

## **Time Consistency, Learning by Doing and Infant-Industry Protection: The Linear Case\***

DERMOT LEAHY

*University of Birmingham and University College, Dublin*

J. PETER NEARY

*University College, Dublin*

---

*Abstract:* This paper examines the implications for strategic trade policy of different assumptions about precommitment in a dynamic oligopoly game with learning by doing. Assuming that demands are linear, we find that the optimal first-period subsidy is increasing in the rate of learning with precommitment but *decreasing* in it if the government cannot precommit to future subsidies. The infant-industry argument is thus reversed in the absence of precommitment.

### I INTRODUCTION

**I**n oligopolistic industries when pure profits are being made, governments typically have an incentive to employ a strategic trade policy. This involves precommitment to subsidies or tariffs that are designed to shift rents to home firms or to the government itself. However, if firms move before governments they may be able to influence the government's policy choices. It has been shown by de Meza (1986) and Neary (1994) that, if an export subsidy is optimal, governments should offer larger subsidies to lower-cost firms. If firms can influence their costs in advance they can then influence the

Paper presented at the Eighth Annual Conference of the Irish Economic Association.

\*The second author's work on this paper forms part of the International Economic Performance Programme of the Centre for Economic Performance, London School of Economics.

optimal subsidy that they receive. As noted by Leahy (1992), firms that experience learning by doing can benefit by playing strategically against governments for higher subsidies.

This paper examines the implications for strategic trade policy of different assumptions about precommitment and the timing of moves in a dynamic oligopoly game with learning by doing. Similar issues of timing and precommitment have been much discussed in the macroeconomics literature under the title of time consistency. This paper is among the first to discuss such issues in the context of microeconomic policy.<sup>1</sup>

Learning by doing has often been cited as a justification for infant-industry protection. The traditional infant-industry argument in a competitive context relies on the existence of an additional distortion such as capital-market imperfections or positive externalities generated by the protected industry. Our concern here is whether the existence of learning by doing strengthens the case for strategic trade policy and to what extent different move orders affect the government's optimal policy.

The plan of the paper is as follows. Section II sets up the basic two-period export subsidy model with learning by doing and explains the different assumptions made about the timing of moves. Section III considers the benchmark case where governments and firms precommit in the first period to their second-period output and subsidy levels. In Section IV we examine a model in which firms and governments are unable to precommit to future actions. We regard this "time-consistent" case as both more interesting and more plausible than the benchmark precommitment equilibrium. Section V is a short conclusion.

## II THE MODEL

This model builds on Brander and Spencer (1985), extending it to two periods linked by learning by doing. Period 1 is the infant phase in which the home industry is learning while period 2 represents the mature phase in which all learning has ceased. Assume a single home firm which produces outputs  $x_1$  and  $x_2$  in periods 1 and 2 respectively and  $m$  symmetric foreign firms each of which produces  $y_1$  and  $y_2$ . The number of foreign firms is fixed and aggregate foreign outputs are represented by  $Y_1 = my_1$  and  $Y_2 = my_2$  in each period. We assume that firms play Cournot and sell a homogeneous commodity on a third market where they face a time-invariant linear demand function:

$$p_t = a - b(x_t + Y_t) \quad t = 1, 2 \quad (1)$$

1. See also Maskin and Newbery (1990).

where  $a$  and  $b$  are constants.

The home firm has constant marginal cost  $c_1 = c$  in period 1 but its second-period marginal cost falls in first-period output:

$$c_2 = c - \lambda x_1, \quad (2)$$

where  $\lambda$  is a positive constant. The foreign firms are assumed to have reached maturity before period 1 and have constant marginal cost  $c^*$  in both periods. All firms maximise the discounted sum of first- and second-period profits. For the home firm this is:

$$\pi = \pi_1 + \rho\pi_2, \quad \pi_t = (p_t - c_t + s_t)x_t, \quad t = 1, 2, \quad (3)$$

where  $\rho$  is the discount factor and  $s_t$  is the subsidy set by the home government in period  $t$ . (We assume throughout that the foreign government does not offer a subsidy.) The home government maximises the discounted sum of first- and second-period welfare. Period- $t$  welfare, as in Brander and Spencer (1985), equals profits less subsidy payments:

$$W = W_1 + \rho W_2, \quad (4)$$

$$W_t = \pi_t - s_t x_t = (p_t - c_t)x_t.$$

We assume *intratemporal* precommitment by the government throughout, in the sense that it commits to its period- $t$  subsidy before firms choose their period- $t$  outputs. However we consider two alternative assumptions about *intertemporal* precommitment giving rise to two different equilibria, which are referred to as the *precommitment equilibrium* and the *sequence equilibrium* respectively. In the precommitment equilibrium all agents take decisions for both periods at the start of period 1. That is, they precommit intertemporally. By contrast, in the sequence equilibrium no agent can precommit intertemporally. In each period the government first sets its subsidy and the firms then set their outputs.

### III THE PRECOMMITMENT EQUILIBRIUM

This is modelled as a two-stage game in which the government chooses its first- and second-period subsidies in the first stage and the firms choose their outputs in the second stage. Following standard practice the game is solved by backward induction. In the second stage the home firm chooses  $x_1$  and  $x_2$  to maximise the profit function (3) while a representative foreign firm chooses  $y_1$  and  $y_2$  to maximise  $\pi^* = \pi_1^* + \rho\pi_2^*$ . The period- $t$  first-order condition for a

typical foreign firm is  $p_t - c^* - by_t = 0$ . An explicit expression for the foreign reaction function can be obtained by using (1) in this first-order condition to obtain:

$$bY_t = mby_t = \frac{m}{m+1}(a - c^* - bx_t). \quad (5)$$

The second-period first-order condition for the home firm is:

$$p_2 - c_2 + s_2 - bx_2 = 0. \quad (6)$$

When choosing its first-period output the home firm must take into account the effect of first-period output on its second-period marginal cost. The first-period first-order condition is:

$$\frac{d\pi}{dx_1} = \frac{\partial\pi_1}{\partial x_1} + \rho \frac{\partial\pi_2}{\partial c_2} \frac{dc_2}{dx_1} = p_1 - c + s_1 - bx_1 + \rho\lambda x_2 = 0. \quad (7)$$

Compared to the home firm's second-period first-order condition and the first-order conditions of the foreign firms, there is an additional term in  $\lambda$  on the right-hand side of (7). This additional term is positive and it implies that marginal revenue is set below marginal cost in the first period. The home firm has an incentive to produce beyond the point of short-run profit-maximisation in order to move down its learning curve and further reduce its second-period costs. The home and foreign first-order conditions can now be solved simultaneously to show that the two home outputs are completely determined by the two subsidies.

We turn next to the first stage of the game. The home government moves first, choosing the first- and second-period subsidies in the knowledge of how home and foreign outputs will respond. Given that the two home outputs depend only on the subsidies they can be controlled directly. Therefore, it is possible to model the government's problem as one of choosing  $x_1$  and  $x_2$  in order to maximise welfare. The two first-order conditions are:

$$\begin{aligned} \text{(i)} \quad \frac{dW}{dx_1} &= p_1 - c - bx_1 \left\{ 1 + \frac{dY_1}{dx_1} \right\} + \rho\lambda x_2 \\ &= a - c - bx_1 \left\{ 2 - \frac{m}{m+1} \right\} - bY_1 + \rho\lambda x_2 = 0. \\ \text{(ii)} \quad \frac{dW}{dx_2} &= p_2 - c_2 - bx_2 \left\{ 1 + \frac{dY_2}{dx_2} \right\} \\ &= a - c - bx_2 \left\{ 2 - \frac{m}{m+1} \right\} - bY_2 + \lambda x_1 = 0. \end{aligned} \quad (8)$$

Making use of the home firm's first-order conditions yields the following expression for the optimal subsidies:

$$s_t^o = \frac{m}{m+1} bx_t \quad t = 1, 2. \quad (9)$$

This is identical in form to the optimal subsidy in the static Brander-Spencer game. Learning by doing in the precommitment equilibrium alters the *value* of the optimal subsidy (since output  $x_t$  is higher). However, it does not change the basis for intervention, which continues to be one of reducing the home firm's costs in order to shift profits towards it and away from the foreign firm. To obtain explicit expressions for outputs substitute (9) into the firm's first-order conditions and solve to obtain:

$$bx_1 = \frac{2+(m+1)\rho\gamma}{\Delta} \phi, \quad bx_2 = \frac{2+(m+1)\gamma}{\Delta} \phi. \quad (10)$$

Here  $\gamma$  equals  $\lambda/b$ , the rate of learning normalised by the size of the market;  $\phi$  equals  $a-(m+1)c+mc^*$ , a measure of the cost-competitiveness of the home firm; and  $\Delta$  equals  $4-(m+1)^2\rho\gamma^2$ , which must be positive for stability. From (10), outputs and hence the optimal subsidies are clearly increasing in the rate of learning. Finally, the envelope theorem can be used to demonstrate that welfare is also increasing in learning:

$$\frac{dW}{d\lambda} = \frac{\partial W}{\partial \lambda} = \rho x_1 x_2 > 0. \quad (11)$$

Summarising these results:

**Proposition 1:** With linear demands and linear learning and when all firms and the government can precommit to future actions, the optimal first- and second-period subsidies, first- and second-period outputs and welfare are all increasing in the rate of learning.

#### IV SEQUENCE EQUILIBRIUM

In this section we consider the four-stage game in which no agent can precommit intertemporally. The second period is just a standard Brander-Spencer game with many foreign firms in which the home government commits to its subsidy before the firms move. The second-period first-order conditions for the firms are the same as in the previous section. Thus second-period output of the foreign firms is given by (5) for the case of  $t=2$ . Use (1) and (5) in the home firm's second-period first-order condition to obtain:

$$x_2 = \frac{\phi + (m+1)(\lambda x_1 + s_2)}{b(m+2)}. \quad (12)$$

Given  $x_1$ , the home government can control  $x_2$  by choice of  $s_2$ , and its second-period first-order condition takes the form of (8(ii)). The optimal second-period subsidy is thus given by (9) for the case of  $t=2$ . The use of (12) in (9) yields:

$$s_2^0 = \frac{m}{m+1} \frac{\phi}{2} + \frac{m}{2} \lambda x_1. \quad (13)$$

This can be viewed as the government's reaction function. An increase in  $x_1$  leads to an increase in  $s_2$ , since it leads to lower costs in period 2 and thus increases the home government's ability to raise welfare by subsidising the firm in that period.

We turn now to the first period. In stage 2 of the game the firms choose their first-period outputs. The foreign first-order conditions take the same form as in the previous section, and foreign output, which depends only on home output, is represented as before by (5). The home firm's first-order condition is now:

$$\frac{d\pi}{dx_1} = \frac{\partial \pi_1}{\partial x_1} + \rho \frac{d\pi_2}{dx_1} = 0, \quad (14)$$

where the total effect of current output on future profits includes the direct effect of cost changes as well as the indirect effects working through both future output and the future subsidy. Following some manipulations this yields:

$$\frac{d\pi}{dx_1} = p_1 - c + s_1 - bx_1 + \rho \lambda x_2 \left\{ 1 + b \frac{dY_2}{dx_2} \frac{dx_2}{dc_2} \right\} + \rho x_2 \frac{ds_2}{dx_1} = 0. \quad (15)$$

Comparing this to the corresponding formula in the precommitment equilibrium (7), it is clear that (15) has two additional terms. The second term in chain brackets is positive and it shows that the home firm has a greater incentive to produce beyond the point of short-run profit maximisation in order to induce the foreign firms to produce less in period 2 and thus raise the second-period profits of the home firm. This strategic incentive was first noted by Fudenberg and Tirole (1983). The final term on the right-hand side which is also positive represents the home firm's strategic incentive to produce more in order to raise the second-period subsidy. The use of the

foreign firms' second-period reaction function (5) and the government reaction function (13) allows the following considerable simplification of (15):

$$a - c + s_1 - 2bx_1 - bY_1 + \rho\lambda(m+1)x_2 = 0. \quad (16)$$

It is now possible to eliminate  $Y_1$  and  $x_2$  using (5), (12) and (13), and so demonstrate that  $x_1$  depends only on  $s_1$  and parameters. In the first stage the government chooses  $s_1$  to maximise welfare. Since  $x_1$  depends on  $s_1$  alone the problem can be modelled as one in which the government chooses home output before foreign firms move. The relevant first-order condition for the government is (8(i)). The use of (16) in (8(i)) gives:

$$s_1^0 = \frac{m}{m+1} bx_1 - m\rho\lambda x_2. \quad (17)$$

To obtain explicit formulae for home output in the first and second periods, substitute  $s_1^0$  and  $s_2^0$  into the home firm's first- and second-period first-order conditions and combine with (5). The resulting expressions are identical to those in (10). It is then straightforward to derive explicit formulae for the optimal first- and second-period subsidies. The expression for  $s_2^0$  is identical to that in the precommitment equilibrium and the formula for  $s_1^0$  is:

$$s_1^0 = \frac{m}{m+1} \left\{ \frac{2 - (m+1)\rho\gamma[1 + (m+1)\gamma]}{\Delta} \right\} \phi. \quad (18)$$

So, in the sequence equilibrium, outputs in both periods and the second-period subsidy are the same as in the precommitment equilibrium. However, a comparison of (9) and (17) reveals that the first-period subsidy under sequence equilibrium is lower than that under precommitment. Moreover, from (18), the optimal first-period subsidy in the sequence equilibrium is *decreasing* in the rate of learning. Welfare is the same in the sequence equilibrium as it is in the precommitment equilibrium since it depends only on outputs. Summarising these results:

**Proposition 2:** With linear demands and linear learning and when no agents can precommit to future actions, the optimal second-period subsidies, first- and second-period outputs and welfare are all increasing in the rate of learning. However, the optimal first-period subsidy is decreasing in the rate of learning.

The first-period subsidy may actually be negative in the sequence equilibrium.

The results for optimal subsidies in period 1 are illustrated in Figures 1 and 2, where the horizontal axis measures the value of  $\gamma$ , the normalised rate of learning. (Units of measurement for the subsidies are chosen such

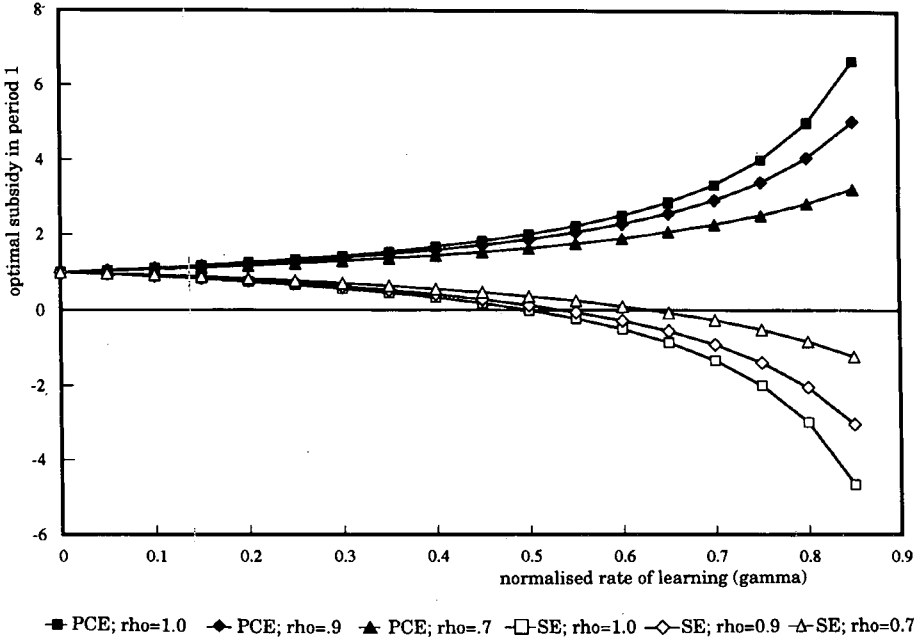


Figure 1: Sensitivity of Optimal Subsidies to Discount Rate.  
 (One foreign firm; optimal one-period subsidy normalised to equal one)

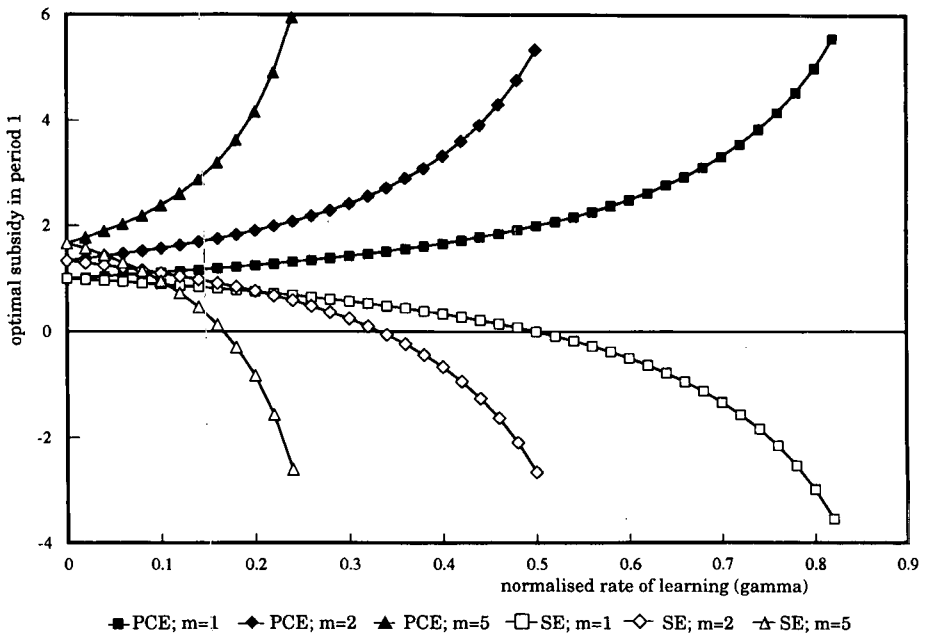


Figure 2: Sensitivity of Optimal Subsidies to Number of Foreign Firms.  
 (No discounting; optimal one-period subsidy with one foreign firm normalised to equal one)



that the static Brander-Spencer value equals unity.) Figure 1 considers the case of only one foreign firm ( $m=1$ ). The optimal subsidy is increasing in  $\gamma$  if precommitment is possible and decreasing in  $\gamma$  otherwise, with both becoming less sensitive to  $\gamma$  as the future is discounted more heavily. Figure 2 fixes the discount factor at unity and considers the effects of increasing the number of foreign firms. The same qualitative conclusions hold, with both subsidies becoming more sensitive to  $\gamma$  as the number of foreign firms increases.

Note that the first-period subsidy is lower than the second-period subsidy in the sequence equilibrium. This is somewhat surprising given that the home firm is learning in the first period but not in the second. It therefore runs counter to intuition derived from a traditional infant-industry argument. Comparing Propositions 1 and 2 it is clear that the extent of precommitment is crucial for the results. If the government cannot precommit, the infant-industry argument is reversed.

## V CONCLUDING REMARKS

This paper has examined the implications for a strategic export subsidy of different assumptions about precommitment. In a linear Cournot model we found that the optimal first-period subsidy is lower if the firms and the government cannot precommit to future actions than if they can and that the optimal subsidy is increasing in learning with precommitment but decreasing in the sequential case. An important policy lesson of our results is that, compared to a static model or one in which agents can precommit to their future actions, the inability of government to precommit may actually reverse the sign of optimal policy. This is because a private sector agent has an incentive to take actions in the present which will increase the subsidy it receives in the future. To counteract this socially wasteful strategic behaviour, the government has an incentive to tax rather than subsidise the agent in the present.

The results of this paper are restricted to the case of linear demands and linear learning. In a companion paper, Leahy and Neary (1994), we show that the results continue to hold under more general assumptions. In future work we hope to apply the insights from the learning by doing case to examine the role of precommitment in affecting optimal policy choice in the presence of other dynamic phenomena, including capacity choice, investment in research and development and natural resource exploitation. Just as the debate on time consistency has caused a major rethink of policy issues in macroeconomics, so the study of precommitment promises to throw new light on the costs and benefits of microeconomic policy in dynamic environments.

## REFERENCES

- BRANDER, J.A., and B.J. SPENCER, 1985. "Export Subsidies and International Market Share Rivalry," *Journal of International Economics*, Vol. 18, pp. 83-100.
- DE MEZA, D., 1986. "Export Subsidies and High Productivity: Cause or Effect?," *Canadian Journal of Economics*, Vol. 19, pp. 347-350.
- FUDENBERG, D., and J. TIROLE, 1983. "Learning by Doing and Market Performance," *Bell Journal of Economics*, Vol. 14, pp. 522-530.
- LEAHY, D., 1992. "Learning by Doing in International Subsidy Games," mimeo., University College Dublin.
- LEAHY, D., and J.P. NEARY, 1994. "Learning by Doing, Precommitment and Infant-Industry Protection," Centre for Economic Research, University College Dublin, Working Paper No. WP94/5.
- MASKIN, E., and D. NEWBERY, 1990. "Disadvantageous Oil Tariffs and Dynamic Consistency," *American Economic Review*, Vol. 80, pp. 143-156.
- NEARY, J.P., 1994. "Cost Asymmetries in International Subsidy Games: Should Governments Help Winners or Losers?," *Journal of International Economics* (forthcoming).