Speckle photography: mixed domain fractional Fourier motion detection

Robert F. Patten, Bryan M. Hennelly, Damien P. Kelly, Feidhlim T. O'Neill, Ying Liu, and John T. Sheridan

School of Electrical, Electronic, and Mechanical Engineering, College of Engineering, Mathematical, and Physical Sciences, University College Dublin, Belfield, Dublin 4, Republic of Ireland

Received July 12, 2005; accepted September 16, 2005

A reflection-based optical implementation of two simultaneous scale-invariant fractional Fourier transforms (FRTs) is used to develop a novel compact speckle photographic system. The system allows the independent determination of both surface tilting and in-plane translational motion from two sequential mixed domain images captured using a single camera. © 2006 Optical Society of America

OCIS codes: 070.2580, 120.6150, 120.3940, 330.4150.

Speckle photography (SP) is a practical means of measuring in-plane translation and tilting motion of optically rough surfaces.[1,2] In-plane translation measurement involves the capture of the intensity of the field reflected from the surface both before and after motion. Numerically calculating the Fourier transform (FT) of the sum or difference of the two sequential images yields a cosinusoidal fringe pattern with spacing inversely proportional to the surface displacement and fringe normal to the direction of motion. In the measurement of tilting, the optical Fourier transform (OFT) of the reflected surface fields is captured and the FT of the sum or difference results in fringe spacing inversely proportional to the magnitude of the rotation. While the imaging technique is insensitive to tilting motion, the OFT technique is insensitive to translation, and thus two systems are required to capture both components of surface motion. Neither technique allows the user to determine the direction of motion.

The fractional Fourier transform[3,4] (FRT) is a linear transform, of which the imaging operation and the FT are special cases.[5–10] Combining the optical implementation of the FRT (OFRT) with SP allows the simultaneous measurement of mixed translation and tilting motions.[11] Using an OFRT system,[12] termed a "fake-zoom lens," variation of both the minimum resolution and the dynamic range of measurement has been demonstrated.[13] Separation of both motion components can be achieved, using images captured in a single FRT domain, if a linear relationship exists between the two types of motion. Otherwise, capture in two different fractional domains is necessary[14] and has been demonstrated.[15] The technique involves the capture of two images first in one domain (OFRT order $a1$) and then two more in the second (OFRT order $a2$), by using a two-lens, scale-invariant OFRT.[16] Correlating these images numerically allows the direction of motion to be determined and also allows decorrelation effects to be observed. In summary this technique requires the capture of four sequential images at a single camera, with a change of OFRT order during capture or the use of two parallel optical systems with different OFRT orders and two cameras, each of which captures two sequential images.

The use of reflective elements in OFRT systems has been discussed theoretically.[17] Light, after crossing a system made up of free-space distances and refracting elements, may be reflected back through the same system by a plane mirror. The effect of the "back" transit can be described by the concatenation of the system components in reverse order, with the input and output planes coincident.[17] An OFRT implementation based on such a geometry is shown in Fig. 1. A folded, single-lens reflection system is arranged about a central beam splitter allowing the input and output planes to be separated. The system distances d_1, $d_2/2$, and d_3 are chosen to generate a scale-invariant OFRT.[16] This geometry offers several potential advantages over the two-lens system: (i) It is more compact; (ii) varying the OFRT order only requires the movement of the lens (L) and plane mirror (M) while maintaining the output plane at a fixed distance from the beam splitter (BS); and (iii) the inherent system error is reduced, as fewer independent components need to be moved when scanning across a range of fractional orders. This is particularly significant if components are mounted on computer-controlled motion stages. Such a metrology system, shown in Fig. 2, allows the simultaneous generation of results in two different fractional domains with coincident input and output planes.

In this Letter we describe the sequential capture and correlation of two images, each containing the OFRT of an input field in two different OFRT domains. It is shown that the system in Fig. 2 allows the independent determination of both surface tilt

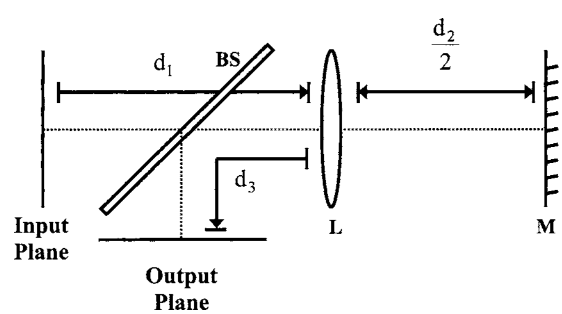

Fig. 1. Folded reflection optical fractional Fourier geometry.

0146-9592/06/010032-3/$0.00

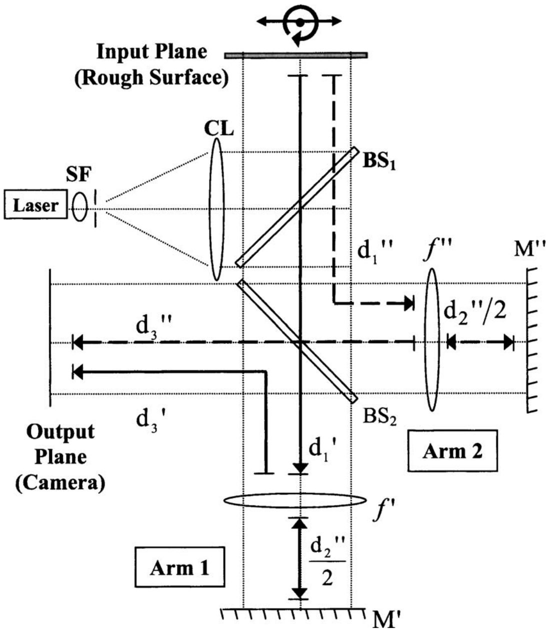

Fig. 2. Mixed domain speckle photography setup.

and in-plane translation with a single camera and the capture of two images.

If the field reflected from a 1D optically rough surface, $u(x)$, undergoes a translation, ξ, and tilt, κ, the resulting field can be described in the spatial domain as

$$u(x) \to u(x-\xi)\exp(+j\kappa x). \tag{1}$$

We define $U_\theta(q)$ to be the FRT of the field.[14] The fractional angle, $\theta = a\pi/2$, where $a=0$ ($q=x$) and $a=1$ ($q=k$), corresponds to the imaging and FT operations, respectively. After motion $U_\theta(q)$ is shifted by an amount $Q_\theta = \xi\cos\theta + \kappa\sin\theta$, where this shift in q is a projection into the FRT domain axis of the actual displacement distance in phase space,[14] $\sqrt{\xi^2+\kappa^2}$. Therefore, following Eq. (1),

$$U_\theta(q) \to U(q-Q_\theta)\exp[+j\Phi_\theta(q)], \tag{2}$$

where $\Phi_\theta(q)=\Phi(q)$ as defined in Ref. 14.

In the folded geometry the first image captured at the camera has the form

$$|U_{\theta 1}(q1)+U_{\theta 2}(q2)|^2 = I_{\theta 1}(q1)+I_{\theta 2}(q2)+U_{\theta 1}(q1) \times U^*_{\theta 2}(q2)+U^*_{\theta 1}(q1)\times U_{\theta 2}(q2), \tag{3}$$

where $I_{\theta i}(qi)=|U_{\theta i}(qi)|^2$. Following motion the second image captured is

$$|U_{\theta 1}(q1-Q_{\theta 1})\exp[+j\Phi_{\theta 1}(q1)]+U_{\theta 2}(q2-Q_{\theta 2}) \times\exp[+j\Phi_{\theta 2}(q2)]|^2 = I_{\theta 1}(q1-Q_{\theta 1})+I_{\theta 2}(q2-Q_{\theta 2}) + 2\,\mathrm{Re}(U_{\theta 1}(q1-Q_{\theta 1})U^*_{\theta 2}(q2-Q_{\theta 2}) \times\exp\{+j[\Phi_{\theta 1}(q1)-\Phi_{\theta 2}(q2)]\}), \tag{4}$$

where $I_{\theta i}(qi-Q_{\theta i})=|U_{\theta i}(qi-Q_{\theta i})|^2$.

In order to define the initial state of the surface (the origin) we calculate the autocorrelation of Eq. (3). Cross correlations are normalized with respect to the autocorrelation.

The correlation of a perfectly random function, $f(x)$, with a shifted version of itself is a shifted delta function $f(x)\otimes f(x-Q)=\delta(x-Q)$. The spatial intensity distributions (speckle fields) captured by our camera are not completely random.[18–20] However, we initially assume that they can be treated as such and thus greatly simplify our analysis. As will be seen, we can explain to the first order many of our experimental observations, in particular those involving correlation peak location. We assume that

$$\frac{I_{\theta j}(qj)\otimes I_{\theta i}(qi-Q_{\theta i})}{I_{\theta j}(0)\otimes I_{\theta j}(0)} = \begin{cases} 1 & i=j \quad \text{and} \quad qi=Q_{\theta i}, \\ 0 & i\neq j \quad \text{or} \quad qi\neq Q_{\theta i}. \end{cases} \tag{5}$$

Examining the correlation of Eqs. (3) and (4) we see that we might reasonably expect two correlation peaks to arise at $q1=Q_{\theta 1}$ and independently at $q2=Q_{\theta 2}$. Because of our assumption regarding the random nature of the fields, all other correlations are zero. For example, examine the term

$$[U_{\theta 1}(q1)\times U^*_{\theta 2}(q2)]\otimes(U_{\theta 1}(q1-Q_{\theta 1})U^*_{\theta 2}(q2-Q_{\theta 2}) \times\exp\{+j[\Phi_{\theta 1}(q1)-\Phi_{\theta 2}(q2)]\}).$$

A correlation peak could exist only if simultaneously $q1=Q_{\theta 1}$, $q2=Q_{\theta 2}$, and $\Phi_{\theta 1}(q1)=\Phi_{\theta 2}(q2)$.

The experimental system (Fig. 2) was implemented using a National Laser Company Ar-ion laser, expanded and collimated through a spatial filter (SF) and collimating lens (CL). Using the first beam splitter, (BS_1), illumination was normal to the test surface.[21] This avoided any changes to the system sensitivity to tilt due to illumination angle.[2] Two independent OFRT arms of different fractional order were arranged about BS_2. The focal lengths of both lenses were equal, $f=f'=f''=20$ cm. The distances used in arm 1 were $d_1'=d_3'=f$ and $d_2'=2f$ and in arm 2, $d_1''=d_3''=1.5f$ and $d_2''=2.8f$. Therefore the first OFRT ($a1=2$) was an imaging system, insensitive to surface rotations (tilting), while the second ($a2=1.409$) was sensitive to both rotation and tilting. The camera used was a Sony XSCE-50. An Oriel 13048 Rotation Stage mounted on an Oriel Encoder Mike Translation Stage was used to displace the test surface. A typical translation error of $\sim 1\ \mu$m and a rotation error of $\sim 100\times 10^{-6}$ rad have been noted.[15] Further errors may arise due to incorrectly positioned lenses and mirrors.[15]

In Figs. 3(a) and 3(b) we present experimental results. The correlations are shown as a function of the number of camera pixels displaced. (A conversion factor of 8.33×10^{-6} m converts pixels to meters.) The normalized autocorrelation peak appears at the center of both figures. In generating Fig. 3(a) the surface was displaced right by 200 μm between capturing the two images. Two peaks appear, shifted by different amounts but in the same direction. With no

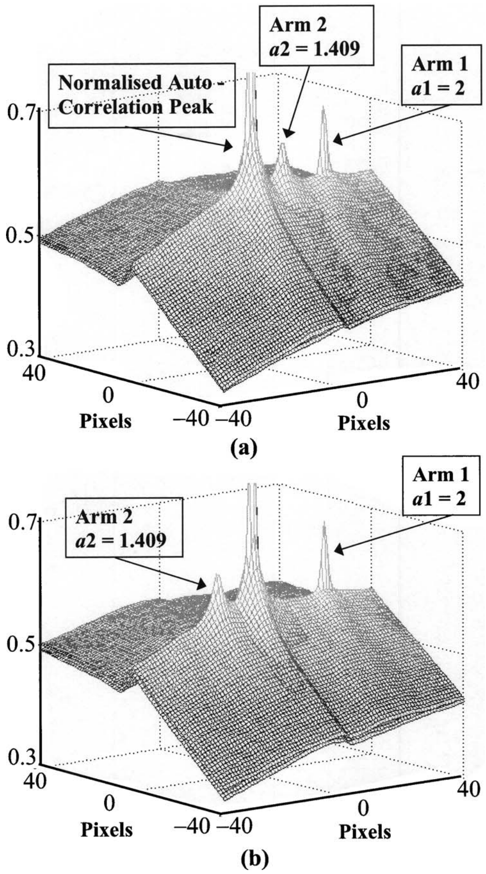

Fig. 3. Displacement of (a) +200 μm and (b) +200 μm and rotation of −540 μrad.

a priori knowledge regarding the surface motion, we determine the magnitude and direction of the translation to be 199 ± 1 μm to the right and that negligible tilting has occurred (a value of 62×10^{-6} rad is estimated).

For Fig. 3(b) the surface was again displaced by 200 μm but also rotated counterclockwise by 540 $\times 10^{-6}$ rad. The correlation peak corresponding to the imaging system ($a1=2$) is almost unaffected, appearing once again to the right. However, the peak corresponding to the OFRT of order $a2=1.409$ now appears to the left of the origin. In this case solving the two simultaneous equations[14] allows the determination of both the translation (199 ± 1 μm to the right) and rotation (clockwise 661 ± 121 μrad). Experimental errors include the effects of uncertainty due to the mechanical translation and/or rotation stages and also the effects of inaccurate positioning of the optical components (errors in the OFRT order values).

Examining these experimental results, the speckle fields are clearly not completely random, i.e., no delta functions appear. In particular, we note that the correlation is never zero and that the three correlation peaks, which are clearly observable, have different shapes (widths). This cannot be simply attributed to an imbalance in the intensities in the two arms of the system, but arises because of the different statistical properties of the fields in the different fractional Fourier domains.[18–20]

A novel speckle photographic system has been implemented and results, which illustrate system performance, have been provided. A first-order model has been proposed, the deficiencies of which, arising primarily due to the neglected statistical properties of the speckle fields, have been briefly discussed.

We acknowledge Enterprise Ireland, Science Foundation Ireland, and the Irish Research Council for Science, Engineering and Technology, through the Research Innovation and Proof of Concept Funds, and the Basic Research and Research Frontiers Programs. J. T. Sheridan's e-mail address is john.sheridan@ucd.ie.

References

1. P. K. Rastogi, *Speckle Metrology*, R. S. Sirohi, ed. (Dekker, New York, 1993) pp. 50–58.
2. H. Tiziani, Opt. Commun. **5**, 271 (1972).
3. V. Namias, J. Inst. Math. Appl. **25**, 241 (1980).
4. A. C. McBride and F. H. Kerr, IMA J. Appl. Math. **39**, 159 (1987).
5. D. Mendlovic and H. M. Ozatkas, J. Opt. Soc. Am. A **10**, 1875 (1993).
6. H. M. Ozatkas and D. Mendlovic, J. Opt. Soc. Am. A **10**, 2522 (1993).
7. A. W. Lohmann, J. Opt. Soc. Am. A **10**, 2181 (1993).
8. R. G. Dorsch, Appl. Opt. **34**, 6016 (1995).
9. H. M. Ozaktas, Z. Zalevsky, and M. Alper Kutay, *The Fractional Fourier Transform with Applications in Optics and Signal Processing* (Wiley, Chichester, 2001).
10. J. T. Sheridan and R. Patten, Opt. Lett. **25**, 448 (2000).
11. J. T. Sheridan and R. Patten, Optik (Stuttgart) **111**, 329 (2000).
12. A. W. Lohmann, Opt. Commun. **115**, 427 (1995).
13. R. Patten, J. T. Sheridan, and A. Larkin, Opt. Eng. **40**, 1438 (2001).
14. J. T. Sheridan, B. Hennelly, and D. Kelly, Opt. Lett. **28**, 884 (2003).
15. D. P. Kelly, B. M. Hennelly, and J. T. Sheridan, Appl. Opt. **44**, 2720 (2005).
16. L. Z. Cai and Y. Q. Wang, Opt. Laser Technol. **34**, 249 (2002).
17. K. B. Wolf and G. Krotzsch, J. Opt. Soc. Am. A **19**, 1191 (2002).
18. J. W. Goodman, *Laser Speckle and Related Phenomena*, J. C. Dainty, ed. (Springer, 1984), Chap. 2.
19. R. S. Hansen, H. T. Yura, and S. G. Hanson, J. Opt. Soc. Am. A **14**, 3093 (1997).
20. H. T. Yura, B. Rose, and S. G. Hanson, J. Opt. Soc. Am. A **15**, 1160 (1998).
21. J. M. Diazdelacruz, Appl. Opt. **44**, 2250 (2005).