 and 3 show that R&D and output are lower (or, equivalently, R&D is lower and price is higher) when firms behave strategically and β exceeds $\bar{\beta}$.

C. *Effects of Strategic Behavior with R&D Cooperation*

Suppose now that firms cooperate in their choice of R&D levels, though they continue to compete at the second stage. Following d'Aspremont and Jacquemin (1988), we assume that the cooperative level of R&D is chosen to maximize joint profits, but that cooperation does not affect the value of the spillover parameter β. (The implications of relaxing this assumption are considered in Section V, subsection C.) Once again, we must distinguish between FCE, where firms commit to both R&D levels and second-period actions, and GCE, where R&D levels are chosen in the anticipation of their strategic effects in the second-period game.

Consider first the case of FCE. Cooperation implies that the optimal level of R&D by each firm maximizes industry profits $\Pi = \Sigma\pi^j$:

$$(13)\quad \frac{\partial\Pi}{\partial x_i} = \frac{\partial\pi^i}{\partial x_i} + (n-1)\frac{\partial\pi^j}{\partial x_i} = \mu^C_F q - \Gamma' = 0, \quad \forall i; \qquad \mu^C_F \equiv \xi.$$

Private (μ^C_F) and social (ξ) returns to R&D coincide: cooperation fully internalizes the externality arising from R&D spillovers. However, this ignores any strategic motive. By contrast, in GCE, each firm takes account of the full effect of its choice of R&D on industry profits. As in the case of no cooperation [(9) above] this adds extra terms:

$$(14)\quad \frac{d\Pi}{dx_i} = \frac{d\pi^i}{dx_i} + (n-1)\frac{d\pi^j}{dx_i} = \frac{\partial\Pi}{\partial x_i} + (n-1)\pi^i_j\left[\frac{da_i}{dx_i} + (n-1)\frac{da_j}{dx_i}\right] = 0, \quad \forall i.$$

Using the results in the Appendix, this becomes:

$$(15)\quad \frac{d\Pi}{dx_i} = \mu^C_G q - \Gamma' = 0, \quad \forall i;$$

$$\mu^C_G \equiv \phi\xi,$$

$$\phi \equiv 1 + (n-1)\frac{\pi^i_j q_a}{q\Delta} < 1.$$

The coefficient of ξ is less than unity: with cooperation, the marginal return to R&D is lower in GCE than in FCE. Once again, we require regularity conditions similar to those assumed in Proposition 1 for this to hold globally:

PROPOSITION 2: *Given Assumptions 2, 3, and 4, then, when firms cooperate in their choice of R&D, the levels of output and R&D are lower with strategic behavior than without (i.e., in GCE than in FCE).*

D. *Effects of R&D Cooperation*

We can now consider the effects of cooperation itself. The results of the last two subsections are summarized in Table 1, which gives the marginal private return to R&D in each of the four equilibria, denoted by μ^k_l, $k = N, C$; $l = F, G$. Recalling that ξ is the social marginal return to R&D per unit output, we can see that no cooperation with FCE and cooperation with GCE lead to underinvestment in R&D, whereas no cooperation with GCE may lead to either underinvestment or overinvestment. [The exact condition for the latter is derived in (25) below.] We have already compared the results across columns in Table 1, and now wish to compare across rows.

The first row suggests why cooperation is superficially desirable: in FCE, it ensures that each firm takes account of the effects of its R&D on the costs of all other firms. Hence, with zero spillovers it gives rise to the same equilibrium and, with strictly positive spillovers, it leads to higher output and R&D (provided again that the regularity conditions hold). However, the GCE comparison in the second row shows that this conclusion is complicated by strategic behavior. Comparing the two expressions at a given point, we see that, when firms behave strategically (i.e., in GCE), the marginal private return to R&D with cooperation, μ_G^C, is greater than that without, μ_G^N, if, and only if, β exceeds a new threshold value $\bar{\beta}'$:

$$(16)\quad \mu_G^C - \mu_G^N = (n-1)(\alpha+\phi)(\beta-\bar{\beta}')\theta,$$

$$\bar{\beta}' \equiv \frac{\alpha}{\alpha+\phi}.$$

Assuming that ϕ is positive (so that a strategic cooperative has some incentive to engage in R&D), $\bar{\beta}'$ lies strictly between zero and one. (In the d'Aspremont-Jacquemin case of homogeneous-product Cournot competition with linear demand, both it and $\bar{\beta}$ equal 0.5.) Finally, as in previous subsections we may extend this local comparison to a global comparison. Under the same regularity conditions as before, we obtain:

PROPOSITION 3: *Given Assumptions 2, 3, and 4, then, when firms do not behave strategically, cooperation leads to more output and R&D, provided spillovers are strictly positive; but when they do behave strategically, it leads to less output and R&D unless spillovers are sufficiently high that β is greater than $\bar{\beta}'$ (where both β and $\bar{\beta}'$ are evaluated at the equilibrium corresponding to the regime opposite to that specified in Assumption 4).*

Propositions 2 and 3 allow the equilibria with R&D cooperation to be located in Figures 1 to 3, with both output (or price) and R&D levels ranked by the locations of the corresponding equilibria.[6]

E. *Industry Profits*

Having examined how different assumptions about strategic behavior and R&D cooperation affect output and R&D, we turn to evaluate the different equilibria from both private and social perspectives. Consider first industry profits. Totally differentiating industry profits in symmetric equilibria yields:

$$(17)\quad d\Pi = [\pi_i^i + (n-1)\pi_j^i]n\,da + (\xi q - \Gamma')n\,dx.$$

Setting the coefficients of *da* and *dx* equal to zero shows that a cartel, which seeks to maximize industry profits, chooses output (or, equivalently, price) such that marginal *industry* profitability is zero; but for that level of output it chooses the efficient level of R&D. Moreover, we can draw iso-profit curves in (a, x) space which are horizontal where they cross the efficient R&D locus (along which ξq equals Γ'), and vertical where they cross the cartel period-2 equilibrium locus (along which $\pi_i^i + (n-1)\pi_j^i$ equals zero). Two such curves, centered on the cartel outcome A^M, are shown in Figures 4 and 5. The curve HH in these diagrams, as in Figures 1 to 3, represents the oligopoly equilibrium locus (i.e., the locus of points at which the marginal profitability of second-period actions is zero). The line MM represents the cartel period-2 equilibrium locus (the locus of points which equate marginal cost to industry marginal revenue), and must lie below HH in Cournot competition and above it in Bertrand competition. Finally, the line R represents the efficient R&D locus, on

[6] A comprehensive list of the location of all four equilibria for intermediate values of β requires a comparison of the values of μ along the diagonals of Table 1. Details of this in the homogeneous-product Cournot case are given in the Appendix to an earlier version of this paper, Leahy and Neary (1995).

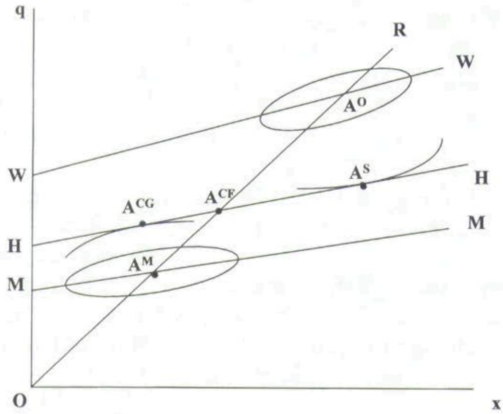

FIGURE 4. ISO-PROFIT AND ISO-WELFARE LOCI IN COURNOT COMPETITION

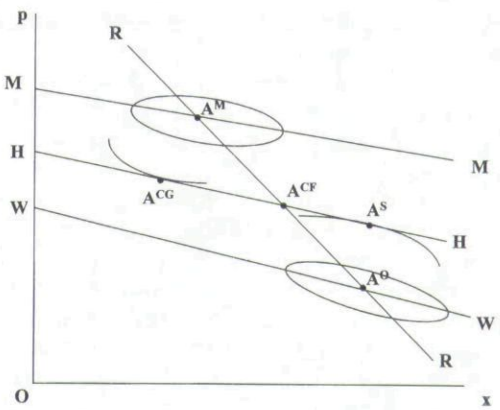

FIGURE 5. ISO-PROFIT AND ISO-WELFARE LOCI IN BERTRAND COMPETITION

which must lie both the cartel equilibrium A^M and the nonstrategic cooperative equilibrium A^{CF}. Changes in the spillover parameter shift all the loci in Figures 4 and 5 but they do not affect the qualitative relationships between the loci shown in the diagram.

In order to rank the level of profits in the different equilibria, we first need to establish which of the oligopoly equilibria maximizes industry profits. To do this, we use the period-2 first-order condition, $\pi^i_i = 0$, and its slope, (7), to simplify (17):

$$(18) \quad \frac{1}{n}\frac{d\Pi}{dx} = (n-1)\frac{\pi^i_j q_a}{\Delta}\xi + \xi q - \Gamma'$$

$$= \phi\xi q - \Gamma'.$$

From (15), this is the first-order condition for R&D in the "Cooperation GCE" equilibrium. Hence, we may conclude:

PROPOSITION 4: *Across all oligopoly equilibria, industry profits are maximized when R&D is chosen both cooperatively and strategically.*

To rank the levels of profits in other equilibria, we must assume that industry profits are quasi-concave in x along HH. If this holds, profits fall monotonically as we move away from A^{CG} in either direction along the HH locus, and the rankings of equilibria given in Section II, subsection D, allow us to rank the levels of profits by inspection of Figures 1 to 5.

F. *Social Welfare*

The result that R&D cooperation when firms play strategically maximizes oligopoly profits raises crucial questions about the policy stance towards cooperation. However, before considering these, we must examine the levels of welfare. Following standard convention, we measure welfare as the sum of consumer surplus, $u(\{\mathbf{q}\}) - \Sigma p_i q_i$ (where the aggregate utility function depends on the vector of quantities $\{\mathbf{q}\}$), and industry profits, net of subsidy payments:

$$(19) \quad W(q, x) = u(\{\mathbf{q}\}) - c(x)nq - n\Gamma(x).$$

As with industry profits, we differentiate this totally, setting $u_i = p$:

$$(20) \quad dW = (p - c)nQ_a da + (\xi q - \Gamma')n dx,$$

where $Q_a = \partial q_i / \partial a_i + (n-1)\partial q_j / \partial a_i$ is positive (equal to one) in Cournot competition and negative in Bertrand competition. Naturally, first-order conditions for a welfare maximum are that price equal marginal cost and that the marginal social return to R&D equal its marginal cost. Away from the optimum, we can draw iso-welfare contours which are horizontal where they cross the efficient R&D locus (where ξq equals Γ'), and vertical where they cross the $p = c$ locus. The latter, denoted WW in Figures 4 and 5, lies above the HH locus in (q, x) space and below it in (p, x) space, since the socially optimal level of output exceeds the oligopoly level for a given level of R&D. Two such iso-welfare curves, centered on the social optimum A^O, are shown in Figures 4 and 5.

As with profits, it is helpful to determine where welfare is maximized along the HH locus. This is done by substituting from equation (7) into (20) to obtain:

$$(21)\qquad \frac{1}{n}\frac{dW}{dx} = \mu^S q - \Gamma' = 0,$$

$$\text{where: } \mu^S \equiv \left[1 + (p - c)\frac{q_a Q_a}{q\Delta}\right]\xi.$$

μ^S is the social return to R&D per unit output at the second-best optimum. The coefficient of ξ is greater than one, implying that for given output the second-best optimum requires *overinvestment* in R&D. Moreover, μ^S is greater than the marginal private return to R&D per unit output in all of the four oligopoly equilibria, with the possible exception of the "No-Cooperation GCE" case when actions are strategic substitutes and spillovers are low. [Equation (27) below gives the exact condition.] As with earlier results, these local comparisons imply global rankings under appropriate regularity conditions:

PROPOSITION 5: *If actions are chosen noncooperatively, welfare is maximized when the level of R&D is such that its marginal return per unit output equals μ^S. Given Assumptions 2, 3, and 4, this second-best optimum has higher levels of output and R&D than any of the free-market oligopoly outcomes, except for the "No-Cooperation GCE" equilibrium when actions are strategic substitutes and β is low.*

Geometrically, the second-best optimum in Figures 4 and 5 is the point of tangency A^S of HH with the highest attainable iso-welfare locus. Moreover, provided the welfare function is quasi-concave in x along HH, we may rank the welfare levels in different equilibria in terms of their distance from A^S. Inspection of the diagrams shows that the ranking of the equilibria with respect to welfare is almost exactly the reverse of the ranking with respect to profits when spillovers are low. By contrast, when spillovers are high, the two rankings are more similar. Keeping this in mind, we proceed to derive rules for optimal intervention in the next section.

III. Attaining the First-Best Optimum

When we come to consider optimal intervention, it is immediately obvious that *R&D policy alone cannot attain the first-best optimum.* With two targets to control (the levels of output and R&D of the representative firm), two instruments are required. If we assume that the second instrument is an output subsidy, its optimal value must be such that the gap between price and marginal cost is eliminated, i.e.,

$$(22)\qquad s^O = bq > 0,$$

where b is (the absolute value of) the slope of the representative firm's inverse demand function (equal to $-\partial p_i/\partial q_i$ in Cournot competition and $(-\partial q_i/\partial p_i)^{-1}$ in Bertrand competition). This formula holds irrespective of the order of moves of firms and government and irrespective of whether firms cooperate on R&D or not.

Turning to R&D policy, assume first that the government can commit to the optimal output subsidy before firms choose their R&D. The optimal R&D policy then follows immediately from the results of the last section. With a subsidy, the profit-maximizing condition for R&D in each of the four equilibria becomes:

$$(23)\qquad \mu_l^k q + \sigma_l^k = \Gamma', \qquad k = N, C;$$

$$l = F, G,$$

TABLE 2—FIRST-BEST OPTIMAL SUBSIDIES TO R&D IN DIFFERENT EQUILIBRIA

	No cooperation on R&D	Cooperation on R&D
FCE	$(\xi - \theta)q \geq 0$	0
GCE	$(n-1)(\alpha+1)(\beta - \tilde{\beta}')\theta q \gtrless 0$	$(1-\phi)\xi q > 0$
SE	$\sigma_G^N - \phi\Psi' q \gtrless 0$	$\sigma_G^C - \phi n \Psi' q \gtrless 0$

Notes: $\tilde{\beta}' \equiv \frac{\alpha}{\alpha+1}\bar{\beta} < 1$; $\Psi' \equiv \frac{\partial s^O}{\partial x_i}$.

where the marginal return to R&D per unit output can be read from Table 1. Since the first-best optimum requires that the marginal cost of R&D, Γ', equal its marginal social return ξq, the optimal R&D subsidy must be:

$$(24) \quad \sigma_l^k = (\xi - \mu_l^k)q, \qquad k = N, C; \quad l = F, G.$$

The exact values of this expression in each of the four equilibria are given in the first two rows of Table 2. All are necessarily nonnegative, except for σ_G^N:

$$(25) \quad \sigma_G^N > 0 \quad \text{IFF} \quad \beta > \tilde{\beta}' \equiv \frac{\alpha}{\alpha+1}\bar{\beta}.$$

The threshold value of β is strictly positive if, and only if, actions are strategic substitutes. [Recall equation (11).] This implies that, if firms behave strategically but noncooperatively and competition is normal Cournot, R&D should be *taxed* if there are no spillovers. However, the threshold is likely to be small: it is always less than $\bar{\beta}$ in absolute value and it is algebraically less than $1/(n+1)$ in homogeneous-product Cournot competition. So there is always some level of spillovers which justify subsidization and R&D should *always* be subsidized in normal Bertrand competition. Summarizing:

PROPOSITION 6: *If the government can commit to the optimal output subsidy, then the optimal R&D subsidy is nonnegative, except in the "No-Cooperation GCE" equilibrium when $\beta < \tilde{\beta}'$ (which requires that actions are strategic substitutes and β is low).*

We now wish to compare the values of the subsidies in the different equilibria. Unlike Propositions 1 to 5, such comparisons do not require any regularity conditions. All the subsidies are evaluated at the same point and so their values may be compared directly. And, since the optimal subsidies are directly related to the marginal returns to R&D by equation (24), comparison between them is straightforward. By applying the results of earlier sections, we may immediately state:

PROPOSITION 7: *When firms do not cooperate on their choice of R&D, strategic behavior implies a higher optimal subsidy if, and only if, $\beta > \bar{\beta}$; when they do cooperate, strategic behavior always implies a higher optimal subsidy; cooperation without strategic behavior requires no subsidy; and, given strategic behavior, cooperation requires a higher subsidy than no cooperation if, and only if, $\beta < \bar{\beta}'$.*

How are these results affected if the government cannot commit to an output subsidy, the case we call Sequence Equilibrium? Firms now anticipate that the output subsidy will be set by equation (22).[7] But the right-hand side of this depends on the levels of R&D. Hence, firms have a strategic incentive to alter their R&D in order to increase their output subsidy. The government in turn, anticipating this incentive, should take it into account in setting its R&D subsidy. Even though x is chosen before the output subsidy, the government can still achieve the GCE optimum since it has two instruments at its disposal and only two distinct targets, x and q (as in Leahy and Neary, 1994).

The resulting fully time-consistent optimal subsidies are derived in the Appendix and given in the third row of Table 2. We use the

[7] As already noted, we confine attention to subgame perfect equilibria. Cases where the government first announces the GCE output subsidy and then reneges are considered in a related model by Leahy and Neary (1996).

parameter Ψ' to represent the derivative of the optimal output subsidy s^O with respect to each firm's level of R&D: $\Psi' \equiv \partial s^o/\partial x_i$. The precise form this takes depends on the details of the second-period game. However, it is likely to be positive, implying that the output subsidy is a *strategic complement* for R&D. This is because the optimal output subsidy depends on the gap between marginal revenue and price; since higher R&D reduces the latter (by lowering marginal cost which encourages an increase in output) it can only lower the optimal subsidy if it causes a more than compensating fall in marginal revenue.[8] If Ψ' is positive, there is an additional payoff to investment in R&D, which mandates a lower value of the R&D subsidy to restrain firms from this strategic overinvestment. With cooperation, both the extra incentive and the necessary corrective by the government are greater, depending on $n\Psi'$ rather than Ψ'. In both cases, with and without R&D cooperation, the optimal subsidy is lower in SE than in the corresponding GCE case if, and only if, Ψ' is positive. Summarizing:

PROPOSITION 8: *If the government cannot commit to an output subsidy, then the first-best optimum can still be achieved. With or without R&D cooperation, the optimal R&D subsidies are unambiguously lower than with government commitment if, and only if, Ψ' is positive, implying that the output subsidy is a strategic complement for R&D.*

The optimal subsidies may be negative in SE both with and without cooperation on R&D.

IV. Optimal Second-Best Subsidies to R&D

Suppose now that it is not possible to subsidize output.[9] Welfare is related to R&D and period-2 actions in the same way as before. However, the only instrument now available to the government is the R&D subsidy σ. This alters the incentive for investment in R&D but cannot affect the period-2 first-order condition, $\pi_i^i = 0$. Hence the best outcome that policy can achieve is the second-best optimum, where the marginal return to R&D equals $\mu^S q$. Combining (21) and (23), the R&D subsidy which attains this outcome in each of the four oligopoly equilibria is given by:

$$(26) \quad \bar{\sigma}_l^k = (\mu^S - \mu_l^k)q, \qquad k = N, C; \quad l = F, G.$$

Since μ^S exceeds ξ, it must also exceed each of the μ_l^k, with the possible exception of μ_G^N. The exact condition for the latter is as follows:

$$(27) \quad \bar{\sigma}_G^N > 0 \quad \text{IFF} \quad \beta > \bar{\beta}'' \equiv \frac{(n-1)(\alpha+1)\tilde{\beta}' - \alpha'}{(n-1)(\alpha+1+\alpha')},$$

$$\text{where: } \alpha' \equiv (p-c)\frac{q_a Q_a}{q\Delta} > 0.$$

The new threshold $\bar{\beta}''$ must be negative if actions are strategic complements; while if they are strategic substitutes it may still be negative and must be less than $\tilde{\beta}'$, which we have already seen in Section III is likely to be small. Summarizing:

PROPOSITION 9: *The second-best optimal subsidies to R&D are positive, except for the "No-Cooperation GCE" equilibrium when actions are strategic substitutes and β is low.*

---

[8] For example, in homogeneous-product Cournot competition, Ψ' equals $(1 + r)\xi/n^2$, where r equals nqp''/p', a measure of the concavity of the market demand curve. Hence Ψ' is positive except when demand is highly convex. For a similar result in tariff theory, see Brander and Spencer (1984) and Ronald W. Jones (1987).

[9] Suzumura (1992) also considers the possibility that output subsidies are not feasible. However, he deals with it by using what he calls a "Second-Best Welfare Function," defined as $W^S(x) = W[x, q(x)]$, where $q(x)$ is the solution to the Cournot oligopoly output equilibrium condition setting marginal cost equal to marginal revenue. Geometrically, this amounts to looking at values of welfare along the HH locus in Figures 1 and 2. Suzumura's approach is equivalent to ours in a linear model. (See Section V, subsection A.) In more general contexts, our approach has the advantage of permitting an exact comparison of the subsidies in all four equilibria at the same second-best optimal point.

In addition, the rankings of the optimal subsidies which held in the first-best optimum continue to hold in the second-best optimum. For completeness, we state this as follows:

PROPOSITION 10: *The rankings of subsidies given in Proposition 7 for the first-best optimum also hold at the second-best optimum.*

Like Propositions 6 to 8, these two are completely general and do not require any regularity conditions (though, of course, the parameters must now be evaluated at the second-best optimum itself). However, although μ^S must exceed ξ at any given point, we cannot compare the first- and second-best R&D subsidies in the same manner. Nor can we adapt the techniques used earlier, since Assumption 4 does not rule out the possibility that the values of μ_i^k at A^O and $\bar{A}$ may differ considerably. However, in the special linear case of Section V, subsection A, below, when these parameters as well as μ^S and ξ are all constant, we can be sure that each second-best optimal subsidy is greater than the corresponding first-best optimal subsidy.

V. Linear Cournot Competition and Cooperative Synergies

In this section, we explore the properties of the model in more detail for special functional forms and then discuss the consequences of relaxing a key assumption.

A. *Cournot Competition with Linear Demands and Quadratic Costs of R&D*

Consider the case of homogeneous-product Cournot competition with linear demands, so $p = a - bQ$, with a and b constant. On the cost side, we assume that the marginal cost of output is linear in R&D, so (1) becomes: $c_i = c_0 - \theta(x_i + \beta X_{-i})$, with c_0, θ, and β constant. In symmetric equilibria this simplifies to: $c = c_0 - \xi x$, assumed nonnegative. As for the costs of R&D itself, we assume they are quadratic: $\Gamma(x) = \gamma x^2/2$, with γ constant.

Under these assumptions, the expressions derived for the general case simplify considerably: see the Appendix, Table A1. Comparisons between equilibria now depend on only three parameters: the number of firms n; the spillover parameter β; and a new parameter measuring the relative effectiveness of R&D, defined as $\eta \equiv \theta^2/b\gamma$. Table A1 helps resolve some ambiguous results. For example, it shows that the level of R&D at the second-best optimum is always less than at the first-best optimum. (Details are in the Appendix.) It is also possible to obtain a feel for the quantitative implications of the model with the help of Figure 6. This illustrates the relationship between welfare in each equilibrium (relative to that in the social optimum) and the spillover parameter β, assuming $n = 2$ and $\eta = 0.4$.

The first conclusion suggested by Figure 6 is that the level of welfare in the second-best optimum closes only about half of the gap between the levels of welfare in the cartelized equilibrium and the first-best optimum. To see how robust this is, Figure 7 shows for different values of n how $(W^S - W^M)/(W^O - W^M)$ varies as a function of a composite parameter χ^2, which depends positively on *both* the effectiveness of R&D η and the spillover parameter β.[10] As Figure 7 shows, with large numbers of firms, welfare in the second-best optimum is close to the first-best level, and so R&D policy has the potential to close much of the gap between the cartel and the first-best levels. However, for highly concentrated industries (small n), R&D policy alone (in the absence of competition policy) is at best a limited tool for improving market performance. The potential effectiveness of R&D policy is dramatically lower when R&D is highly effective and spillovers are high.

The second conclusion suggested by Figure 6 concerns the benefits of cooperation. In the absence of strategic behavior, cooperation

[10] Comparisons between all the equilibria, such as those in Figure 6, depend on three parameters, n, β, and η. However, if we exclude equilibria in which R&D is chosen noncooperatively, as in Figures 7 and 8, the comparisons depend only on two parameters: n and χ^2, which equals $\xi^2/nb\gamma$, or, alternatively, $\{1 + (n-1)\beta\}^2\eta/n$. The range for β in Figure 6 (0.0 to 1.0) corresponds to a range for χ^2 in Figures 7 and 8 of 0.2 to 0.8. The explicit expressions underlying Figures 7 and 8 are given in the Appendix.

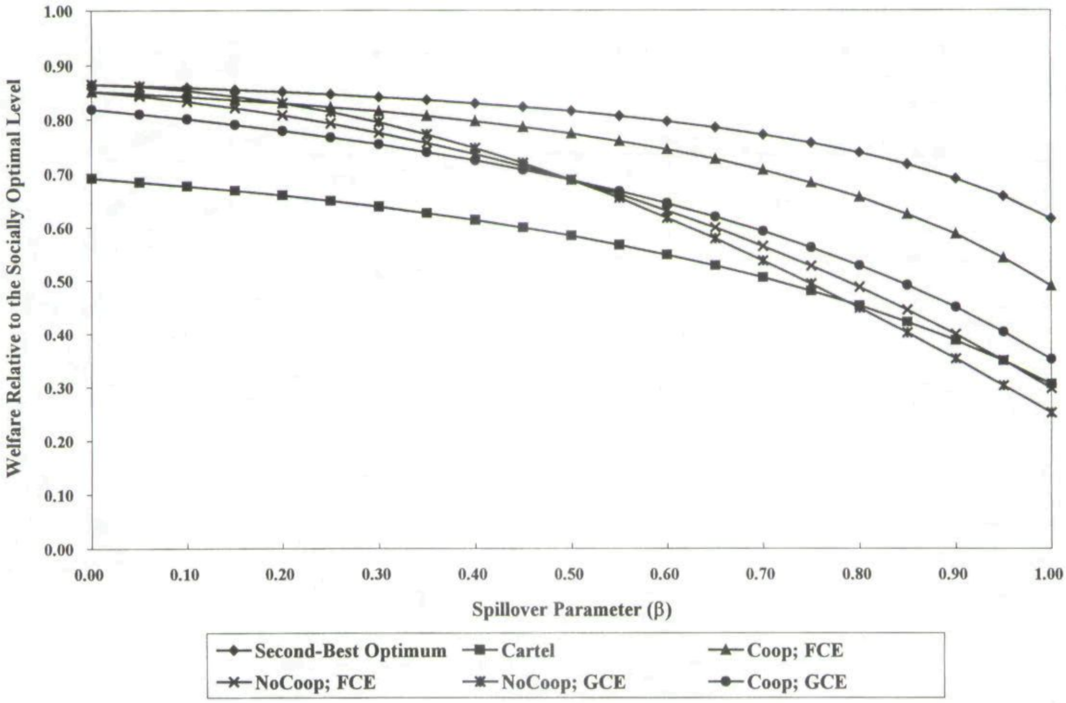

FIGURE 6. LEVELS OF WELFARE IN DIFFERENT EQUILIBRIA (LINEAR DEMAND; QUADRATIC COSTS OF R&D; $\eta = 0.4$; $n = 2$)

Notes: These curves are exact for the case of Cournot competition with linear demand, quadratic costs of R&D, and $\eta = 0.4$. Each curve shows, for $n = 2$, the level of welfare in the corresponding equilibrium relative to that at the social optimum, as a function of the spillover parameter, β.

greatly enhances market performance: in FCE, the cooperative equilibrium always leads to a higher level of welfare than the noncooperative one and the gap is significant when spillovers are high. However, when firms behave strategically, the superiority of cooperation is much reduced. In GCE, cooperation leads to higher welfare than no cooperation whenever β exceeds 0.5 (as we know from the theoretical results). But cooperation does not help that much, leading for high spillovers to a level of welfare only slightly higher than the cartel level. Figure 8 shows that this result is robust: except for cases with ineffective R&D, low spillovers, and large numbers of firms, strategic cooperation leads to welfare levels which are not much better than the cartel level. Recalling from Proposition 4 that firms always have an incentive to cooperate, this suggests that the case for encouraging cooperation is much weaker than previous studies suggest: such encouragement is both limited in its potential for raising welfare and likely to be redundant in any case.

B. *Stability of the Linear Model*

It is necessary to check that any set of parameter values is consistent with stability. The details of this are lengthy and are relegated to the Appendix. In all cases, stability is more likely the lower is η, i.e., the higher the cost of, and the lower the effectiveness of, R&D. Heuristically, higher values of η impart an element of increasing returns to the model, increasing the incentive for each firm to deviate from the symmetric equilibrium. As for increases in β, they tend to enhance stability when firms do not cooperate but to make it less likely when they do.

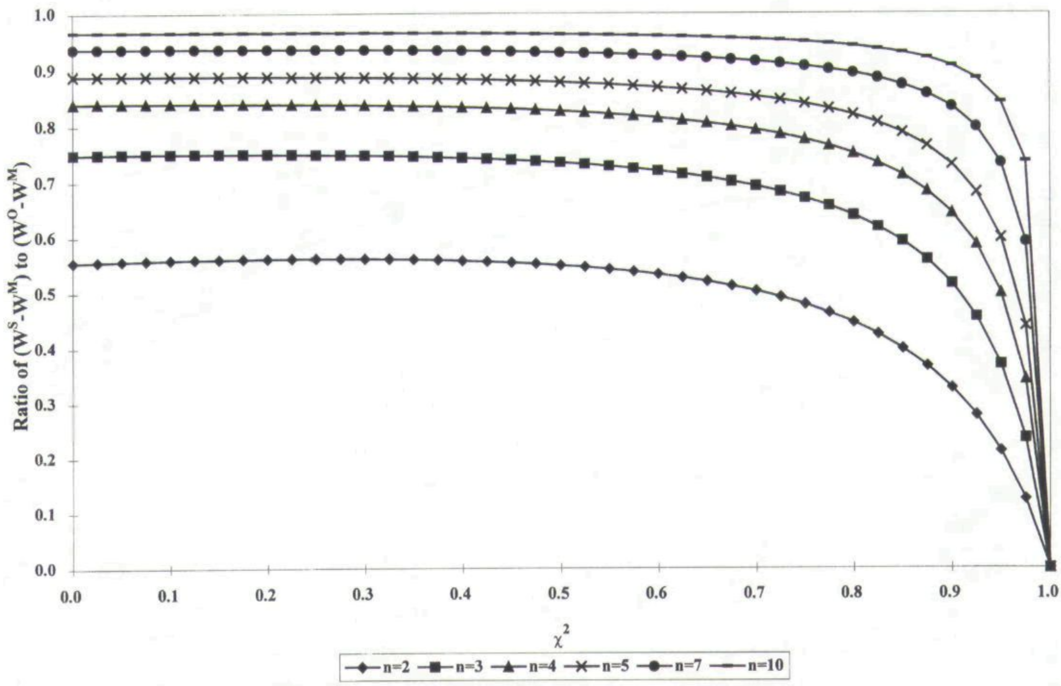

FIGURE 7. MAXIMUM WELFARE GAIN FROM R&D POLICY ALONE

Notes: Same assumptions as Figure 6, except that η and n are now variables. Each curve shows, for different levels of n, the ratio of $(W^S - W^M)$ to $(W^O - W^M)$ as a function of χ^2.

Instances of instability should be interpreted as a failure of the assumptions of the model, especially that of symmetry, requiring most plausibly some entry or exit of firms from the industry.

C. *Cooperative Synergies*

So far, we have assumed that the spillover parameter β is unaffected by the decision to cooperate. This is clearly unrealistic, and a number of authors have considered the implications of allowing β to rise when cooperation occurs, reflecting cooperative synergies.[11] Kamien et al. (1992) go so far as to describe as an "R&D cartel" the type of cooperation which we have considered so far (in which β is unaffected), reserving the term "cooperation" for the case where spillovers are complete ($\beta = 1$).

How are our results affected if R&D is subject to cooperative synergies? The first point to note is that all our conclusions concerning the effects of strategic behavior for a *given* degree of cooperation are clearly unaffected. Thus it remains true that strategic behavior tends to reduce output, R&D, and welfare in most cases, except the Brander-Spencer benchmark case of strategic substitutes, low spillovers, and no cooperation. Our conclusions about the effects of cooperation itself must be amended, of course. The expressions given in Table 1 continue to hold, but comparisons across rows must allow for higher values of β, and hence of ξ, when firms cooperate on their R&D. This naturally tends to make cooperation more attractive from a welfare point of view. However, it also makes it

[11] See, for example, Michael Katz (1986) and Massimo Motta (1994). Yannis Katsoulacos and David Ulph (1994) construct a model in which the spillover parameter is endogenized.

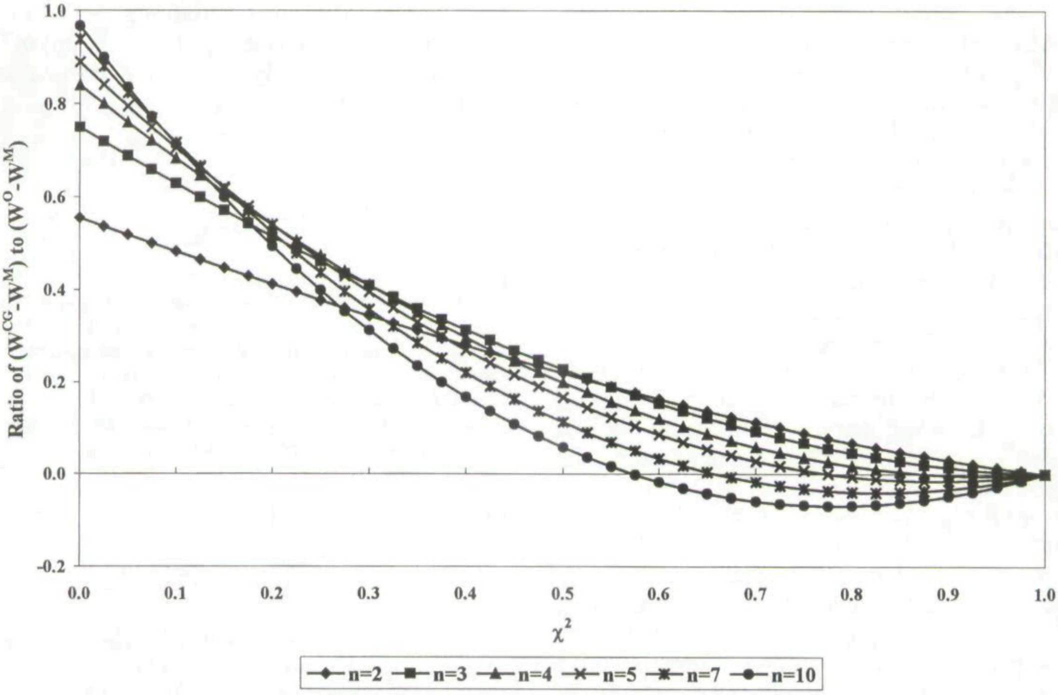

FIGURE 8. WELFARE GAIN FROM STRATEGIC COOPERATION ON R&D

Notes: Same assumptions as Figure 7. Each curve shows, for different levels of n, the ratio of $(W^C_G - W^M)$ to $(W^O - W^M)$ as a function of χ^2.

more privately profitable, reinforcing the view that policy intervention to encourage cooperation is likely to be redundant whether or not it is desirable. Finally, the conclusions drawn from the simulations of the linear case in Figure 6 also continue to hold for high β, while Figures 7 and 8 are unaffected by cooperative synergies.

VI. Summary and Conclusions

This paper has considered a general model of an oligopolistic industry, in which firms first invest in R&D and then produce output. Our objective has been to present a unified treatment of a number of issues in order to establish the principles which should govern public intervention in industries where R&D is important. In particular, we have sought to disentangle the influences of strategic behavior and R&D cooperation on the levels of output, R&D, and welfare; to compare the private profitability and social performance of different equilibria; and to calculate explicitly the optimal subsidies to output and R&D under alternative assumptions. Since our model allows for arbitrary numbers of firms and general functional forms in both output and price competition, it encompasses and generalizes earlier treatments of this topic.

In comparing the levels of output, R&D, profits, and welfare in different equilibria, the paper makes three technical contributions. First, Propositions 1 to 3 and 5 make global comparisons between these levels directly, subject to mildly restrictive regularity conditions (essentially, that the Seade [1980] stability condition hold along a path between the two equilibria to be compared). Second, Propositions 6 to 10 compare the optimal subsidies which are required to attain either the first-best optimum (where both output and R&D subsidies can be used) or the second-best optimum (where only R&D subsidies are available). These results rely on local comparisons only and so need far fewer qualifications than those of Propositions 1 to 3 and 5. Third, we show that the difference between output and price competition is not a crucial determinant of the results. What matters is the extent of R&D spillovers and whether firms' second-period actions (outputs in Cournot competition, prices in Bertrand competition) are strategic substitutes or complements.

Turning to our substantive contributions, we have stressed the need to distinguish the implications of strategic behavior on the one hand from those of R&D cooperation on the other. Concerning the former, our results show that strategic behavior tends to *reduce* output, R&D, and welfare and to mandate higher subsidies in all cases except that considered by Brander and Spencer (1983). The exception is when firms choose their R&D levels noncooperatively, R&D spillovers are low and firms' actions are strategic substitutes. In all other cases, strategic behavior generates outcomes which are unambiguously less desirable from a social perspective.

As for R&D cooperation, its superficial attractiveness is highlighted by the fact that it unambiguously raises output, R&D, and welfare (eliminating the need for any R&D subsidy) when firms do *not* behave strategically. However, with strategic behavior, cooperation is less attractive: only when spillovers are high does it raise welfare and so require a lower subsidy.

Our results do not overturn the finding of d'Aspremont and Jacquemin that R&D cooperation is socially desirable when spillovers are high. However, they cast doubt on both its relevance and its usefulness. The result is less relevant because industry profits are *always* higher when firms choose their R&D strategically and cooperatively. Indeed, with higher spillovers, cooperation is more attractive from *both* private and social perspectives. So intervention to encourage cooperation is likely to be least needed when cooperation itself is socially desirable. As for the usefulness of the result, with its implication that R&D cooperation should be encouraged (or facilitated by relaxing antitrust legislation), our simulations for the linear Cournot case suggest that the payoff from doing so is likely to be low and that the welfare cost of lax competition policy is likely to be high.

APPENDIX

PROOF OF LEMMA 1:

We give a stability proof for a more general case, which is also useful in the section dealing with stability below. Define λ and ρ as the derivatives of the marginal profitability of firm i with respect to its own action and to the action of any other firm j, respectively: $\lambda \equiv \pi_{ii}$ and $\rho \equiv \pi_{ij}$. From Dixit [1986 equation (36-ii)], necessary conditions for stability in a symmetric n-firm oligopoly are that:

$$\text{(A1)} \quad \text{(i)}\ \lambda < 0 \quad \text{and}$$

$$\text{(ii)}\ (-1)^n(\lambda - \rho)^{n-1}[\lambda + (n-1)\rho] > 0.$$

Moreover, from Seade (1980 Theorem 1), $\lambda + (n-1)\rho > 0$ is *sufficient* for *in*stability and so $\lambda + (n-1)\rho < 0$ is *necessary* for stability. (We rule out the knife-edge case $\lambda + (n-1)\rho = 0$, since convergence to equilibrium will not occur.) Combining this with (A1) (i) gives a general necessary condition: $\lambda < \text{Min}\{0, -(n-1)\rho\}$. And combining it with (A1) (ii) when n is even gives: $\lambda < \text{Min}\{\rho, -(n-1)\rho\}$. As for sufficient conditions, Frank H. Hahn (1962) implies: $\lambda < \rho < 0$; and Seade (1980 Appendix) implies: $|\lambda| > (n-1)|\rho|$. Combining these with (A1) (ii) gives a general sufficient condition: $\lambda < \text{Min}\{\rho, -(n-1)\rho\}$. Hence:

LEMMA A1: In a symmetric n-firm oligopoly model: (i) $\lambda < \text{Min}\{0, -(n-1)\rho\}$ *is always necessary for stability; (ii)* $\lambda < \text{Min}\{\rho, -(n-1)\rho\}$ *is always sufficient; and (iii) when n is even,* $\lambda < \text{Min}\{\rho, -(n-1)\rho\}$ *is both necessary and sufficient.*

Corollaries of Lemma A1 are that the Seade sufficient condition: (a) is necessary when $n = 2$; but (b) does not nest the Hahn condition when $n > 2$. Applying Lemma A1 to the period-2 game: $\lambda - \rho < 0$ by Assumption 1 and $\lambda + (n-1)\rho = -\Delta$, which gives Lemma 1.

Derivation of Equations (7) and (15) and Proofs of Lemma 2 and Proposition 8

We begin by totally differentiating the period-2 first-order condition of a typical firm i:

$$\text{(A2)} \quad \theta q_a dx_i + \beta\theta q_a dX_{-i} + \pi^i_{ii} da_i$$

$$+ (n-1)\pi^i_{ij} da_j + q_a ds = 0, \quad \forall i.$$

TABLE A1—LEVELS OF OUTPUT, R&D, AND WELFARE IN COURNOT COMPETITION WITH LINEAR DEMANDS AND QUADRATIC COSTS OF R&D

	Output (q)	R&D (x)	Welfare (W)
First-best optimum	$\frac{1}{1-\chi^2}\frac{A}{nb}$	$\frac{1}{1-\chi^2}\frac{\xi A}{nb\gamma}$	$\frac{1}{1-\chi^2}\frac{A^2}{2b}$
Cartel	$\frac{1}{2-\chi^2}\frac{A}{nb}$	$\frac{1}{2-\chi^2}\frac{\xi A}{nb\gamma}$	$\frac{3-\chi^2}{(2-\chi^2)^2}\frac{A^2}{2b}$
Oligopoly	$\frac{n}{n+1-n\tilde{\mu}\chi}\frac{A}{nb}$	$\frac{n}{n+1-n\tilde{\mu}\chi}\frac{\mu A}{nb\gamma}$	$\frac{n(n+2-n\tilde{\mu}^2)}{(n+1-n\tilde{\mu}\chi)^2}\frac{A^2}{2b}$

Notes: $A \equiv a - c_0$; $\chi \equiv \xi/\sqrt{(nb\gamma)} = [1+(n-1)\beta]\sqrt{(\eta/n)}$; $\tilde{\mu} \equiv \mu/\sqrt{(nb\gamma)} = (\mu/\theta)\sqrt{(\eta/n)}$; $\eta \equiv \theta^2/b\gamma$. For each oligopoly equilibrium, the appropriate value of μ, the marginal private return to R&D per unit output, should be read from Table 1, with $\pi_{ii} = -2b$ and $\pi_{ij} = -b$, and so $\alpha = \phi = 2/(n+1)$ and $\overline{\beta} = 0.5$.

For the second-best optimum, the values of output, R&D, and welfare are given by the corresponding formula for the oligopoly equilibria, with μ set equal to μ^s. Specializing the general formula given in (21) to the linear case gives $\mu^s = (n+2)\xi/(n+1)$.

TABLE A2—THRESHOLD VALUES OF η CONSISTENT WITH STABILITY IN COURNOT GCE WITH LINEAR DEMANDS AND QUADRATIC COSTS OF R&D

Stability condition	Description	Threshold values of η	
		Without cooperation	With cooperation
$\lambda < -(n-1)\rho$	Necessary; necessary and sufficient for $\beta > 0.5$	$\frac{(n+1)^2}{2[1+(n-1)\beta][n-(n-1)\beta]}$	$\frac{(n+1)^2}{2[1+(n-1)\beta]^2}$
$\lambda < \rho$	Sufficient; necessary and sufficient for $\beta < 0.5$ and n even	$\frac{n+1}{2(1-\beta)[n-(n-1)\beta]}$	$\frac{n+1}{2(1-\beta)[1+(n-1)\beta]}$
$\lambda < 0$	Necessary; only relevant for $\beta < 0.5$ and n odd	$\frac{(n+1)^2}{2[n-(n-1)\beta]^2}$	$\frac{(n+1)^2}{2[1+(n-1)\beta][n-(n-1)\beta]}$

Consider now two shocks to a symmetric equilibrium. The first is a uniform increase in R&D by each firm with no government intervention; so $dX_{-i} = (n-1)dx_i$, $da_i = da_j$, $ds = 0$, and firm subscripts and superscripts can be suppressed. Solving (A2) in this case gives (7), the slope of the HH schedule. The second shock, the subject of Lemma 2, is an increase in R&D by firm i alone, so $dX_{-i} = 0$. Now we need an additional equation, obtained by totally differentiating the first-order condition of a typical firm j whose R&D does *not* increase:

$$\text{(A3)} \qquad \beta\theta q_a dx_i + \pi^j_{jj} da_j + \pi^j_{ji} da_i + (n-2)\pi^j_{jk} da_k + q_a ds = 0, \qquad i \neq j \neq k.$$

Although this shock is asymmetric, the symmetry of the initial equilibrium allows us to set $da_j = da_k$, $\pi^j_{jj} = \pi^i_{ii}$, and $\pi^j_{jk} = \pi^i_{ij} = \pi^j_{ji} = \pi_{ij}$. As for ds, it equals zero in Section II and, as discussed in Section III, $\Psi' dx_i$ in SE. Solving equations (A2) and (A3) then yields:

$$\text{(A4)} \quad (\pi_{ii} - \pi_{ij})\Delta da_i = [\{\pi_{ii} - \pi_{ij} + (n-1)(1-\beta)\pi_{ij}\}\theta + (\pi_{ii} - \pi_{ij})\Psi']q_a dx_i,$$

$$\text{(A5)} \quad (\pi_{ii} - \pi_{ij})\Delta da_j = [(\beta\pi_{ii} - \pi_{ij})\theta + (\pi_{ii} - \pi_{ij})\Psi']q_a dx_i.$$

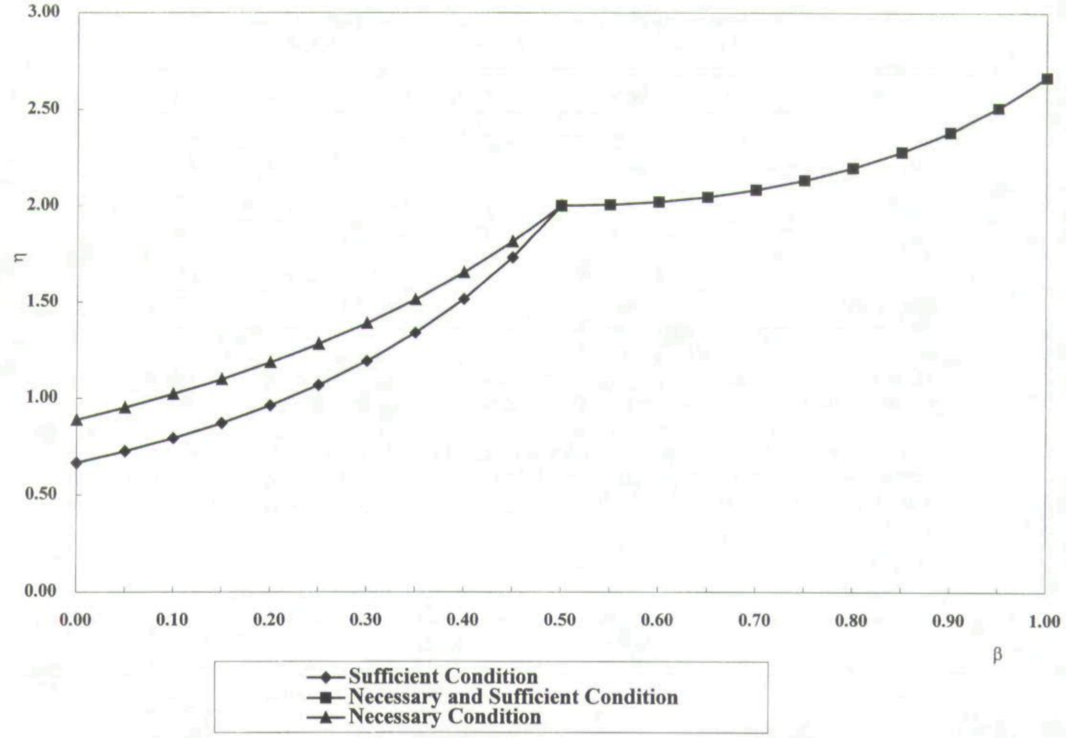

FIGURE A1. MAXIMUM VALUES OF η CONSISTENT WITH STABILITY IN THE LINEAR (N-GCE) GAME: $n = 3$

Equation (A5) alone with $\Psi' = 0$ gives equation (10) and Lemma 2; while equations (A4) and (A5) combined with $\Psi' = 0$ give equation (15). Repeating these exercises with Ψ' nonzero gives the optimal subsidies under SE in the third row of Table 2. Proposition 8 follows immediately.

Proofs of Propositions 1, 2, 3, and 5

We give a general proof, which applies to all of these propositions. In each case, we wish to compare two equilibria: (x^1, a^1) which satisfies $\mu^1(x^1, a^1)q^1 = \Gamma'(x^1)$, the first-order condition for maximization of $\pi^1(x_1, \cdots x_n)$ with respect to x_i; and (x^2, a^2) which satisfies $\mu^2(x^2, a^2)q^2 = \Gamma'(x^2)$, the first-order condition for maximization of $\pi^2(x_1, \cdots x_n)$ with respect to x_i. Both equilibria lie along HH, so from Assumption 2, a and x are monotonically related: $a = a(x)$; while q equals a in Cournot equilibria and is monotonically decreasing in a in (symmetric) Bertrand equilibria. This allows us to eliminate a and q, writing $m(x) \equiv \mu[x, a(x)]$, $q = q(x)$ and $g(x) \equiv \Gamma'(x)$.

The problem may now be stated compactly. Given: two first-order conditions, (a) $m^1(x^1)q(x^1) = g(x^1)$; and (b) $m^2(x^2)q(x^2) = g(x^2)$; a local ranking of μ's at the first equilibrium, (c) $m^1(x^1) > m^2(x^1)$; Assumption 3, (d) equilibrium of each type is unique; and Assumption 4 (the Seade condition) holding for the profit function of the second equilibrium, (e) $\partial^2\pi^2/\partial x_i\,\partial x < 0$ at x^1, x^2, and all intermediate points; we wish to prove that $x^1 > x^2$. The proof is immediate. From (b), (d), and (e): $m^2(x)q(x) < g(x)$ for all x if, and only if, $x > x^2$. But from (a) and (c): $m^2(x^1)q(x^1) < g(x^1)$. Hence $x^1 > x^2$.

The assumptions made do not rule out the possibility of the local ranking of the μ's being reversed at the second equilibrium: (f) $m^1(x^2) < m^2(x^2)$. However, in that case, Assumption 4 cannot hold for the profit function of the first equilibrium; i.e., $\partial^2\pi^1/\partial x_i\,\partial x < 0$ cannot hold. For, if it did, we would have from (a) and (d): $m^1(x)q(x) < g(x)$ for all x if, and only if, $x > x^1$. But, since $x^1 > x^2$, this implies that $m^1(x^2)q(x^2) > g(x^2)$. From (b), this in turn implies that $m^1(x^2) > m^2(x^2)$, which contradicts (f).

Linear Demands and Quadratic Costs of R&D

Table A1 gives the values of output, R&D, and welfare under the assumptions of Section V, subsection A. The results are most conveniently expressed in terms of two new parameters χ and $\tilde{\mu}$, which equal ξ and μ, respectively, deflated by $\sqrt{(nb\gamma)}$. For each oligopoly equilibrium the appropriate value for μ should be read from Table 1, with $\pi_{ii} = -2b$ and $\pi_{ij} = -b$, and so $\alpha = \phi = 2/(n+1)$ and $\bar{\beta} = 0.5$.

From Table A1, R&D at the first-best optimum is greater than at any of the oligopoly equilibria if, and only if, $\mu < (n+1)\xi/n$. This holds in all cases (including the second-best optimum) except for μ_G^N when $n > 2$ and β is low.

Figure 7 is based on the following expression:

$$\text{(A6)} \qquad \frac{W^S - W^M}{W^O - W^M} = 1 - \frac{(2-\chi^2)^2}{(n+1)^2 - n(n+2)\chi^2}.$$

This equals $1 - 4/(n+1)^2$ at $\chi^2 = 0$, rises very slightly until it reaches its maximum value at $\chi^2 = 1/n(n+2)$, and then falls monotonically to reach zero when χ^2 itself attains its maximum permissible value of unity. The expression underlying Figure 8 is:

$$\text{(A7)} \quad \frac{W^{CG} - W^M}{W^O - W^M} = 1 - (2-\chi^2)^2 \times \frac{(n+1)^2 + n(n-1)(n^2+n+2)\chi^2}{\{(n+1)^2 - 2n\chi^2\}^2}.$$

Like (A6), this equals $1 - 4/(n+1)^2$ at $\chi^2 = 0$. It then falls steadily (actually falling below zero for high χ^2 when n is greater than 3) to reach zero at $\chi^2 = 1$.

Stability

With linear demands and quadratic costs of R&D, the first-order condition for R&D in all cases is simply: $\mu q - \gamma x = 0$, where μ and γ are constants. To apply Lemma A1 to the linear R&D game under GCE, differentiate this with respect to x_i, and substitute from the linear versions of (A4) and (A5):

$$\text{(A8)} \qquad \lambda = -\gamma + \mu \frac{dq_i}{dx_i} = -\gamma + \mu \frac{n-(n-1)\beta}{b(n+1)}\theta,$$

$$\text{(A9)} \qquad \rho = \mu \frac{dq_j}{dx_i} = -\mu \frac{1-2\beta}{b(n+1)}\theta.$$

Substituting the appropriate values of μ for the noncooperative and cooperative cases, we can calculate threshold values for η consistent with the stability conditions. The results are given in Table A2 and are illustrated for the noncooperative case when $n = 3$ in Figure A1. All of the latter are increasing in β, while those for the cooperative case are decreasing in β, except for the sufficient condition in the range $\beta \in \{(n-2)/2(n-1), 0.5\}$. Finally, the result of Henriques (1990) emerges as a special case of ours: assuming $n = 2$ and $\eta = 1$ with no cooperation gives a threshold value for β of $(3 - \sqrt{7})/2 \approx 0.177$.

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