The Macroeconomics of Dr. Strangelove

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This paper examines the weapons-accumulation decisions of two adversarial countries in the context of a deterrence / conflict initiation game embedded in an overlapping-generations model. The demographic structure permits analysis of both within- and between-country intergenerational externalities caused by past weapons-accumulation decisions, as well as of intragenerational externalities from the adversary's current weapons accumulation. Zero accumulation is a possible equilibrium with both noncooperative and cooperative behavior. Countries may also accumulate weapons to the point where conflict initiation never occurs. Pareto-improving policies are generally available, but international cooperation need not be Pareto-improving. (JEL D62, D74, E21)

The Cold War has ended, and President George Bush has declared the beginning of a "new world order." This new world order has so far been characterized by the intensification of regional disputes, civil strife, and civil war; its implications for arms proliferation and arms control are still unclear. This paper considers both superpower conflict and the transition to international cooperation in the context of a model that emphasizes intergenerational linkages. Specifically, we explore the weapons-accumulation decisions of two adversarial countries in the context of a deterrence/conflict initiation game embedded in an overlapping-generations model. The demographic structure of the overlapping-generations model allows the analysis of both between- and within-country intergenerational external effects caused by past weapons-accumulation decisions, while the two-country setting permits consideration of international external effects caused by the adversary country's current weapons-accumulation decision. We analyze the noncooperative equilibrium and then consider the effects of a regime change characterized by international cooperation, but where the intergenerational externalities remain.

We emphasize intergenerational externalities in our model because they are intrinsically hard to internalize: those who bear their cost or receive their benefit are without representatives in the present generation. Thus their internalization requires intervention by long-lived government institutions. We also analyze the problem of such institutions.

The analysis draws on a number of diverse economic literatures. The model utilizes the overlapping-generations framework of Maurice Allais (1947) and Paul A. Samuelson (1958), since this demographic structure lends itself to analysis of situations where agents' actions have consequences beyond their own lifetimes. Positive intergenerational external effects play a role in models of endogenous growth (see e.g., Paul M. Romer, 1986; Nancy L. Stokey, 1988), while intergenerational diseconomies are the focus of Todd Sandler's (1982) work on intergenerational club goods. In this paper, as in work on environmental externalities (John et al., 1992), both positive and negative intergenerational effects interact. Finally, the model draws upon the game-theoretic arms-control literature (see e.g., Michael D. Intriligator and Dagobert L.

Brito, 1984; Steven J. Brams and D. Marc Kilgour, 1985; Frederick van der Ploeg and Aart J. de Zeeuw, 1990).

The main findings of the model are that equilibria in which neither country accumulates weapons are possible outcomes of both noncooperative and cooperative versions of the game and that countries may choose to accumulate weapons to the point where conflict initiation is so dangerous that it never occurs. The dynamic adjustment of the model to a steady state can be interpreted as an arms race: successive generations engage in individually rational but socially suboptimal arms accumulation. For second-best reasons, neither within-country intergenerational cooperation nor international cooperation necessarily leads to smaller weapons stocks or to Pareto improvements.

I. The Environment

A. *Preliminaries*

Consider an infinite-horizon world with two identical countries (called A and B). In each country, a new generation (called generation t) of agents is born at each date $t = 1, 2, \ldots$. There is no population growth, and each generation is normalized to be of size 1. Each member of generation t lives for one period with certainty, and for two periods with probability $\phi(\cdot)$. Further, each has preferences defined over consumption when young, $c^y(t)$, and when old, $c^o(t+1)$. These preferences are represented by a von Neumann-Morgenstern utility function:

$$(1) \quad U(c^y(t)) + \phi(\cdot)V(c^o(t+1))$$

where $\phi(\cdot)$ is derived below. Assume that $U(\cdot)$ and $V(\cdot)$ are both increasing, twice continuously differentiable, and strictly concave. Also assume that

$$\lim_{c^y \to 0} U'(\cdot) = \infty$$

$$\lim_{c^o \to 0} V'(\cdot) = \infty$$

$$V(0) \geq 0.$$

Each member of each generation $t \geq 1$ in each country is endowed with $e > 0$ units of the economy's single good when young and nothing when old. At date $t = 1$ each country collectively possesses a stock of $W(1) \geq 0$ weapons.

Any portion of the date-t endowment can be consumed at date t, yielding utility at t, or used to provide for future utility. To accomplish the latter, the consumption good can either be saved or used to add to the home economy's weapon stock. Each young agent has access to a storage technology, which is the sole vehicle for saving. Storage of $s(t)$ units of the good at date t yields $Rs(t)$ units of the good at date $t+1$, where $R > 0$ is the gross technological rate of return. Alternatively, units of the good can be converted into an equivalent number of units of weapons, $w(t)$. Accumulation of weapons at t adds to the stock of weapons that will be carried into date $t+1$, $W(t+1)$. The stock of weapons evolves according to

$$(2) \quad W(t+1) = (1-\delta)W(t) + w(t)$$

given the depreciation rate $\delta \in (0,1]$ and $W(1)$. Weapons accumulation affects utility by altering the probability of survival, $\phi(\cdot)$, as discussed in the next subsection.

Since a primary focus of this paper is on external effects across generations, we abstract from the well-understood free-rider problems within a country and within a generation. We assume that the agents alive in each country at the beginning of each date elect a government for a one-period term. This government behaves myopically, carrying out policies made solely for the welfare of agents alive during its term in office. The government has the power to levy lump-sum taxes and transfers. Specifically, it levies lump-sum taxes on the young to achieve the desired stock of weapons. That is, an agent's choice of weapons can be interpreted as arising from the collective provision of national defense, a public good. This alloca-

tion could be achieved as a Lindahl equilibrium.[1]

B. *The Game Between Countries*

Each generation of young agents plays a one-shot game against its counterparts in the other country. At the start of the period, young agents choose the amount of their endowment that they wish to convert to weapons, taking as given the equivalent choice in the other country and the inherited stocks of weapons in both countries. From equation (2), these decisions give rise to the current weapons stocks (W^A, W^B) in the two countries. (For ease of presentation, we suppress the time index in this subsection.)

At the end of the period there is a "potential for conflict." Specifically, there is a realization of a random variable that determines the probability that one side or the other will win, should either side decide to initiate a conflict.[3] A conflict has three characteristics. First, it has a winner and a loser: that is, there is a utility gain from winning and loss from losing. Second, we assume for simplicity that a conflict does not deplete either country's stock of weapons. Third, a conflict can escalate into a full-scale catastrophic war.[4]

Each side decides whether or not to initiate conflict. The gain from winning and loss from losing are proportional to utility when old and are the same no matter which side initiates. If the conflict does escalate, then there is mutual annihilation, giving zero utility in old age. The risk of this outcome generates the cost of initiating a conflict.

If either side initiates conflict, the probability of annihilation is $q = q(W^A, W^B)$, with $q_1 > 0$, $q_2 > 0$, $q_{11} < 0$, $q_1(x,x) = q_2(x,x) \forall x$ and $q(0,0) = 0$. The probability that country A wins a conflict is

$$p = \eta + f(W^A, W^B) \qquad \eta \sim U[0,1]$$

where η is a random variable determining the potential for conflict, and $f_1 > 0$, $f_2 < 0$, $f_{11} < 0$, $f(x,y) \equiv -f(y,x) \forall x,y$, and $f(x,x) = 0 \forall x$.[5] The probability that country B wins is $1 - p$. The $q(\cdot)$ function captures the idea that increased weapons stocks in either country make any conflict potentially more dangerous. The $f(\cdot)$ function captures the idea that a country's likelihood of winning a conflict is higher, the more weapons it possesses and the fewer weapons possessed by its adversary. Accumulation that increases one country's probability of winning a conflict decreases the probability that the other wins, and each country's probability of winning equals $\frac{1}{2}$ when the countries possess equal weapons stocks. We assume that $f(\cdot)$ and $q(\cdot)$ are continuous and twice-differentiable. Provided that annihilation does not occur, the agents in the victorious country receive utility when they are old equal to $V(\cdot)(1 + \alpha)$, and the agents on the losing side have utility when they are old equal to $V(\cdot)(1 - \alpha)$, where α is a positive parameter.[6]

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[1] Note that while young agents would like to levy lump-sum taxes on the old, such a policy would not be achievable as a Lindahl equilibrium. We are grateful to a referee for clarifying the role of the government in this model.

[2] The term "conflict" should be interpreted liberally; we think of it as any situation in which the two countries engage in contentious behavior, ranging perhaps from a verbal dispute at the United Nations or an expulsion of diplomats to a confrontational stand-off like the Cuban missile crisis or the Berlin blockade or even a limited war like Vietnam or Afghanistan.

[3] We assume that this probability is independent of which side initiates, that is, there is no first-mover advantage. In the problem as set out, the sides will never choose to initiate simultaneously.

[4] These last two assumptions together suggest that if a conflict escalates, weapons stocks and all else are annihilated, while in a limited conflict the weapons destroyed are those that would otherwise have depreciated, so the stock bequeathed to the next generation is not affected.

[5] Assume that if p as defined exceeds 1, it takes the value 1, and if p as defined is less than 0, it takes the value 0. Since we restrict attention to symmetric equilibria, these restrictions are never relevant for our analysis.

[6] The augmentation (diminution) of utility as a result of winning (losing) a conflict is analogous to the payment of tribute by the losing side to the winning side. While Michelle R. Garfinkel (1990) makes the transfer of goods explicit, we model the tribute as a psychological gain (loss).

Now consider country A's decision of whether or not to initiate conflict. If B initiates, A's decision is irrelevant. Therefore we need only consider the case in which B does not initiate. Country A's expected utility when old if it initiates is

$$(1-q)p(1+\alpha)V(\cdot)$$
$$+(1-q)(1-p)(1-\alpha)V(\cdot)$$

and it is $V(\cdot)$ if country A does not initiate. Thus, country A will choose to initiate if and only if

$$p > [1+z(q)]/2$$

where $z(q) = q/[\alpha(1-q)]$. It is indifferent between initiating and not initiating if

$$\eta = -f(\cdot) + [1+z(q)]/2.$$

Similarly, country B is indifferent between initiating and not initiating if

$$1-p = [1+z(q)]/2$$

$$\Rightarrow \eta = 1 - f(\cdot) - [1+z(q)]/2.$$

The probability that neither side initiates is then the probability that η lies between these two values, which simply equals $z(q)$. Since $z(q)$ is increasing in q and $q(\cdot)$ is increasing in both its arguments, it follows that greater weapons accumulation by either side makes conflict less likely to occur, but more likely to be catastrophic if it does occur.[7]

Now consider the first-period weapons choice. Country A chooses weapons (w) and saving (s) to maximize

$$U(c^y) + \phi(W^A, W^B)V(c^o)$$

(subject to the budget constraints discussed

below), where

$$\phi(W^A, W^B)$$
$$= z(q) + [1-z(q)](1-q)$$
$$\times \{E(p)(1+\alpha) + [1-E(p)](1-\alpha)\}$$
$$= z(q) + (1-\rho q)[1+2\alpha f(\cdot)]$$
$$\rho = (1+\alpha)/\alpha.$$

(Country B solves an analogous problem.) The first term $[z(q)]$ is the probability that neither country initiates. The second term is the probability that someone initiates $(1-z)$, multiplied by the probability that the world nevertheless survives $(1-q)$, multiplied by the expected gain from conflict, which in turn depends upon the expected probability of victory, $E(p) = \frac{1}{2} + f(\cdot)$. In a symmetric equilibrium, $f(\cdot) = 0$, so the expected gain from conflict is zero; in this case $\phi(\cdot)$ can be interpreted as the survival probability. Note that, since $f(\cdot)$ and $q(\cdot)$ are continuous and differentiable, so too is $\phi(\cdot)$. The behavior of the economy depends upon the properties of $\phi(\cdot)$, which are discussed in detail in Section III.

C. The Agent's Optimization Problem

The representative agent in country A (and symmetrically for the representative agent in country B) takes as given the return on storage, R, the stocks of weapons at the beginning of period t, $W^A(t-1)$ and $W^B(t-1)$, and the weapons choice of country B, $w^B(t)$. The agent chooses $c^{Ay}(t)$, $c^{Ao}(t+1)$, $w^A(t)$, and $s^A(t)$ to maximize

$$(3) \quad U(c^{Ay}(t))$$
$$+ \phi(W^A(t), W^B(t))V(c^{Ao}(t+1))$$

subject to

$$(4) \quad c^{Ay}(t) + w^A(t) + s^A(t) \le e$$

$$(5) \quad c^{Ao}(t+1) \le s^A(t)R$$

$$(6) \quad W^A(t) = (1-\delta)W^A(t-1) + w^A(t)$$

and $c^{Ay}(t), c^{Ao}(t+1), w^A(t), s^A(t) \ge 0$.

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[7]For an interior solution to the problem set out here, $z(q)$ must be less than 1 [$\Rightarrow q < \alpha/(1+\alpha)$]. For the present, we assume that this condition holds and discuss its implications later in the paper.

Utility maximization yields the following first-order conditions for an interior optimum:

(7) $-U'(\cdot) + \phi(\cdot)V'(\cdot)R = 0$

(8) $-U'(\cdot) + \phi_1(\cdot)V(\cdot) = 0.$

The restrictions on the utility function ensure that (7) is always satisfied. A necessary condition for (8) is that ϕ_1 be positive; that is, an increase in w^A benefits country A. The assumptions of the model so far are not sufficient to ensure that ϕ_1 is always positive. If ϕ_1 is negative, agents will not invest in weapons, and as shown in Section III, an equilibrium with zero investment in weapons is possible. Henceforth, we assume that ϕ_1 is positive unless otherwise noted.[8]

The individual equates the marginal utility of consumption in youth with the expected marginal utility of consumption in old age [equation (7)]: ϕR can be thought of as an implicit interest rate. Equations (7) and (8) together reveal that the individual also equates the return on saving to the return on weapons accumulation:

$$\phi_1 V = \phi R V'.$$

The second-order conditions from individual maximization require that

$$\tilde{\Delta} \equiv \det \begin{bmatrix} U'' + \phi V''R^2 & U'' + \phi_1 V'R \\ U'' + \phi_1 V'R & U'' + \phi_{11}V \end{bmatrix} > 0$$

$$\Rightarrow U''(\phi V''R^2 - \phi_1 V'R)$$

$$+ \phi_{11}V(U'' + \phi V''R^2)$$

$$- \phi_1 V'RU' \left[\frac{U''}{U'} + \frac{\phi_1}{\phi} \right] > 0.$$

Sufficient for this is that $U''/U' + \phi_1/\phi < 0$ and $\phi_{11} < 0$.

II. The Steady State

A symmetric steady-state Nash equilibrium is given by the quintuple for country A, $\{c^y, c^o, w, W, s\}$, and the identical quintuple for country B, $\{c^y, c^o, w, W, s\}$, such that the following statements hold.

(i) Agents maximize (3) subject to (4)–(6), given R and W.
(ii) The goods market clears:

(9) $[c^y + w + s] + c^o = e + sR.$

(iii) The stock of weapons is constant:

(10) $\delta W = w.$

The equilibrium can be characterized by the two first-order conditions, (7) and (8), and constant weapons stocks, (10). Total differentiation of the first-order conditions evaluated at an interior steady state yields the following system:

(11) $\begin{bmatrix} u+\cdots & u-a \\ u-\zeta & u-b \end{bmatrix} \begin{bmatrix} ds \\ dw \end{bmatrix}$

$$\begin{bmatrix} (1-sr)/R & u & aw/\delta & \phi_\alpha/\phi \\ \zeta s/R & u & bw/\delta & \phi_{1\alpha}/\phi_1 \end{bmatrix} \begin{bmatrix} dR \\ de \\ d\delta \\ d\alpha \end{bmatrix}$$

where

$u \equiv -U''/U' > 0$

$\iota \equiv -V''R/V > 0$

$\zeta \equiv \phi_1/\phi > 0$

$a \equiv (\phi_1 + \phi_2)/\delta\phi \gtrless 0$

$b \equiv (\phi_{11} + \phi_{12})/\delta\phi_1 \gtrless 0$

$\phi_\alpha \equiv \partial\phi/\partial\alpha$

$\phi_{1\alpha} \equiv \partial\phi_1/\partial\alpha.$

For stability the determinant of the left-

hand-side matrix, Δ, must be positive. This implies that

$$\Delta \equiv u(v + \zeta) + a(u - \zeta) - b(u + v) > 0.$$

A sufficient condition for $\Delta > 0$ is $a(u - \zeta) > 0$ and $b < 0$.

III. Equilibria

A. Properties of $\phi(\cdot)$

We begin this section by discussing the properties of $\phi(\cdot)$, since these are central to a characterization of the model's equilibria. Recall that $\phi(\cdot)$ is the weight attached to old agents' utility and depends upon both the probability of survival to old age and the gain or loss from any conflict that might occur:

$$\phi(W^A, W^B) = z(q) + (1 - \rho q)(1 + 2\alpha f).$$

Recall also that $z(q) = q/[\alpha(1 - q)]$ so that $z' > 0$, $z'' > 0$. We restrict our attention to symmetric equilibria, in which $q_1 = q_2$, $f_1 = -f_2$, and $f = 0$. Then,

$$(12) \quad \phi_1 = (z' - \rho)q_1 + 2\alpha(1 - \rho q)f_1$$
$$- 2\alpha \rho f q_1$$
$$= (z' - \rho)q_1 + 2\alpha(1 - \rho q)f_1$$

$$(13) \quad \phi_2 = (z' - \rho)q_2 + 2\alpha(1 - \rho q)f_2$$
$$- 2\alpha \rho f q_2$$
$$= (z' - \rho)q_2 + 2\alpha(1 - \rho q)f_2$$

$$(14) \quad \frac{d\phi(\cdot)}{dw} = \phi_1 + \phi_2 = 2(z' - \rho)q_1.$$

The first term in equation (12) captures two effects. A higher weapons stock in country A increases the probability of catastrophe if either side initiates $(-\rho q_1)$ but also therefore increases the risk associated with initiating, making initiation less likely to occur $(z'q_1)$. The second term in equation (12) captures the fact that a higher weapons stock benefits country A in the
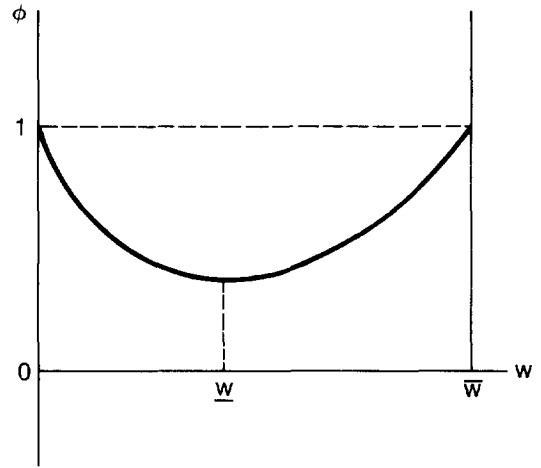

FIGURE 1. THE PROBABILITY OF SURVIVAL FOR SYMMETRIC WEAPONS STOCKS

event of conflict; thus, it is positive. Considering equation (13), note that the first term is identical to the first term of equation (12) and reflects the same intuition. A higher weapons stock in country B is costly to country A, however; hence the second term in equation (13) is negative (since $f_2 < 0$).

The behavior of $\phi(\cdot)$, evaluated at a steady state (with $w = \delta W$), is illustrated in Figure 1 under the assumption of symmetry across countries: $w^A = w^B = w$. At $w = 0$, $\phi(0,0) = 1$ because $q(0,0) = 0$, $z(0) = 0$, and $f(0,0) = 0$. That is, when neither country has any weapons, conflict is certain but costless because annihilation is impossible. Thus, the probability of survival is unity. As w increases, the danger inherent in any conflict increases, but the likelihood of conflict falls. The first effect dominates for low weapons stocks; the second effect dominates at high weapons stocks. The probability of survival is also unity if agents never initiate conflict because the associated risk of annihilation is so great. Specifically, let $\overline{w}$ be defined by $q(\overline{w}/\delta, \overline{w}/\delta) = 1/\rho$. Then $z(q) = 1$ and $\phi(\overline{w}/\delta, \overline{w}/\delta) = 1$. This possibility resembles the doctrine of "mutually assured destruction" under which deterrence works perfectly because initiation of a conflict carries too great a risk of annihilation.
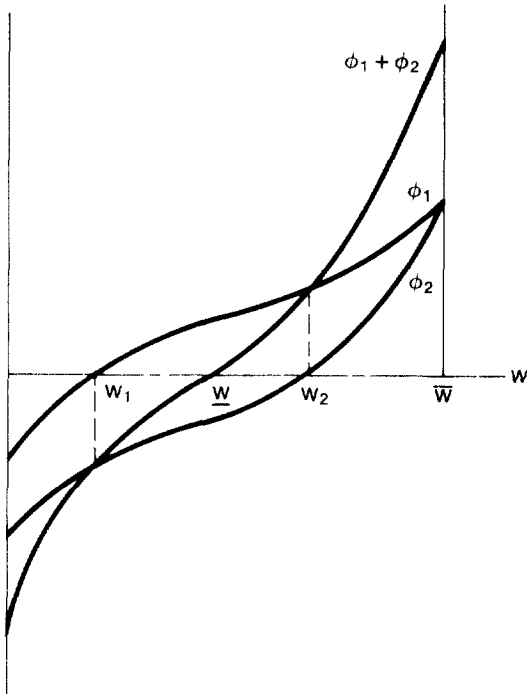
FIGURE 2. DERIVATIVES OF THE SURVIVAL
PROBABILITY FUNCTION.

Figure 2 shows the partial derivatives of $\phi(\cdot)$ [for arbitrary assumptions on the second derivatives of $\phi(\cdot)$]. It is easily confirmed that at $w = 0$, $z'(\cdot) - \rho = -1$, implying that $\phi_1(\cdot) > 0$, $\phi_2(\cdot) < 0$, and $\phi_1(\cdot) + \phi_2(\cdot) < 0$. Figure 2 is drawn such that $\phi_1(\cdot) < 0$ at $w = 0$. At $w = \overline{w}$, $z'(\cdot) - \rho = 1 + \alpha > 0$, implying that $\phi_1(\cdot)$ and $\phi_2(\cdot)$ are positive and equal. From our assumptions on $f(\cdot)$ and $q(\cdot)$, $\phi_1(\cdot)$ lies strictly above $\phi_2(\cdot)$ for $w < \overline{w}$. We thus divide the range of w into four regions. In the region $0 < w < w_1$, both ϕ_1 and ϕ_2 are negative. (Since ϕ_1 may be positive for all w, this region may be empty.) For $w_1 < w < \underline{w}$, ϕ_1 is positive, ϕ_2 is negative, and their sum $(\phi_1 + \phi_2)$ is negative. The function $\phi(\cdot)$ attains its minimum at $\underline{w}$, where $\phi_1 + \phi_2 = 0$. In the region $\underline{w} < w < w_2$, ϕ_1 is positive, ϕ_2 is negative, and their sum is positive. Finally, for $w_2 < w < \overline{w}$, both ϕ_1 and ϕ_2 are positive.

Figures 1 and 2 are drawn such that $\phi(\cdot)$ is single-troughed and $\phi_1(\cdot)$ and $\phi_2(\cdot)$ are monotonically increasing in w; this is not necessarily implied by our assumptions on $f(\cdot)$ and $q(\cdot)$, as can be seen by examining equations (15)–(18):

$$(15) \quad \phi_{11} = (z' - \rho)q_{11} + 2\alpha(1 - \rho q)f_{11}$$
$$+ z''(q_1)^2 - 4\alpha\rho f_1 q_1$$

$$(16) \quad \phi_{12} = (z' - \rho)q_{12} + 2\alpha(1 - \rho q)f_{12}$$
$$+ z''q_1q_2 - 2\alpha\rho f_2 q_1 - 2\alpha\rho f_1 q_2$$
$$= (z' - \rho)q_{12} + 2\alpha(1 - \rho q)f_{12}$$
$$+ z''q_1q_2$$

$$(17) \quad \phi_{11} + \phi_{12} = 2\big[(z' - \rho)q_{11} + \alpha(1 - \rho q)$$
$$\times (f_{11} + f_{12}) + z''(q_1)^2 - 2\alpha\rho f_1 q_1\big]$$

$$(18) \quad \phi_{22} = (z' - \rho)q_{22} + 2\alpha(1 - \rho q)f_{22}$$
$$+ z''(q_2)^2 - 4\alpha\rho f_2 q_2.$$

From equations (15)–(18), ϕ_{11}, ϕ_{22}, and ϕ_{12} can all be of either sign, as can the sum $(\phi_{11} + \phi_{12})$.

B. Steady-State Equilibria

Our primary concern is with the equilibrium weapons stock and associated survival probability. We thus let $s(w)$ be implicitly defined by the steady-state version of the first-order condition (7):

$$-U'(c - s - w)$$
$$+ \phi(w/\delta, w/\delta)V'(sR)R = 0.$$

Increases in w increase the marginal utility of first-period consumption; this effect tends to increase saving. Below $\underline{w}$, increases in w decrease $\phi(\cdot)$, which tends to discourage saving. the opposite is true above $\underline{w}$.[9] Hence $s'(w)$ is negative below $\underline{w}$ and is of ambiguous sign above $\underline{w}$.[10]

[9] The effects of changes in the survival probability on saving in our model are the theoretical counterparts of Joel B. Slemrod's (1986, 1990) finding that an increased threat of nuclear war is associated with decreased saving. We consider the relationship between our results and Slemrod's further in Section IV.

[10] Specifically, $s'(w) = (a - u)/(a + t) \gtrless 0$.

The two first-order conditions can be combined to yield

$$(19) \quad \phi_1(\cdot)/\phi(\cdot) = V'(\cdot)R/V(\cdot).$$

Steady-state equilibria for this economy can be represented by graphing the two sides of equation (19) as functions of the steady-state weapons stock, w. The right-hand side of (19) is everywhere positive, and it slopes upward if and only if $s'(w) < 0$.

Figures 3 and 4 illustrate the cases of a single interior equilibrium and multiple interior equilibria, respectively. For purposes of exposition, we make the simplifying assumptions that $\phi_1(\cdot)/\phi$ and $\phi_2(\cdot)/\phi$ are increasing in w, and for diagramatic simplicity, we draw them as linear. While a less well-behaved function might divide the range of w into more than four regions, each region would still have the characteristics of one of the four described above. The equilibrium in Figure 3 is stable, as are the two outer equilibria in Figure 4.[11]

Since the right-hand side of (19) is everywhere positive, an interior equilibrium is not possible in the region $0 < w < w_1$. A corner solution with zero weapons accumulation is possible, however, if $V'R/V$ lies above ϕ_1/ϕ at $w = 0$ (Fig. 5), so that the nonnegativity constraint on weapons accumulation is binding. Note that if $\phi_1(0,0) < 0$, then this corner solution must be an equilibrium.

FIGURE 3. A UNIQUE STEADY-STATE EQUILIBRIUM

FIGURE 4. MULTIPLE STEADY-STATE EQUILIBRIA

[11]Two distinct types of multiplicity may arise. First, there may be multiple steady states, but the game between countries may still have a unique equilibrium at any given date. Such multiplicity can be attributed to strategic complementarities between past generations' accumulation decisions and the present generations' decisions $(\phi_{11} + \phi_{12} > 0)$ the more weapons that were accumulated in the past, the greater the return to accumulation in the present. The final steady state then depends upon initial weapons stocks. Alternatively, there may be multiple equilibria in a given time period, with the associated possibility of coordination failure. For example, each country might devote a lot of its endowment to weapons simply because the other is doing so, even if there is a better Nash equilibrium with less investment in weapons. This possibility arises if there are strategic complementarities within the period $(\phi_{12} > 0)$: the more weapons accumulated by one country, the higher the return to weapons accumulation in the other country. See Russell W. Cooper and John (1988) for more discussion of strategic complementarities and coordination failures
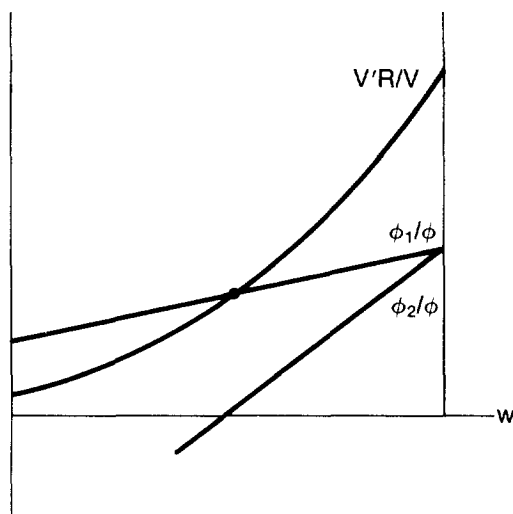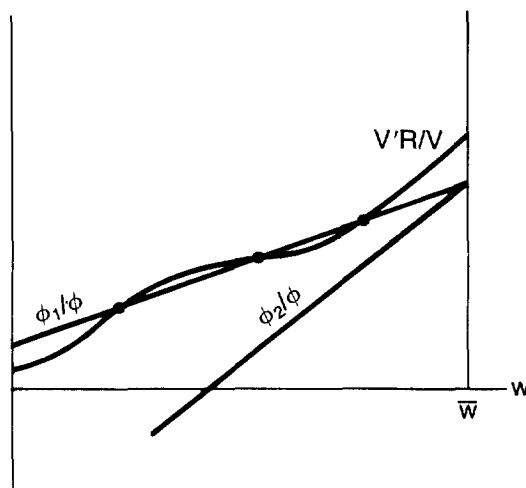
Another possible corner solution has agents accumulating weapons to such an extent that initiating conflict is never beneficial and so will never occur because the risk of annihilation is too great; that is, weapons are accumulated up to $\bar{w}$. In this no-conflict equilibrium, ϕ_1/ϕ lies above $V'R/V$ at $w = \bar{w}$, as is shown in Figure 6.
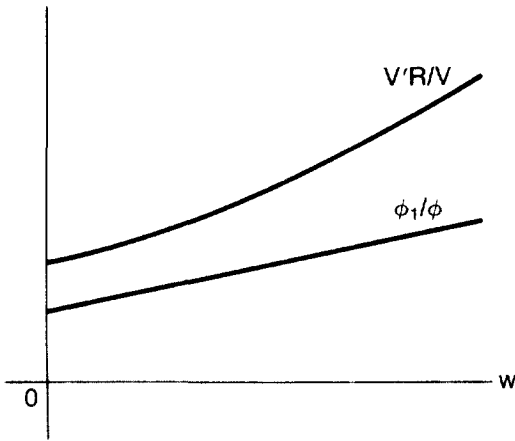
FIGURE 5. AN EQUILIBRIUM WITHOUT WEAPONS
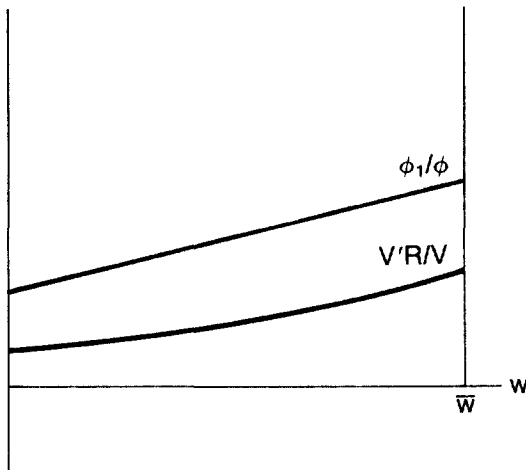ACCUMULATION

FIGURE 6. A NO-CONFLICT EQUILIBRIUM

Arms races, such as have been observed throughout history, can be interpreted in our model as a dynamic adjustment to a steady state with positive accumulation. Starting from some weapons stock below equilibrium, successive generations in each country engage in weapons investment that augments the weapons stock. Given one country's investment in weapons, the self-interest of the other country dictates that it should also engage in investment. Such investment is socially counterproductive below $\underline{w}$, in the sense that symmetric changes

in weapons stocks simply decrease the survival probability without providing an advantage to either country. The consequence is that, below $\underline{w}$, each generation bequeaths a more dangerous world to generations that follow. If the steady-state weapons stock exceeds $\underline{w}$, then the post-$\underline{w}$ buildup actually makes the world safer because of the deterrent effect of possible annihilation.

IV. Comparative Statics

Now consider how steady-state saving and weapons accumulation are affected by changes in the exogenous variables of the model. In the following comparative-static experiments, we restrict our attention to symmetric interior stable equilibria; thus we consider the symmetric response of saving and weapons accumulation to, for example, an equal change in the return on storage in both countries. That is, we analyze the effects on equilibrium of changes in the characteristics of the world.[12] Many of the results are ambiguous, reflecting the facts that the effective interest rate, $\phi(\cdot)R$, is endogenous even though the return on saving is fixed and that the assumptions of our model place few restrictions on the second-derivative properties of $\phi(\cdot)$.

PROPOSITION 1: *(i) Increases in the return to storage have an ambiguous effect on saving and weapons accumulation. (ii) If $V(\cdot)$ is logarithmic, then increases in the return to storage increase weapons accumulation but still have an ambiguous effect on saving.*

PROOF:
(i) From (11),

$$\frac{ds}{dR} = \frac{1}{\Delta R}[(b-u)(sv-1)+s\zeta(a-u)] \gtrless 0$$

$$\frac{dw}{dR} = \frac{1}{\Delta R}[u(sv-1)+\zeta(su+1)] \gtrless 0.$$

[12] The comparative-static results for the zero-weapons and zero-conflict equilibria are straightforward, since the weapons stock does not respond to small changes in the parameters.

(ii) If $V(\cdot)$ is logarithmic, then $sv = 1$ and

$$\frac{ds}{dR} = \frac{1}{\Delta R}\left[s\zeta(a-u)\right] \lessgtr 0$$

$$\frac{dw}{dR} = \frac{1}{\Delta R}\left[\zeta(su+1)\right] > 0.$$

When the return on saving rises, consumption in old age is more attractive. Agents can effectively shift consumption into the future in two ways: by increasing future consumption directly through saving, or indirectly by accumulating weapons. An increase in the return on saving increases the value of consumption in old age, encouraging agents to accumulate more weapons to benefit from this higher consumption. For $w < \underline{w}$ $(a < 0)$, this extra accumulation reduces $\phi(\cdot)$ and so reduces the effective return on saving, thus discouraging saving. For $w > \underline{w}$, the saving response is ambiguous. The ambiguity stems partly from the familiar income and substitution effects. In the special case in which $V(\cdot)$ is logarithmic, these effects cancel.

PROPOSITION 2: *Increases in the depreciation rate of the weapons stocks have an ambiguous effect on saving and weapons accumulation.*

PROOF:
From (11),

$$\frac{ds}{d\delta} = \frac{w}{\delta\Delta}\left[u(b-a)\right] \gtrless 0$$

$$\frac{dw}{d\delta} = \frac{w}{\delta\Delta}\left[a(u-\zeta)-b(v+u)\right] \gtrless 0.$$

Increases in δ decrease the aggregate weapons stock (i.e., $dW/d\delta < 0$), affecting both ϕ and ϕ_1, and thus a and b, respectively. For $w > \underline{w}$ the probability of survival, $\phi(w/\delta, w/\delta)$, is increasing in w so that $a > 0$. Additional restrictions are needed to sign $ds/d\delta$ and $dw/d\delta$.

PROPOSITION 3: *Increases in the endowment of the young have an ambiguous effect on saving and increase weapons accumulation.*

PROOF:
From (11),

$$\frac{ds}{de} = \frac{1}{\Delta}\left[u(a-b)\right] \gtrless 0$$

$$\frac{dw}{de} = \frac{1}{\Delta}\left[u(v+\zeta)\right] > 0.$$

Increases in the endowment encourage weapons accumulation. The effect on saving depends upon how the increased accumulation of weapons affects the implicit interest rate $\phi(\cdot)R$. If the economy is in the region where weapons accumulation increases world safety $(w > \underline{w})$, then $a > 0$, so the increased accumulation raises the effective return on saving and encourages saving. For $b > 0$, however, increased accumulation increases the marginal return on accumulation (ϕ_1), making saving a relatively less attractive way to increase expected old-age utility.

PROPOSITION 4: *Increases in the return to winning a conflict, α, have an ambiguous effect on both saving and weapons accumulation.*

PROOF:
From (11),

$$\frac{ds}{d\alpha} = \frac{1}{\Delta}\left[\frac{\phi_\alpha}{\phi}(u-b)-\frac{\phi_{1\alpha}}{\phi_1}(u-a)\right] \gtrless 0$$

$$\frac{dw}{d\alpha} = \frac{1}{\Delta}\left[\frac{\phi_{1\alpha}}{\phi_1}(u+v)-\frac{\phi_\alpha}{\phi}(u-\zeta)\right] \gtrless 0$$

since $\phi_\alpha < 0$ and $\phi_{1\alpha} \gtrless 0$.

This result resembles that in Proposition 2, reflecting the fact that changes in δ and changes in α affect agents' decisions through their effect on $\phi(\cdot)$. The resemblance is apparent when it is noted that ϕ_α and $\phi_{1\alpha}$

play the same role as a and b in Proposition 2.

An increase in α increases the likelihood of initiation and hence reduces the survival probability (i.e., $\phi_\alpha < 0$). Such a change can be interpreted as a decline in inherent world safety. The increased likelihood of annihilation has the direct effect of reducing saving, consistent with Slemrod's (1986, 1990) findings that the increased threat of nuclear war is accompanied by a decrease in saving. Other factors in our model can make the overall impact of a change in α be of either sign, however. In particular, changes in α have an ambiguous effect on the marginal return to accumulation ($\phi_{1\alpha} \gtrless 0$) and so may encourage or discourage saving through this channel.

William Shepherd (1988) argued that the United States and the (former) Soviet Union were inherently safe countries, in the sense that neither had much to gain from initiating conflict. He therefore concluded that weapons accumulation in these countries was inconsistent with the needs of deterrence. In our model, however, the ambiguous effect of α on ϕ_1 permits high weapons accumulation to arise in inherently safe economies (when $\phi_{1\alpha} < 0$). Nevertheless, if decreases in safety increase the return to accumulation, so that $\phi_{1\alpha} > 0$, and if $u - \zeta > 0$, then we find that weapons accumulation is lower in safer worlds.

V. Welfare Analysis

One reason for adopting an overlapping-generations framework is that it permits explicit analysis of intergenerational welfare effects. In this model there are three distinct sources of externality. First, contemporaneous weapons accumulation in country A affects the welfare of agents in country B, by its effect both on the likelihood of conflict and the expected gain from any conflict that does occur. Second, since weapons outlast the agents who invest in them, the accumulation decisions of previous generations in country A affect the weapons stock and, hence, the utility of the current generation in country A. Third, the accumulation decisions of previous generations of country A

affect the utility of the current generation in country B. These externalities differ depending upon the equilibrium weapons stock. The presence of externalities leads to the conclusion that the Nash equilibrium may not be Pareto-efficient.

The only link between generations in this model is through the effect of weapons accumulation, so the depreciation rate of the weapons stock, δ, provides a measure of the extent of the intergenerational externality. In particular, notice that if $\delta = 1$, the link between generations disappears, and there is no intergenerational externality; successive generations simply play a series of unconnected one-shot games. The appropriate magnitude of δ depends on how quickly weapons become obsolete and on the length of a period.

A. First Best

The first-best symmetric outcome in this model arises if a social planner can prevent the initiation of conflict. At any positive symmetric weapons stocks, conflict is a negative-sum game by the assumptions on $f(\cdot)$ and $q(\cdot)$. If the planner can enforce noninitiation, moreover, there is no reason to accumulate weapons, so a zero stock of weapons is first-best. We henceforth assume that noninitiation is never enforceable and so restrict attention to the effects of resource reallocations. Not surprisingly, even in the absence of enforceable noninitiation, the first-best symmetric allocation in this model entails zero accumulation of weapons, since this implies a survival probability of unity. Complete multilateral disarmament starting at some date τ when agents possess positive weapons stocks [i.e., setting $w(\tau) = 0$ in both countries for all $t \geq \tau$] need not be Pareto-improving, however. These results are stated as Proposition 5.

PROPOSITION 5: (i) Zero weapons accumulation ($w = 0$) is a necessary condition for a first best symmetric equilibrium. (ii) If $\underline{w} \geq w(\tau) > 0$, or if δ is sufficiently large, then complete disarmament [$w(t) = 0 \; \forall \; t \geq \tau$] is Pareto-improving; if $\underline{w} < w(\tau)$ and δ is small, complete disarmament will be Pareto-
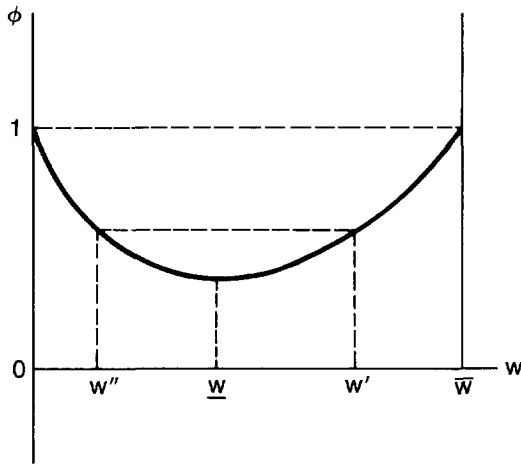
FIGURE 7. THE FIRST-BEST EQUILIBRIUM

improving if and only if

$$U(e - \underline{w} - s(\underline{w})) + \phi(\underline{w}/\delta, \underline{w}/\delta)V(s(\underline{w})R)$$

$$\geq U(e - w(\tau) - s(w(\tau)))$$

$$+ \phi(w(\tau)/\delta, w(\tau)/\delta)V(s(w')R)$$

where $s(\cdot)$ indicates the optimal choice of saving for a given w.

PROOF:

(i) Consider first any $w' > \underline{w}$ as shown in Figure 7. Then by the continuity of $\phi(\cdot)$ there exists a $w'' < \underline{w}$ such that $\phi(w''/\delta, w''/\delta) = \phi(w'/\delta, w'/\delta)$. At w'', the probability of survival is the same as at $w = w'$, but fewer resources are devoted to weapons, so consumption is higher, and all agents can be made better off. Now consider w such that $\underline{w} \geq w > 0$. Then a decrease in w increases the survival probability and increases resources available for consumption. By the continuity of $\phi(\cdot)$ and $\phi(0,0) = 1$, the first-best equilibrium occurs at $w = 0$.

(ii) If the initial weapons stock is below $\underline{w}$, then weapons depletion increases welfare, as noted in (i). If the initial weapons stock exceeds $\underline{w}$ and δ is small, disarmament initially decreases world safety by encouraging initiation of conflict and so will only be Pareto-improving if the resources freed by nonaccumulation outweigh the cost of a lower survival probability. In particular, if $\delta \approx 0$, this must be true at $\underline{w}$, which is the condition stated in the proposition. If δ is large, however, the economy may "jump over" the minimum point $\underline{w}$; indeed, if $\delta = 1$, multilateral disarmament immediately moves the economy to its first-best equilibrium.

The first-best equilibrium in this model is identical to that in van der Ploeg and de Zeeuw's (1990) model. The overlapping-generations framework, however, complicates the welfare analysis of disarmament because different generations are affected differently. Proposition 5 illustrates one important conclusion of this paper: if prior accumulation decisions have placed the economy above $\underline{w}$, then intergenerational externalities complicate any attempts at Pareto-improving disarmament (presuming, of course, that weapons cannot be costlessly destroyed).

As a straightforward corollary of Proposition 5, we note that if the economy exhibits multiple symmetric equilibria in the range $[0, \underline{w}]$, then these equilibria are inversely Pareto-ranked by the stock of weapons. Further, if the corner solution $w = 0$ is an equilibrium, then it is evidently Pareto-superior to any other equilibrium. Henceforth, we abstract from the possibility of multiple equilibria.

B. *Pareto Improvements with Intergenerational Transfers*

Proposition 5 shows that disarmament need not be Pareto-improving, since it may be costly to early generations. Pareto improvements are generically possible if we permit intergenerational transfers, but such improvements may involve additional accumulation of weapons rather than disarmament. First, note that we can write steady-state utility as a function of steady-state w

as follows:

$$\Upsilon(w) = U(e - w - s(w))$$

$$+ \phi(w/\delta, w/\delta)V(s(w)R)$$

where $s(w)$ indicates optimal saving. We know from Proposition 5 that this has a global maximum at $w = 0$. Differentiating and appealing to the envelope theorem, we obtain

$$\Upsilon'(w) = -U'(\cdot) + [(\phi_1 + \phi_2)/\delta]V(\cdot)$$

$$= (V(\cdot)/\phi)$$

$$\times [(\phi_1 + \phi_2)/\phi\delta - V'(\cdot)R/V(\cdot)]$$

using the first-order condition $U'(\cdot) = \phi R V'(\cdot)$.

If the economy is initially in noncooperative equilibrium, then $\phi_1/\phi = V'(\cdot)R/V$, implying

$$\Upsilon'(w) = V(\cdot)[(\phi_1/\phi)(1 - \delta)/\delta + (\phi_2/\phi)/\delta]$$

$$= \frac{V(\cdot)}{\phi}\left[(\phi_1 + \phi_2)\left(\frac{1 - \delta}{\delta}\right) + \phi_2\right]$$

The first term is the intergenerational externality arising from higher inherited weapons stocks from both countries, while the second term is the within-generation across-country externality. The intergenerational externality is negative (i.e., higher weapons stocks reduce the welfare of future generations) for $w < \underline{w}$, and it is positive for $w > \underline{w}$. The within-generation externality is negative for $w < w_2$, and positive for $w > w_2$. Since $w_2 > \underline{w}$, it follows that both externalities are negative if the economy is in a noncooperative equilibrium below $\underline{w}$. Higher steady-state weapons stocks in this region are thus associated with lower welfare. Conversely, both externalities are positive if the equilibrium entails a weapons stock in excess of w_2, in which case higher equilibrium weapons stocks are associated with higher welfare. Propositions 7 and 8 in Section VI show that, in both these cases, Pareto improvements are possible without intergenerational transfers.

In the intermediate region, the intragenerational externality is negative, but the intergenerational externality is positive. If $\Upsilon'(w) > 0$ at a noncooperative equilibrium, then the intergenerational externality dominates, and higher steady-state weapons stocks are associated with higher utility, suggesting that buildup of weapons would be Pareto-improving. Such a conclusion is not quite right, however, because weapons buildup imposes costs on current generations but provides benefits to future generations. As established in Proposition 6, Pareto-improving buildup is possible if

$$\phi_1(1 - \delta) + \phi_2 R = (\phi_1 + \phi_2)[(1 - \delta)/\delta]$$

$$+ \phi_2[1 + (R - 1)/\delta] > 0.$$

This expression differs from $\Upsilon'(w)$ by the addition of $\phi_2(R - 1)/\delta < 0$. A high return to storage or a low depreciation rate thus means disarmament may be Pareto-improving even if $\Upsilon'(w) > 0$.

PROPOSITION 6: *Assume that the economy is dynamically efficient $(R > 1)$ and in a noncooperative equilibrium (w^{NC}) with $\phi_1 + \phi_2 > 0$ and $\phi_2 < 0$ (i.e., $\underline{w} < w^{NC} < w_2$).[13] Pareto improvements are generically possible.*

PROOF:

We first show that if $\phi_1(1 - \delta) + \phi_2 R > 0$, then a small increase in weapons is Pareto-improving with appropriate transfers. Let $w(t) = w^{NC} \; \forall \; t \neq \tau; \; w(\tau) = w^{NC} + \epsilon, \epsilon$ small. Then, starting at date $\tau + 1$, construct transfers $T(t)$ from the young of generation t to the old of generation $t - 1$ (i.e., a transfer from young to old at time t) such that the expected utility of generation $t - 1$ is unchanged. Recall that

$$\phi(W(t + 1), W(t + 1))$$

$$= \phi(W(t)(1 - \delta) + w(t), W(t)(1 - \delta) + w(t)).$$

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[13] These three assumptions rule out the obvious sources of Pareto improvements. See Propositions 7 and 8 for more discussion.

Take a Taylor-series expansion around $w = w^{NC}$ and $T(\tau + 1) = 0$ of the expected utility of generation τ. Using the first-order conditions, the change in expected utility is then

$$\phi_2(\cdot)V(\cdot)\varepsilon + \phi(\cdot)V'(\cdot)T(\tau + 1).$$

Setting $T(\tau + 1)$ so that expected utility is unchanged implies

$$T(\tau + 1) = -\phi_2(\cdot)V(\cdot)\varepsilon / \phi(\cdot)V'(\cdot) > 0.$$

Generation τ is made worse off by the extra weapons accumulation of the other country ($\phi_2 < 0$) and so must be compensated when old by the next generation of young agents.

Similarly, $T(\tau + 2)$ can be defined so that the expected utility of generation $\tau + 1$ is unchanged:

$$(\phi_1 + \phi_2)(1 - \delta)V(\cdot)\varepsilon - U'(\cdot)T(\tau + 1)$$

$$+ \phi V'(\cdot)T(\tau + 2) = 0$$

$$\Rightarrow (\phi_1 + \phi_2)(1 - \delta)V(\cdot)\varepsilon$$

$$- \phi R V'(\cdot)T(\tau + 1)$$

$$+ \phi V'(\cdot)T(\tau + 2) = 0$$

$$\Rightarrow [(\phi_1 + \phi_2)(1 - \delta) + \phi_2 R]V(\cdot)\varepsilon$$

$$+ \phi V'(\cdot)T(\tau + 2) = 0.$$

Likewise, $T(\tau + 3)$ is defined by

$$(\phi_1 + \phi_2)(1 - \delta)^2 V(\cdot)\varepsilon - U'(\cdot)T(\tau + 2)$$

$$+ \phi V'(\cdot)T(\tau + 3) = 0$$

$$\Rightarrow (\phi_1 + \phi_2)(1 - \delta)^2 V(\cdot)\varepsilon$$

$$- \phi R V'(\cdot)T(\tau + 2)$$

$$+ \phi V'(\cdot)T(\tau + 3) = 0$$

$$\Rightarrow \{(\phi_1 + \phi_2)[(1 - \delta)^2 + (1 - \delta)R] + \phi_2 R^2\}V(\cdot)\varepsilon$$

$$+ \phi V'(\cdot)T(\tau + 3) = 0.$$

By repeated substitution, $T(\tau + N)$ is defined by

$$\left[(\phi_1 + \phi_2) \sum_{n=1}^{N} (1 - \delta)^n R^{N-n} + \phi_2 R^N\right]$$

$$\times V(\cdot)\varepsilon + \phi V'(\cdot)T(\tau + N) = 0$$

$$\Rightarrow \left\{(\phi_1 + \phi_2) \sum_{n=1}^{N} [(1 - \delta)/R]^n + \phi_2\right\}$$

$$\times R^N V(\cdot)\varepsilon + \phi V'(\cdot)T(\tau + N) = 0.$$

As $N \to \infty$, and since $(1 - \delta)/R < 1$, the term in braces tends to

$$\frac{(\phi_1 + \phi_2)(1 - \delta)}{R - (1 - \delta)} + \phi_2$$

which is positive since $\phi_1(1 - \delta) + \phi_2 R > 0$ by assumption. It follows that, at some date τ^*, the transfers change sign; that is, members of generation $\tau^* - 1$ make both a positive transfer when young to the old of generation $\tau^* - 2$ and a positive transfer when old to the young of generation τ^*.

Now set $T(t) = 0 \ \forall \ t \geq \tau^*$. The welfare of all generations up to and including generation $\tau^* - 2$ is unchanged by construction. Generation $\tau^* - 1$ is better off since it does not have to make the positive transfer at τ^* that would leave its utility unchanged. All future generations are also better off because they neither make nor receive transfers but do inherit a larger weapons stock. This pattern of transfers is thus Pareto-improving. A similar argument holds if $\phi_1(1 - \delta) + \phi_2 R < 0$; in this case a small decrease in $w(\tau)$ is Pareto-improving.

Proposition 6 establishes that there is some critical value of the weapons stock above which Pareto improvements are (locally) possible by building up weapons and below which Pareto improvements are (locally) possible by reduction of weapons stocks. This critical value is higher, the higher the return on storage and the depreciation rate.

VI. Cooperative Equilibria

A. Cooperation Across Countries

Consider the possibility that members of a given generation in countries A and B cooperate. As in the earlier discussion, we disregard the trivial case of a nonaggression pact whereby agents agree not to initiate conflict (note that such an agreement in general will not be time-consistent) and restrict our attention to the cooperative choice of weapons. We think of such cooperation as a verifiable arms treaty that neither restricts future generations nor precludes conflict in the current period. We assume, in keeping with our symmetry assumptions, that both countries have equal bargaining power. The problem faced by the two countries is to choose both their weapons stocks to maximize (3) subject to (4) and (5), taking as given the inherited weapons stocks in both countries. The first-order conditions for an interior equilibrium in this problem are

$$(20) \qquad -U'(\cdot) + \phi(\cdot)V'(\cdot)R = 0$$

$$(21) \qquad -U'(\cdot) + (\phi_1 + \phi_2)V(\cdot) = 0.$$

These first-order conditions are identical to those from the original problem [equations (7) and (8)], except that there is an additional $\phi_2 V(\cdot)$ term in equation (21). This indicates that the two countries internalize the intragenerational externality. The intergenerational externality remains, however. We consider this case to be of particular interest since intragenerational externalities are intrinsically easier to internalize than external effects across generations.

We now consider the possibility of a regime change at time τ. That is, we suppose that the economy is at a noncooperative steady-state equilibrium (w^{NC}) prior to τ, and that at all dates including and after τ, both countries cooperate on their choice of weapons. Such a regime change could be interpreted as corresponding to the end of the Cold War. The following propositions reveal that both buildup and reduction of weapons stocks are possible responses and

that disarmament need not be Pareto-improving.[14]

PROPOSITION 7: *If $\phi_2 > 0$ at w^{NC} so that $\bar{w} > w^{NC} > w_2$, then agents in both countries will choose cooperatively to build up their stocks of weapons, either to an interior equilibrium, w^C, or to $\bar{w}$. Such buildup is Pareto-improving.*

PROOF:
First consider accumulation to an interior cooperative equilibrium, w^C, as illustrated in Figure 8. Let $\bar{w}(t) = \delta W(t)$. From equation (2),

$$\bar{w}(t+1) = (1-\delta)\bar{w}(t) + \delta w(t).$$

If, at time t, $w^{NC} \leq \bar{w}(t) < w^C$, then the following inequality holds:

$$\frac{\phi_1(\bar{w}(t)/\delta, \bar{w}(t)/\delta) + \phi_2(\bar{w}(t)/\delta, \bar{w}(t)/\delta)}{\phi(\bar{w}(t)/\delta)}$$
$$> \frac{V'(s(\bar{w}(t))R)R}{V(s(\bar{w}(t))R)}.$$

At date t, agents choose their weapons stock, $w(t)$, to equate $(\phi_1 + \phi_2)/\phi$ and $V'R/V$, where $V(\cdot)$ and $\phi(\cdot)$ are functions of $w(t)$ and the inherited weapons stock, $\bar{w}(t)$, which they take as given. If $\delta < 1$, then $(\phi_1 + \phi_2)/\phi$ responds less to changes in $w(t)$ than to changes in steady-state w, implying that the out-of-steady-state $(\phi_1 + \phi_2)/\phi$ passes through $\bar{w}(t)$ and is flatter than the steady state relationship. (In the limit as $\delta \to 0$, this curve becomes horizontal.) Also, if $\delta < 1$, changes in $w(t)$ lead to a smaller increase in $\phi(\cdot)$ than do changes in steady-state w and so have a smaller effect on

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[14] The analysis of Proposition 7 uses our assumption that $(\phi_1 + \phi_2)/\phi$ is upward-sloping. If this relationship slopes downward, but $V'R/V$ (for an individual generation) still slopes upward, then the analysis is almost identical. In this case individual generations might choose weapons investment in excess of w^C, the cooperative equilibrium, but the aggregate weapons stock still will not exceed w^C. If both relationships slope downward, then we cannot rule out the possibility of oscillatory adjustment of the weapons stock.
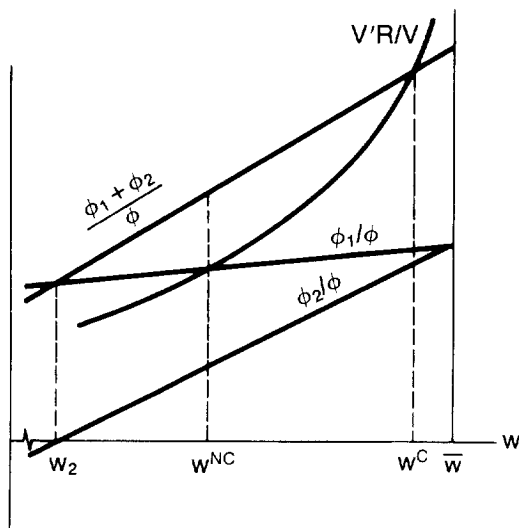
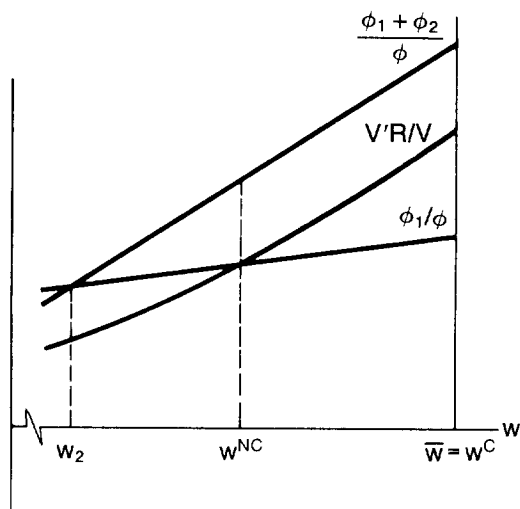FIGURE 8. BUILDUP TO AN INTERIOR
COOPERATIVE EQUILIBRIUM

FIGURE 9. BUILDUP TO A NO-CONFLICT
EQUILIBRIUM

saving through this channel. As a consequence, the out-of-steady-state $V'R/V$ rotates through $\tilde{w}(t)$ and becomes steeper than its steady-state counterpart. By continuity of $V(\cdot)$ and $\phi(\cdot)$, it follows that

$$\tilde{w}(t) < w(t) < w^C$$

$$\Rightarrow \tilde{w}(t) < \tilde{w}(t+1) < w^C.$$

If $\delta = 1$, then there is no intergenerational externality, and the out-of-steady-state and steady-state relationships are coincident. Thus,

$$w(t) = \tilde{w}(t+1) = w^C.$$

At the time of the regime change $\tilde{w}(\tau) = w^{NC}$. Therefore if $\delta < 1$, the two countries cooperatively build up their weapons stocks, which asymptotically approach w^C. If $\delta = 1$, the two countries move immediately to w^C.

Buildup to $\overline{w}$ is illustrated in Figure 9. The analysis is similar, except that individual generations may choose investment in excess of $\overline{w}$, and so $\overline{w}$ is obtained in finite time even if $\delta < 1$. Buildup is Pareto-improving since $\phi_1 + \phi_2 > 0$ for $w^{NC} < w < w^C$: a larger world weapons stock bestows a

positive externality on future generations. Generations after τ are better off since they could always choose the same allocation of resources as at the noncooperative equilibrium and benefit from the larger inherited weapons stock.

If the economy is originally in equilibrium above w_2, then cooperation entails internalization of a positive within-generation externality, so that agents choose to build up their weapons stocks. When the within-generation externality (ϕ_2) is positive, then so too is the across-generation externality $(\phi_1 + \phi_2)[(1 - \delta)/\delta]$. The two countries thus cooperatively agree to forgo more consumption to make conflict less likely, and despite the lower consumption, cooperation is Pareto-improving. This result is reminiscent of Garfinkel's (1990) finding that adversarial countries in a stochastic environment might accumulate weapons in a cooperative equilibrium. Her result obtains because, in general, there do not exist punishments sufficiently harsh to make not deviating from the cooperative agreement incentive-compatible in the absence of accumulation. Similarly, cooperation that leads to disarmament need not be Pareto-improving, as established in the following proposition.
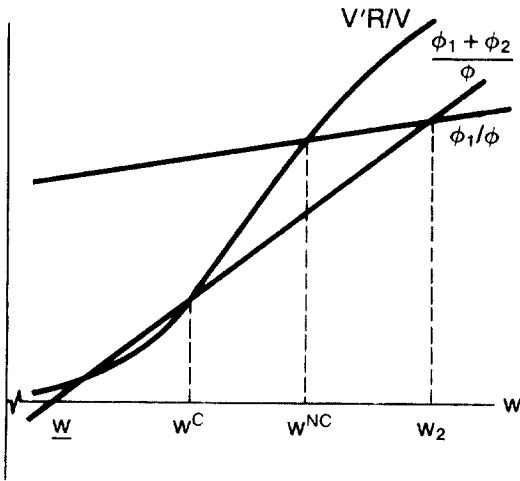
FIGURE 10. REDUCTION OF WEAPONS STOCKS TO
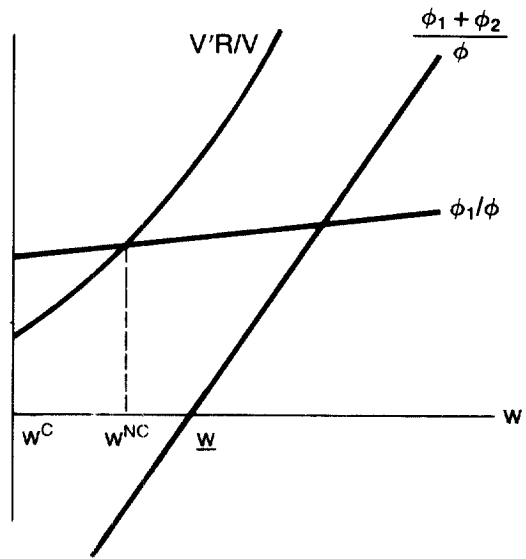AN INTERIOR EQUILIBRIUM

FIGURE 11 REDUCTION OF WEAPONS STOCKS TO
AN EQUILIBRIUM WITHOUT WEAPONS
ACCUMULATION

PROPOSITION 8: *(i) If $\phi_2 < 0$ at w^{NC} and $\underline{w} < w^{NC} < w_2$, then agents in the two countries will choose cooperatively to reduce weapons stocks to an interior equilibrium or to $w = 0$. This reduction need not be Pareto-improving. (ii) If $\phi_2 < 0$ at w^{NC} and $w^{NC} < \underline{w}$, agents in the two countries will cooperatively cease all weapons investment and reduce weapons stocks to $w = 0$. This reduction is Pareto-improving.*

PROOF:

(i) Arms reduction to an interior equilibrium is represented in Figure 10; the analysis is analogous to that in Proposition 7. Since $\phi_2 < 0$ at w^{NC}, agents internalize a negative intragenerational externality and so cooperatively choose to reduce their weapons stocks. Since $w^{NC} > \underline{w}$, $\phi_1 + \phi_2 > 0$, and the intergenerational externality is therefore positive. Disarmament in general imposes costs on subsequent generations by decreasing the survival probability and so need not be Pareto-improving. Similar analysis applies for reduction of weapons stocks to zero; in this case, arms reduction is initially costly to future generations but eventually benefits them.

(ii) Reduction of weapons stocks to zero is represented in Figure 11. Since $V'R / V$

is everywhere positive and $(\phi_1 + \phi_2)/\phi$ is negative. generation τ and all subsequent generations immediately move to the corner solution where $w = 0$. Over time, natural depreciation of weapons allows the aggregate weapons stock to decline asymptotically to zero. Below $\underline{w}$, decreases in the aggregate weapons stock increase the survival probability and so impose positive externalities on subsequent generations. Disarmament is therefore Pareto-improving.

Any individual generation does not take account of the effects of its actions on the safety of future generations. Internalization of the within-generation externality thus need not be Pareto-improving for standard second-best reasons: it worsens the across-generation externality. Note, though, that part (i) of Proposition 8 does *not* imply that disarmament is never Pareto-improving even if the noncooperative weapon stock exceeds $\underline{w}$. First, if $\delta = 1$, then there is no intergenerational externality. Second, even if a generation after τ is made worse off because it inherits a lower weapons stock, it also benefits from the fact that it in turn is able to

internalize the intragenerational externality through international cooperation: a cooperative choice of a smaller weapons stock frees resources for consumption.

Our results are consistent with the model of Intriligator and Brito (1984), which demonstrates that a reduction of weapons stocks may promote war rather than peace.[15] The buildup scenario is also consistent with Intriligator and Brito's (1984) finding that buildup may increase world safety, and in our model, it may increase world welfare as well.

B. Cooperation Across Generations

Suppose that social planners in each country choose the resource allocation for their individual countries for all time, taking the rules of international interaction as given. This imposes within-country intergenerational cooperation while sustaining the potential for international conflict. In the setting of our model, this is perhaps less plausible in that it requires internalization of externalities between agents who are not alive at the same time. It does however allow for comparison with the infinitely-lived-agent models in the arms-control literature.[16] We analyze this problem as a one-shot game in which a social planner in each country treats all generations symmetrically and chooses a constant allocation of resources through time. In this scenario, the planners can ensure that their countries are dynamically efficient and will impose a larger weapons stock on the world, as shown in Proposition 9.

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[15] Recall that in our model, lower weapons stocks are *always* associated with a greater likelihood of conflict, but a smaller probability of disaster should conflict occur. This is because the probability of annihilation, $q(\cdot)$, is increasing in w, but the probability that neither side intitiates, $z(q)$, is increasing in $q(\cdot)$. The trade-off between these two effects gives rise to the behavior of the survival probability, $\phi(\cdot)$, as w varies. In comparing our results to those of Intriligator and Brito (1984), we are interpreting "war" in our model as annihilation, not conflict.

[16] Intergenerational cooperation also might be a result of altruism across generations, as in the literature on Ricardian equivalence.

PROPOSITION 9: *If the intergenerational within-country externality is internalized, then goods will be stored only if storage is dynamically efficient. If the economy is dynamically efficient, the equilibrium weapons stock will be higher than in the noncooperative equilibrium.*

PROOF:

The planner for country A chooses w^A and c^o to maximize

$$U(c^y) + \phi(w^A/\delta, w^B/\delta)V(c^o)$$

subject to

$$c^y = e - c^o - w \qquad \text{if } R < 1$$
$$c^o = R(e - c^y - w) \qquad \text{if } R \geq 1.$$

The first-order conditions yield

$$(22) \quad U'(\cdot) = \begin{cases} \phi V'(\cdot) & \text{if } R < 1 \\ \phi V'(\cdot)R & \text{if } R \geq 1 \end{cases}$$

and

$$(23) \qquad U'(\cdot) = \phi_1 V(\cdot)/\delta.$$

Equation (22) establishes that storage is only undertaken if it is dynamically efficient, that is, if $R \geq 1$.[17] If $R < 1$, the planner achieves the optimal allocation of goods through transfers from young to old. The only difference between equations (23) and (8) is the $1/\delta$ in (23): ϕ_1/δ represents the total effect on country A of present weapons accumulation. As is clear from Figure 12, the equilibrium weapons stock given intergenerational cooperation, w^{IC}, exceeds the noncooperative weapons stock, w^{NC}.[18]

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[17] If the planner discounts the welfare of future generations, or if there is population growth, then this condition is amended in the usual way

[18] If, as can occur, $V''R/V$ is downward-sloping, then it might seem that intergenerational cooperation would reduce weapons accumulation. In this case, however, the equilibria being compared are neither stable nor unique
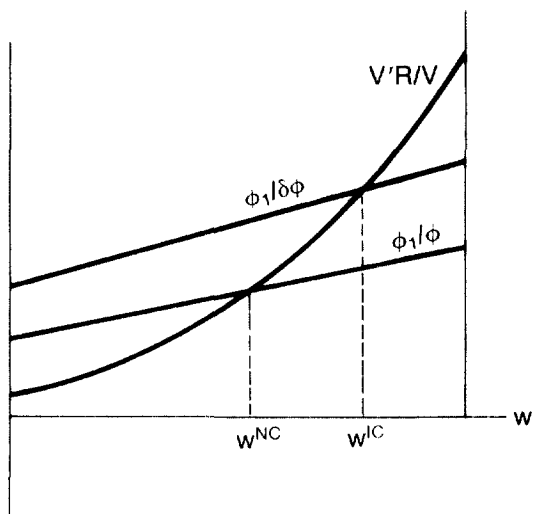
FIGURE 12. AN EQUILIBRIUM WITH
INTERGENERATIONAL COOPERATION

Since old agents' weapons accumulation generates uncompensated benefits for young agents in their own country, it is possible that a social contract of the type proposed by Laurence J. Kotlikoff et al. (1988) could be developed to induce each generation to accumulate more weapons. Notice that, while this is welfare-improving from the perspective of a single country it may actually lower world welfare. Again, this is a second-best result.

VII. Conclusion

In this paper we have developed a two-country overlapping-generations model of deterrence or conflict initiation. Our model emphasizes the intergenerational element of international tensions and so highlights aspects of arms accumulation that have received little attention in the literature. We abstract, however, from some topics that have received more attention in the literature. In particular, successive generations in our model play one-shot games, removing the scope for repeated-game phenomena, such as reputation effects and punishment

strategies. We suspect that real-world behavior is characterized by repeated games with intergenerational externalities.

This paper follows in the tradition of the literature in that we examine the arms race in a two-country world. While such analysis was evidently appropriate during the Cold War era, it is less so today. Our approach remains useful for a number of reasons. First, the world may once again be dominated by two superpowers in the future. Second, and perhaps more importantly, our analysis also can be applied to regional conflicts when devastation of both countries is a possible outcome. Moreover, our model lends itself to a multicountry generalization with coalition formation; such an approach may provide a useful framework for analyzing present-day conflicts. Finally, a lesson of our model is that, although the world may no longer be dominated by two superpowers, the legacy of the Cold War affects current decisions and current welfare.

Despite our emphasis on intergenerational effects, some of our results resemble those of the arms-control literature with infinitely-lived agents. We find that arms-control (reduction) arrangements can either increase or decrease the possibility of international conflict and that international arrangements leading to increases in international weapons stocks are successful both in deterring conflict and in improving world welfare. While we do find that having no weapons is always better than having some weapons, fewer weapons need not be better, even if the reduction in weapons is brought about by an international agreement. Although apparently counterintuitive, this result is firmly grounded in intergenerational trade-offs: the immediate welfare improvements won by reducing the resources devoted to defense may be accompanied by uncompensated welfare losses on future generations. While the end of the Cold War may bring a peace dividend, it does not guarantee a safer world: current generations still bear the consequences of past accumulation decisions. This we see as a cautionary tale for the proponents of the "new world order."

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