

# TOPOGRAPHIC OBJECT RECOGNITION THROUGH SHAPE

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## **ABSTRACT**

Automatic structuring (feature coding and object recognition) of topographic data, such as that derived from air survey or raster scanning large-scale paper maps, requires the classification of objects such as buildings, roads, rivers, fields and railways. The recognition of objects in computer vision is largely based on the matching of descriptions of shapes. Fourier descriptors, moment invariants, boundary chain coding and scalar descriptors are methods that have been widely used and have been developed to describe shape irrespective of position, orientation and scale. The applicability of the above four methods to topographic shapes is described and their usefulness evaluated.

All methods derive descriptors consisting of a small number of real values from the object's polygonal boundary. Two large corpora representing data sets from Ordnance Survey maps of Purbeck and Plymouth were available. The effectiveness of each description technique was evaluated by using one corpus as a training-set to derive distributions for the values for supervised learning. This was then used to reclassify the objects in both data sets using each individual descriptor to evaluate their effectiveness. No individual descriptor or method produced consistent correct classification.

Various models for the fusion of the classification results from individual descriptors were implemented. These were used to experiment with different combinations of descriptors in order to improve results. Overall results show that Moment Invariants fused with the "min" fusion rule gave the best performance with the two data sets. Much further work remains to be done as enumerated in the concluding section.

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## Chapter 1: INTRODUCTION

The Intelligent and Graphical Research Group within the Department of Computer Science at National University of Ireland, Maynooth (NUIM) is researching into the automatic recognition of features and objects on topographic maps. The main application of this work is the automatic structuring of topographic data for computer cartography and GIS systems. The techniques being evaluated can be divided into two broad categories:

- recognition based on isolated shape (described here), and
- recognition based on context.

In shape-based classification, the shape of each object is described using a small number of descriptor values (typically 7 to 15 real numbers). Recognition is based on matching the descriptors of each shape to standard values representing typical shapes and choosing the closest match. Several types of descriptor values have been developed (mostly in the field of computer vision). Research at NUIM so far has concentrated on four techniques:

- scalar descriptors (area, dimension, elongation, number of corners etc.),
- Fourier descriptors,
- moment invariants and
- boundary chain encoding.

These techniques are well understood when applied to images and can be normalised to describe shapes irrespective of position, scale and orientation. They can also be easily applied to vector graphical shapes.

Work carried out to date includes the object recognition and classification of buildings and parcels (from test data provided by the Isle of Man government) using three of the above mentioned techniques namely Fourier descriptors, moment invariants and scalar descriptors. Results indicate that no one shape technique alone is powerful enough for the task - in different situations one technique will perform better than the others and produce significant results (e.g. distinguishing buildings from linear features in built-up areas using the moment invariants method).

In order to test these techniques further, they were evaluated on a corpus of topographic data provided by OSGB using the feature codes (object types) used in the large-scale OS GB topographic database. The most significant aims were to:

- statistically analyse the range of descriptor values obtained by each method both within and between each OS feature type;
- evaluate classification performance of each method on all polygons through comparison with original data;

- investigate possible improvement in performance by evaluating strategies of combining methods; and
- evaluate performance of methods in detecting misclassified features in original data.

This report describes the results of this exercise. It contains the following sections:

1. The main tasks and aims of the project;
2. A description of the implementation and integration of the software modules for individual methods;
3. An evaluation of each method;
4. A comparison between methods;
5. Combination and selection of methods for optimal results;
6. Conclusions;
7. Suggestions for future research derived from the conclusions.

## **Chapter 2: Shape-based Classification**

Topographic data capture for large-scale maps (typically depicted at 1:1250 and 1:2500) consists of two parts: the digitisation of the geometry and the addition of attributes indicating the feature and/or object type being depicted. Whereas the former can be automated using image processing and similar techniques, the latter is often a manual task. One possible means of automation is object recognition through shape.

This project uses shape recognition techniques borrowed from the field of computer vision to describe a measurement of shape to characterise and classify features on maps. The main application of this work is the automatic structuring of topographic data for computer cartography and Geographical Information Systems (GIS).

Recognition of objects is largely based on the matching of description of shapes with a database of standard shapes. Numerous shape description techniques have been developed such as, Fourier descriptors, moment invariants and scalar features (area, number of points, etc.). Previous work has evaluated these techniques on topographic objects as depicted in large-scale mapping. Unlike many applications where the shape categories are very exact (for example, identifying a particular type of aircraft in a scene), this problem requires the classification of a particular shape into a general class of similar object shapes, for example, building, road or parcel. Each technique proved partially successful in distinguishing classes of object although no one technique provided a general solution to the problem. As part of this report these techniques are further evaluated on a real-world problem using a corpus of topographic data provided by Ordnance Survey in Great Britain (OS GB). The data set consists of the features codes (object types) used on the large-scale OS GB topographic database.

This report builds on previous work carried out to produce an accurate combined methodology for the classification of general shapes on maps. The following sections introduce each of the above named shape recognition techniques individually and describe how they are applied as general classifiers to broad classes of topographic shape (buildings, fields and road etc.). The overall implementation of the project and experiment is outlined and sets out the most significant aims of the report. An

evaluation and comparison is made of the effectiveness of each technique in recognising features and objects. A data fusion technique is then proposed and evaluated. This allows the combining of the results of the Fourier descriptor, moment invariants and scalar descriptor techniques respectively, to give an overall score for each candidate object category. The purpose of this report is to draw from our results the main conclusions and see if they are applicable to OS.

The recognition and description of objects plays a central role in automatic shape analysis for computer vision and it is one of the most familiar and fundamental problems in pattern recognition. Common examples are the reading of alphabetic characters in text and the automatic identification of aircraft. Most applications using Fourier descriptors, moment invariants and scalar descriptors for shape recognition deal with the classification of such definite shapes. To identify topographic objects each of the techniques needs to be extended to deal with general categories of shapes, for example houses, parcels and roads.

The data used for the experiments described in the following sections was extracted from vector data sets representing large-scale (1:1250) plans of the Purbeck and Plymouth areas in Great Britain (Ordnance Survey). The data had been pre-processed to extract minimal closed polygons and OS feature codes had been applied. An interpolation method was applied to sample the shape boundary at a finite number (N) of equidistant points. These points are then stored in the appropriate format for processing with each shape description technique. The shapes can then be described using a small set of descriptor values (typically 7 to 10 real numbers). The recognition is based on matching the descriptors of each shape to standard values representing typical shapes and choosing the closest match.

## **2.1 Fourier Descriptors**

### **2.1.1 Background**

Fourier transform theory (Gonzalez and Wintz 1977) has played a major role in image processing for many years. It is a commonly used tool in all types of signal processing and is defined both for one and two-dimensional functions. In the scope of this paper,



the Fourier transform technique is used for shape description in the form of Fourier descriptors. The Fourier descriptor is a widely used all-purpose shape description and recognition technique (Granlund 1972, Winstanley 1998). The shape descriptors generated from the Fourier coefficients numerically describe shapes and are normalised to make them independent of translation, scale and rotation. These Fourier descriptor values produced by the Fourier transformation of a given image represent the shape of the object in the frequency domain (Wallace and Wintz 1980). The lower frequency descriptors store the general information of the shape and the higher frequency the smaller details. Therefore, the lower frequency components of the Fourier descriptors define a rough shape of the original object

### 2.1.2 Theory

The Fourier transform theory can be applied in different ways for shape description. One method works on the change in orientation angle as the shape outline is traversed (Zahn and Roskies 1972), but for the purpose of this paper the following procedure was implemented (Wood 1986). The boundary of the image is treated as lying in the complex plane. So the row and column co-ordinates of each point on the boundary can be expressed as a complex number,  $x + jy$  where  $j$  is sqrt (-1). Tracing once around the boundary in the counter-clockwise direction at a constant speed yields a sequence of complex numbers, that is, a one-dimensional function over time. In order to represent traversal at a constant speed it is necessary to interpolate equi-distant points around the boundary. Traversing the boundary more than once results in a periodic function. The Fourier transform of a continuous function of a variable  $x$  is given by the equation:

$$F(u) = \int_{-\infty}^{\infty} f(u) e^{-j2\pi ux} dx \quad (1)$$

When dealing with discrete images the Discrete Fourier Transform (DFT) is used. So equation (1) transforms to:

$$F(u) = \left( \frac{1}{N} \right) \sum_{x=0}^{N-1} f(u) e^{\frac{-j2\pi x}{N}}$$

(2)

The variable  $x$  is complex, so by using the expansion  $e[-j A] = \cos (A) - j. \sin (A)$  where  $N$  is the number of equally spaced samples, equation (2) becomes:

$$F(u) = \left( \frac{1}{N} \right) \sum_{x=0}^{N-1} f(x + jy) \cdot (\cos(Ax) - j \cdot \sin(Ax)) \quad (3)$$

where  $A = 2\pi u/x$ .

The DFT of the sequence of complex numbers, obtained by the traversal of the object contour, gives the Fourier descriptor values of that shape.

The Fourier descriptor values can be normalised to make them independent of translation, scale and rotation of the original shape. Simply, translation of the shape by a complex quantity having  $x$  and  $y$  components, corresponds to adding a constant  $x + jy$  to each point representing the boundary. Scaling a shape is achieved by multiplying all co-ordinate values by a constant factor. The DFT results in all members of the corresponding Fourier series being multiplied by the same factor. So by dividing each coefficient by the same member, normalisation for size is achieved. Rotation normalisation is achieved by finding the two coefficients with largest magnitude and setting their phase angle equal to zero (Keyes and Winstanley 1999).

### 2.1.3 Fourier Descriptors of cartographic shapes

To apply the Fourier descriptor technique to cartographic data, the points are stored as a series of complex numbers and then processed using the Fourier transform resulting in another complex series of the same length  $N$ . If the formula for the discrete Fourier transform were directly applied each term would require  $N$  iterations to sum. As there are  $N$  terms to be calculated, the computation time would be proportional to  $N^2$ . So the algorithm chosen to compute the Fourier descriptors was the Fast Fourier Transform (FFT) for which the computation time is proportional to  $N \log N$ . The FFT algorithm requires the number of points  $N$  defining the shape to be a power of two. In the case of this project it was decided to use 512 sample points.

The FFT algorithm is applied to these 512 coefficients. The list is normalised for translation, rotation and scale. This results in the first two terms always having the values 0 and 1.0 respectively which makes them redundant for classification. Calculation of the Fourier Spectrum builds a new list and disposes of the Fourier transform list. The result is 510 Fourier descriptor terms.

The nature of the Fourier transform means that general shape information is modelled in the first few terms while the later terms reflect small detail. Therefore in shape classification, a limited number of terms are used. In this project, the first 16 terms are used in the evaluation.

## **2.2 Moment Invariants**

### **2.2.1 Background**

Moment Invariants have been frequently used as features for image processing, remote sensing, shape recognition and classification. Moments can provide characteristics of an object that uniquely represent its shape. Invariant shape recognition is performed by classification in the multidimensional moment invariant feature space. Several techniques have been developed that derive invariant features from moments for object recognition and representation. These techniques are distinguished by their moment definition, such as the type of data exploited and the method for deriving invariant values from the image moments.

It was Hu ( Hu, 1962), that first set out the mathematical foundation for two-dimensional moment invariants and demonstrated their applications to shape recognition. They were first applied to aircraft shapes and were shown to be quick and reliable (Dudani, Breeding and McGhee, 1977). These moment invariant values are invariant with respect to translation, scale and rotation of the shape.

Hu defines seven of these shape descriptor values computed from central moments through order three that are independent to object translation, scale and orientation.

Translation invariance is achieved by computing moments that are normalised with respect to the centre of gravity so that the centre of mass of the distribution is at the origin (central moments). Size invariant moments are derived from algebraic invariants but these can be shown to be the result of a simple size normalisation. From the second and third order values of the normalised central moments a set of seven invariant moments can be computed which are independent of rotation.

### 2.2.2 Theory

Traditionally, moment invariants are computed based on the information provided by both the shape boundary and its interior region (Hu 1962). The moments used to construct the moment invariants are defined in the continuous but for practical implementation they are computed in the discrete form. Given a function  $f(x,y)$ , these regular moments are defined by:

$$M_{pq} = \int \int x^p y^q f(x, y) dx dy \quad (4)$$

$M_{pq}$  is the two-dimensional moment of the function  $f(x,y)$ . The order of the moment is  $(p + q)$  where  $p$  and  $q$  are both natural numbers. For implementation in digital form this becomes:

$$M_{pq} = \sum_x \sum_y x^p y^q f(x, y) \quad (5)$$

To normalise for translation in the image plane, the image centroids are used to define the central moments. The co-ordinates of the centre of gravity of the image are calculated using equation (5) and are given by:

$$\bar{x} = \frac{M_{10}}{M_{00}} \quad \bar{y} = \frac{M_{01}}{M_{00}} \quad (6)$$

The central moments can then be defined in their discrete representation as:

$$\mu_{pq} = \sum_x \sum_y (x - \bar{x})^p (y - \bar{y})^q$$

(7)

The moments are further normalised for the effects of change of scale using the following formula:

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^\gamma}$$

(8)

Where the normalisation factor:  $\gamma = (p + q / 2) + 1$ . From the normalised central moments a set of seven values can be calculated and are defined by:

$$\begin{aligned} \phi_1 &= \eta_{20} + \eta_{02} \\ \phi_2 &= (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2 \\ \phi_3 &= (\eta_{30} - 3\eta_{12})^2 + (\eta_{03} - 3\eta_{21})^2 \\ \phi_4 &= (\eta_{30} + \eta_{12})^2 + (\eta_{03} + \eta_{21})^2 \\ \phi_5 &= (3\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 \\ &\quad - 3(\eta_{21} + \eta_{03})^2] + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03}) \\ &\quad \times [3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \\ \phi_6 &= (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \\ &\quad + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03}) \\ \phi_7 &= (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 \\ &\quad - 3(\eta_{21} + \eta_{03})^2] + (3\eta_{12} - \eta_{30})(\eta_{21} + \eta_{03}) \\ &\quad \times [3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{30})^2] \end{aligned} \quad (9)$$

These seven invariant moments,  $\phi_i$ ,  $1 \leq i \leq 7$ , set out by Hu, were additionally shown to be independent of rotation. However they are computed over the shape boundary and its interior region and so are not easily derived from vector graphics.

### 2.2.3 New moments

For the purpose of this project, an algorithm was implemented that calculates the moment invariants using the shape boundary alone. These can be proven to be invariant under object translation, scale and rotation (Chaur-Chin Chen 1993). Then, using the same notation for convenience, the moment definition in equation (4) can be expressed as:

$$M_{pq} = \int_C x^p y^q ds \quad (10)$$

For  $p, q = 0, 1, 2, 3$ , where  $\int_C$  is the line integral along the curve  $C$  and  $ds = \sqrt{(dx)^2 + (dy)^2}$ . The central moments can be similarly defined as:

$$\mu_{pq} = \int_C (x - \bar{x})^p (y - \bar{y})^q ds \quad (11)$$

Given that the centroids are as in the original method:

$$\bar{x} = \frac{M_{10}}{M_{00}} \quad \bar{y} = \frac{M_{01}}{M_{00}} \quad (12)$$

for a digital image, then equation (11) becomes

$$\mu_{pq} = \sum_{(X,Y) \in C} (x - \bar{x})^p (y - \bar{y})^q \quad (13)$$

Thus the central moments are invariant to translation. These new central moments can also be normalised such that they are scaling invariant.

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^\gamma} \quad (14)$$

where the normalisation factor is:  $\gamma = p + q + 1$ . The seven moment invariant values can then be calculated as before using the results obtained from the computation of equation's (10) to (14) above.

Using the same data sets as in the Fourier descriptor method described earlier, the moments technique is applied. However, for moments the points extracted from the map are stored not as complex numbers but represent the x and y co-ordinates of the polygonal shape. These points are processed by a moment transformation on the outline of the shape, which produces seven moment invariant values that are normalised with respect to translation, scale and rotation using the formulae above. The resulting set of values can be used to discriminate between the shapes.

### **2.3 Scalar Descriptors**

Scalar descriptors are based on scalar features derived from the boundary of an object. They use numerous metrics of the object as shape descriptors. Simple examples of such features include:

- the perimeter length;
- the area of the shape;
- the elongation i.e. ratio of the area of a shape to the square of the length of its perimeter ( $A/P^2$ );
- the number of nodes (junctions) in the boundary;
- the number of (sharp) corners.

Many other scalar descriptors can be devised.

## Chapter 3: Classification

### 3.1 Supervised v Unsupervised Classification

Shape description techniques, such as those described in chapter two, generally characterise an object's shape as a set of real numbers. Classification of objects based on shape therefore consists of comparing these descriptors. Two general forms of classification are possible: unsupervised and supervised.

Unsupervised learning occurs where the distribution of descriptor values of objects in a data-set is analysed. Clusters of objects of similar shape are identified. These are assumed to represent a class of similar objects. In this scheme, the classes identified emerge from the analysis of the data-set and can depend both on that analysis and the data-set in use.

Supervised learning occurs when the classes to which objects are to be assigned are decided beforehand. Values of descriptors that characterise each object class are determined in some way and objects are classified through the similarity of their descriptors to these characteristic values. Supervised learning therefore requires a way to determine some norms for the values of a particular class and a way to measure whether the descriptor values of an unclassified object belong to the group defined by those norms.

A common method to determine the norms for a class is to take a sample of shapes we know to belong to that class and calculate the mean or median values for each descriptor. In addition, a measure of the distribution of values within the sample can be made. Classification then consists to comparing the values of its descriptors with that of the mean, possibly taking into account the distribution for the class.

Given two sets of descriptors, how do we measure their degree of similarity? If two shapes, A and B, produce a set of values represented by  $a(i)$  and  $b(i)$  then the distance between them can be given as  $c(i) = a(i) - b(i)$ . If  $a(i)$  and  $b(i)$  are identical then  $c(i)$



will be zero. If they are different then the magnitudes of the coefficients in  $c(i)$  will give a reasonable measure of the difference. It proves more convenient to have one value to represent this rather than the set of values that make up  $c(i)$ . The easiest way is to treat  $c(i)$  as a vector in a multi-dimensional space, in which case its length, which represents the distance between the planes, is given by the square root of the sum of the squares of the elements of  $c(i)$ . In this way classification can be performed by choosing the class mean that is closest to the shape to be classified.

Earlier work on this project used this distance measure in classification with some limited success (Keyes and Winstanley 1999, 2000). However, this method takes no account of the distribution of descriptor values for each class. Therefore it was decided to incorporate the information given by the distribution using Bayesian statistics.

### **3.2 Classification using Bayes Theorem**

Bayesian statistics allows us to use the distribution of the values for each descriptor for each class of object in determining the probability that a particular object belongs to that class. Given a particular value for a descriptor, we can calculate the *likelihood* of that value occurring in the distribution of values for a particular class. Applying Bayes theorem, we can calculate from this the probability of the object belongs to that class. We can calculate such a probability for each class. We then decide that the object belongs to the class for which it that descriptor gives the highest probability.

The objective is to design classifiers that will classify an object in the most probable of the classes given. For example, in the experiment described later in this report, our classification task has six classes, Buildings, Defined Natural Land Cover, Multiple Surface Land, General Unmade Land, Made Road and Road Side,  $\omega_1, \dots, \omega_6$  respectively, and an unknown feature type taken from the data-set ( for example a building) represented by the feature vector  $x$ . From this the conditional or posteriori probabilities  $P(\omega_i | x), i = 1, 2, \dots, 6$  can be formed which represent the probability that the unknown feature type belongs to the respective class  $\omega_i$  given that the

corresponding feature vector takes on the value  $x$ . To calculate the posteriori probabilities, Bayes decision theory principles are applied.

The first step involves the calculation of the prior probabilities  $P(\omega_i)$  for each class. Take for example the Building class  $\omega_1$  and Defined Natural Land Cover (Defined Land) class  $\omega_2$ . Then,  $P(\omega_1)$  and  $P(\omega_2)$  denote the probabilities of a feature type belonging to either class  $\omega_1$  or  $\omega_2$  respectively before we have considered any descriptor values. As we have a previously classified data-set, we can estimate this *priori* probabilities as:

$$P(\omega_1) = \frac{\text{NumberOfBuildings}}{\text{TotalNumberOfFeatures}}$$

$$P(\omega_2) = \frac{\text{NumberOfDefindLAnd}}{\text{TotalNumberOfFeatures}}$$

Given these probabilities  $P(\omega_1)$  and  $P(\omega_2)$  the first criterion for deciding whether an observed feature type is of type Building or Defined Land would simply be to take the class with the larger probability, which can be written as:

$$\text{if } P(\omega_1) \geq P(\omega_2) \text{ then } \omega_1$$

$$\text{if } P(\omega_1) < P(\omega_2) \text{ then } \omega_2$$

Better probability results can generally be obtained by considering additional information about the features such as the mean and standard deviation of each class. Let this additional information be identified by the descriptor vector  $\bar{x}$  (using feature vector to represent more than one single measured feature). Using this information the conditional probabilities  $P(\omega_i | \bar{x})$  discussed earlier can be formed. The classification criterion can now be described as:

$$\text{if } P(\omega_1 | \bar{x}) > P(\omega_2 | \bar{x}) \text{ then decide } \omega_1$$

and

$$\text{if } P(\omega_2 | \bar{x}) > P(\omega_1 | \bar{x}) \text{ then decide } \omega_2$$

Bayes laws can be applied to these conditional probabilities to redefine them in terms of their density functions, which are denoted by  $f(\bar{x}|\omega_1)$  and  $f(\bar{x}|\omega_2)$ . The derivation of the new classification criterion, now in terms of the conditional density functions  $f(\bar{x}|\omega_1)$  and  $f(\bar{x}|\omega_2)$  states that

$$P(\omega_i | \bar{x}) = \frac{f(\bar{x} | \omega_i) P(\omega_i)}{\sum_{k=1}^2 f(\bar{x} | \omega_k) P(\omega_k)}$$

So equation above can be rewritten as:

$$\text{if } \frac{P(\omega_1) f(\bar{x} | \omega_1)}{\sum_{k=1}^2 f(\bar{x} | \omega_k) P(\omega_k)} \geq \frac{P(\omega_2) f(\bar{x} | \omega_2)}{\sum_{k=1}^2 f(\bar{x} | \omega_k) P(\omega_k)} \text{ then } \omega_1$$

$$\text{if } \frac{P(\omega_1) f(\bar{x} | \omega_1)}{\sum_{k=1}^2 f(\bar{x} | \omega_k) P(\omega_k)} < \frac{P(\omega_2) f(\bar{x} | \omega_2)}{\sum_{k=1}^2 f(\bar{x} | \omega_k) P(\omega_k)} \text{ then } \omega_2$$

Bayes decision rule is obtained by eliminating the denominator and is as follows:

$$\text{if } P(\omega_1) f(\bar{x} | \omega_1) \geq P(\omega_2) f(\bar{x} | \omega_2) \text{ then } \omega_1$$

$$\text{if } P(\omega_1) f(\bar{x} | \omega_1) < P(\omega_2) f(\bar{x} | \omega_2) \text{ then } \omega_2 \quad ?$$

$$L(\bar{x}) = \frac{f(\bar{x} | \omega_2)}{f(\bar{x} | \omega_1)}$$

$$T = \frac{P(\omega_1)}{P(\omega_2)}$$

From the conditional density functions a likelihood ratio  $L(h)$  and threshold  $T$  can be obtained. Using these functions the above criterion now be expressed as:

$$T \geq L(\bar{x}) \text{ then } \omega_1$$

$$T < L(\bar{x}) \text{ then } \omega_2$$

which reads *if*  $T \geq L(\bar{x})$  then decide  $\omega_1$  *else* decide  $\omega_2$ .

This criterion can be generalised quite easily to situations involving more than two classes and multiple dimensional feature spaces. So, let  $k$  be the number of classes involved in this project which equals six and using the respective conditional density functions  $f(\bar{x} | \omega_i)$  the Bayesian classification can now be written as follows:

$$\text{if } f(\bar{x} | \omega_i)P(\omega_k) = \underset{k=1,k}{\text{Max}}\{f(\bar{x} | \omega_k)P(\omega_k)\} \text{ then select } \omega_i$$

### 3.3 Implementing Bayesian Classification

Applying Bayes Theorem to classification therefore requires:

- the calculation of prior probabilities of each class occurring
- the modelling of a distribution function of the likelihoods of values occurring for each class

Both of these were estimated through an analysis of the classification of a data-set provided by Ordnance Survey. The distribution function for each descriptor was approximated as a normal curve, modelled from the means and standard deviations calculated from the data-set.

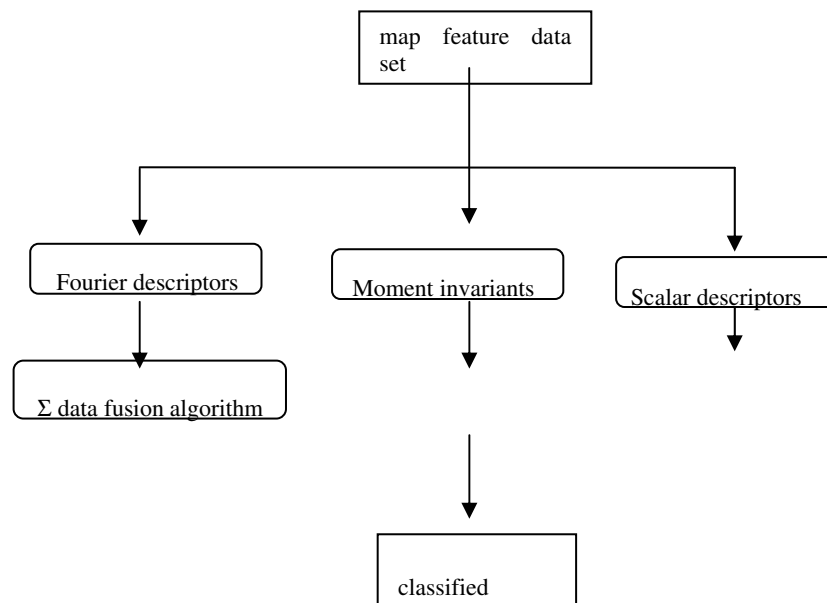
Using Bayesian classification a class can be assigned to each object based on the value of one descriptor. This is accompanied by a probability estimate that the classification is correct. We are evaluating three shape description methods, each containing several descriptors (25 descriptors in all). If, as is likely, these disagree as to the classification, we require a method of combining them to produce an overall consensus as to the correct classification.

## Chapter 4: Combining Classifiers

When setting out to design a shape recognition system the ultimate goal is to achieve the best possible classification performance. Attaining this goal involves the application of suitable classification schemes/techniques to the problem. Traditionally an analysis of the results produced by each technique became the basis for choosing one of the classifiers as a final solution. However, it has been observed in many studies that although one technique would yield the best performance, the set of shapes miss-classified by the different classifiers would not necessarily overlap. This suggests that different classifier techniques can offer complementary descriptions of the shapes to be classified, which leads to the combining of the classifiers for improved performance.

### 4.1 The Fusion Model

Using and combining multiple learned classification models for increasing accuracy and efficiency is an area attracting much interest recently. The central problem involved is how to integrate several classifiers (or “experts”) to produce a single final classification. Figure 1, illustrates the decision combination topology used in this report.



**Figure 1, Decision combination topology used for fusing the results of three shape recognition methods.**

The approach taken here to the fusion of the recognition techniques used follows a classifier combination scheme developed by Kittler et al[1998].

## 4.2 Theory

The fusion of individual classifiers is based on a theoretical framework set out in Bayes theorem. Before fusion can take place probabilities must be assigned or calculated indicating the likelihood that a particular object belongs to each available class.

Considering the classification problem, where  $Z$  is to be assigned to one of  $m$  possible classes  $(\omega_1, \dots, \omega_m)$ , assume there are  $R$  classifiers each representing the given pattern by a distinct measurement vector, the measurement vector used by the  $i$ th classifier being denoted by  $x_i$ . In the measurement space each class  $\omega_k$  is modelled by the probability density function  $p(x_i | \omega_k)$  and its priori probability of occurrence is denoted  $P(\omega_k)$ . The models are considered to be mutually exclusive which means that only one class can be associated with each object. According to Bayes theorem, given measurements  $x_i, i = 1, \dots, R$ , the pattern,  $Z$ , should be assigned to the class  $\omega_j$  provided the a posteriori probability of the interpretation is maximum, i.e.

$$\begin{aligned} \text{assign } Z \rightarrow \omega_j \quad \text{if} \\ P(\omega_j | x_1, \dots, x_R) = \max_k P(\omega_k | x_1, \dots, x_R) \end{aligned} \tag{15}$$

The Bayesian decision rule (15) states that in order to use all the available information correctly to reach a decision, it is essential to compute the probabilities of the various hypotheses by considering all the measurements simultaneously. This is however a very expensive computation therefore rule (15) is simplified and expressed in terms of decision support computations performed by the individual classifiers, each exploiting only the information conveyed by the vector  $x_i$ . Rewriting the a posteriori probability  $P(\omega_k | x_1, \dots, x_R)$  using Bayes theorem we have:

$$P(\omega_k | x_1, \dots, x_R) = \frac{p(x_1, \dots, x_R | \omega_k)P(\omega_k)}{p(x_1, \dots, x_R)} \quad (16)$$

where  $p(x_1, \dots, x_R)$  is the unconditional measurement joint probability density. This can be expressed in terms of the conditional measurement distributions as follows:

$$p(x_1, \dots, x_R) = \sum_{j=1}^m p(x_1, \dots, x_R | \omega_j)P(\omega_j) \quad (17)$$

Therefore, in the following placing the concentration only on the numerator terms of (16).

#### 4.2.1 The Product Rule

The measurement  $p(x_1, \dots, x_R | \omega_k)$  represents the joint probability distribution of the descriptor values extracted by the classifiers. Treating the representations used as conditionally statistically independent (as outlined by Kittler et al ), the following can be obtained,

$$p(x_1, \dots, x_R | \omega_k) = \prod_{i=1}^R p(x_i | \omega_k) \quad (18)$$

where  $p(x_i | \omega_k)$  is the measurement model of the  $i$ th representation. Substituting from (18) and (17) into (16) gives:

$$P(\omega_k | x_1, \dots, x_R) = \frac{P(\omega_k) \prod_{i=1}^R p(x_i | \omega_k)}{\sum_j^m P(\omega_j) \prod_{i=1}^R p(x_i | \omega_j)} \quad (19)$$

and using (19) in (15) gives the following decision rule.

*assign*  $Z \rightarrow \omega_j$  *if*

$$P(\omega_j) \prod_{i=1}^R p(x_i | \omega_j) = \max_{k=1}^m P(\omega_k) \prod_{i=1}^R p(x_i | \omega_k)$$

(20)

Putting this in terms of the a posteriori probabilities yielded by the respective classifiers:

*assign*  $Z \rightarrow \omega_j$  *if*

$$P^{-(R-1)}(\omega_j) \prod_{i=1}^R p(\omega_k | x_i) = \max_{k=1}^m P^{-(R-1)}(\omega_k) \prod_{i=1}^R p(\omega_k | x_i) \quad (21)$$

The decision rule in (21) quantifies the likelihood of a hypothesis by combining the a posteriori probabilities produced by the individual classifiers by means of a product rule. It can be a severe rule of combining the classifier outputs as a single descriptor to inhibit a particular interpretation by outputting a close to zero probability for it.

#### 4.2.2 Sum Rule

Kittler et al [1998] developed a scheme for the fusion of individual classifiers called the *sum rule* which he based on the above theoretical framework. Considering the decision rule in (21) and based on the assumption that the a posteriori probabilities computed by the respective classifiers will not deviate dramatically from the prior probabilities, the posteriori probabilities can then be expressed as:

$$P(\omega_k | x_i) = P(\omega_k)(1 + \delta_{ki}) \quad (22)$$

where  $\delta_{ki}$  satisfies  $\delta_{ki} \ll 1$ . Substituting (22) for the posteriori probabilities in (21) gives:

$$P^{-(R-1)}(\omega_j) \prod_{i=1}^R P(\omega_k | x_i) = P(\omega_k) \prod_{i=1}^R (1 + \delta_{ki}) \quad (23)$$

Expanding the product and neglecting any terms of second and higher order, we can approximate the right hand side of (23) as:

$$P(\omega_k) \prod_{i=1}^R (1 + \delta_{ki}) = P(\omega_k) + P(\omega_k) \sum_{i=1}^R \delta_{ki} \quad (24)$$



By substituting (24) and (22) into (21) the sum decision rule is obtained.

$$\begin{aligned} & \text{assign } Z \rightarrow \omega_j \text{ if} \\ & (1-R)P(\omega_j) + \sum_{i=1}^R P(\omega_k | x_i) = \max_{k=1}^m [(1-R)P(\omega_k) + \sum_{i=1}^R P(\omega_k | x_i)] \end{aligned} \quad (25)$$

The sum of the classifiers  $R$  is obtained for each class  $\omega_k$  and the likelihood class computed by taking the maximum a posteriori probabilities produced by the sum combination scheme.

### 4.3 Classifier Combination

The product and sum decision rules in (21) and (25) form the basic schemes for classifier combination. Many combination strategies can be developed from these rules by noting that:

$$\prod_{i=1}^R P(\omega_k | x_i) \leq \min_{i=1}^R P(\omega_k | x_i) \leq \frac{1}{R} \sum_{i=1}^R P(\omega_k | x_i) \leq \max_{i=1}^R P(\omega_k | x_i) \quad (26)$$

This shows that the product and sum rules can be approximated by the upper or lower bounds suggested by (26), as appropriate. Also the hardening of the a posteriori probabilities  $P(\omega_k | x_i)$  to produce binary valued functions  $\Delta_{ki}$  as

$$\Delta_{ki} = \begin{cases} 1 & \text{if } P(\omega_k | x_i) = \max_{i=1}^R P(\omega_j | x_i) \\ 0 & \text{otherwise} \end{cases} \quad (27)$$

result in the combining of a decision outcome rather than just the combining of posteriori probabilities. From these approximations the following rules can be constructed. All the combination schemes and their relationship are represented in Figure 2.

### 4.3.1 Majority Vote Rule

Using the sum rule from (25) and the above assumption of equal priors and by hardening the probabilities according to (27) gives:

$$\begin{aligned} & \text{assign } Z \rightarrow \omega_j \text{ if} \\ & \sum_{i=1}^R \Delta_{ji} = \max_{k=1}^m \sum_{i=1}^R \Delta_{ki} \end{aligned} \quad (28)$$

When calculating the majority vote rule for each class  $\omega_k$ , the sum on the right hand side counts the votes received for the individual classifiers. The class, which receives the largest number of votes, is selected as the majority decision and the final single classification.

### 4.3.2 Min Rule

Starting with the product rule in (21) and bounding the product of posteriori probabilities from the above we obtain

$$\begin{aligned} & \text{assign } Z \rightarrow \omega_j \text{ if} \\ & P^{-(R-1)}(\omega_j) \min_{i=1}^R P(\omega_j | x_i) = \max_{k=1}^m P^{-(R-1)}(\omega_k) \min_{i=1}^R P(\omega_k | x_i) \end{aligned} \quad (29)$$

which under the assumption of equal priors simplifies to

$$\begin{aligned} & \text{assign } Z \rightarrow \omega_j \text{ if} \\ & \min_{i=1}^R P(\omega_j | x_i) = \max_{k=1}^m \min_{i=1}^R P(\omega_k | x_i) \end{aligned} \quad (30)$$

The min rule combination scheme quantifies the likelihood of a given shape belonging to a particular class by determining the minimum a posteriori probability for each class  $\omega_k$ . The final decision is then based on the maximum of the obtained minimum probabilities for each individual classifier.

### 4.3.3 Max Rule

Starting from the sum rule in (25) and approximating the sum by the maximum of the posteriori probabilities gives

$$\text{assign } Z \rightarrow \omega_j \text{ if}$$

$$(1 - R)P(\omega_j) + R \max_{i=1}^R P(\omega_j | x_i) = \max_{k=1}^m [(1 - R)P(\omega_k) + R \max_{i=1}^R P(\omega_k | x_i)] \quad (31)$$

which, under the assumption of equal priors, simplifies to

$$\begin{aligned} & \text{assign } Z \rightarrow \omega_j \text{ if} \\ & \max_{i=1}^R P(\omega_j | x_i) = \max_{k=1}^m \max_{i=1}^R P(\omega_k | x_i) \end{aligned} \quad (32)$$

This strategy obtains a decision by computing the maximum posteriori probability for each class and then taking the maximum of these values.

#### 4.3.4 Median Rule

Under the assumption of equal priors the sum rule in (25) can be viewed to be computing the average a posteriori probability for each class over all the classifier outputs,

$$\begin{aligned} & \text{assign } Z \rightarrow \omega_j \text{ if} \\ & \frac{1}{R} \sum_{i=1}^R P(\omega_j | x_i) = \max_{k=1}^m \frac{1}{R} \sum_{i=1}^R P(\omega_k | x_i) \end{aligned} \quad (33)$$

That is, the rule assigns a shape to the class in which the average posteriori probability is maximum. However it is possible that a classifier might output an a posteriori probability for some class which is an outlier. Such an output would affect the average, which could lead to an incorrect decision. Another robust method for finding the mean is the median. The following rule bases the combined decision on the median of the posteriori probabilities.

$$\begin{aligned} & \text{assign } Z \rightarrow \omega_j \text{ if} \\ & \text{med}_{i=1}^R P(\omega_j | x_i) = \max_{k=1}^m \text{med}_{i=1}^R P(\omega_k | x_i) \end{aligned} \quad (34)$$

#### **4.4 Implementing Data Fusion**

All the methods of data fusion described above were implemented. Each shape was classified by individual descriptors with accompanying measure of certainty or confidence. These were then fused in each of 8 methods and then the resulting classifications measured against the known classes the object belonged to.

## Chapter 5: Experimental Results and Conclusions

Two data sets were provided by Ordnance Survey to evaluate shape classification on topographic data: Purbeck and Plymouth. Because supervised learning was being used, it was necessary to have an example data set to derive the statistics to provide the likelihood distributions for each descriptor for each class of object. It was decided to use the Purbeck data set for this purpose.

The Purbeck data for all the polygons representing six of the most common feature types (Table 1) were extracted. The four scalar descriptors for each polygon were calculated directly from these boundaries. Each boundary was sampled at 128 points and this sampled boundary used to calculate the Fourier descriptors (FDs) and moment invariants (MI). The sampling results in 128 FDs but it is known that most of the shape information is contained in the first few. Therefore the first 16 were used with FD(0) and FD(1) being redundant due to normalisation.

	<b>Label used here</b>	<b>OS Type</b>	<b>OS Description</b>
1	building	2210321	Building (Type A)
2	defined land	1900300	Defined Natural Land Cover
3	multiple surface land	2400339	Multiple Surface Land
4	unmade land	1400342	General Unmade Land
5	road	2610330	Made Road
6	roadside	2610331	Roadside Unknown Land

**Table 1: Object types used in classification experiment.**

Therefore for each polygon we have 25 descriptor values (four scalars, seven moment invariants and fourteen Fourier Descriptors (FD(2) to FD(16))). The values for each descriptor obtained by all polygons for each of the six chosen feature codes in the Purbeck data were statistically analysed to obtain measures of the mean and standard deviation. A normal distribution was assumed. These distributions were then used to

classify each polygon in both data sets using each individual descriptor (i.e. 25 results per polygon).

The individual results from individual descriptors for each polygon were then fused using each method described in chapter 4 producing an overall result treating all 25 descriptors equally. This was also done for each of the three descriptor types. Finally, the results for each descriptor type were then fused to produce an overall result from all descriptors.

Detailed results are tabulated in appendix 1 (Purbeck) and 2 (Plymouth). Table 2 shows the performance of the individual classifiers on the Plymouth data set. It can be seen that as expected performance was variable depending on descriptor(s) and fusion method used. Best performer was Moment Invariants fused using the Min rule (81% correctly classified). Poorest were most of the descriptors using the Sum rule adjusted (i.e. normalised) (2%) which is symptomatic of the theoretically weak basis for this method [Kittler 1998]. It is also note-worthy that fusing all methods using the techniques described here produces poorer performance than Moment Invariants alone.

7.3 Performance of fused descriptors over all selected features									
Number of polygons processed: 8837									
7.3.1 All 25 Descriptors									
ALL		majority	max	min	median	sum	sum adj	product	product adj
	number	5978	5974	7047	5992	5992	186	6938	6603
	percent	68	68	80	68	68	2	79	75
7.3.2 Scalar Descriptors									
SCALARS		majority	max	min	median	sum	sum adj	product	product adj
	number	6195	6010	6592	6223	6185	196	6381	6552
	percent	70	68	75	70	70	2	72	74
7.3.3 Fourier Descriptors									
FOURIERS		majority	max	min	median	sum	sum adj	product	product adj
	number	5940	5958	5789	5921	5949	175	5837	5815
	percent	67	67	66	67	67	2	66	66
7.3.4 Moment Invariants									
MOMENTS		majority	max	min	median	sum	sum adj	product	product adj
	number	6192	6051	7122	6281	6119	6026	7004	6973
	percent	70	68	81	71	69	68	79	79
7.3.5 Majority of 3 methods									
MAJORITY		majority	max	min	median	sum	sum adj	product	product adj
	number	6102	6025	6544	6134	6076	198	6294	6419
	percent	69	68	74	69	69	2	71	73

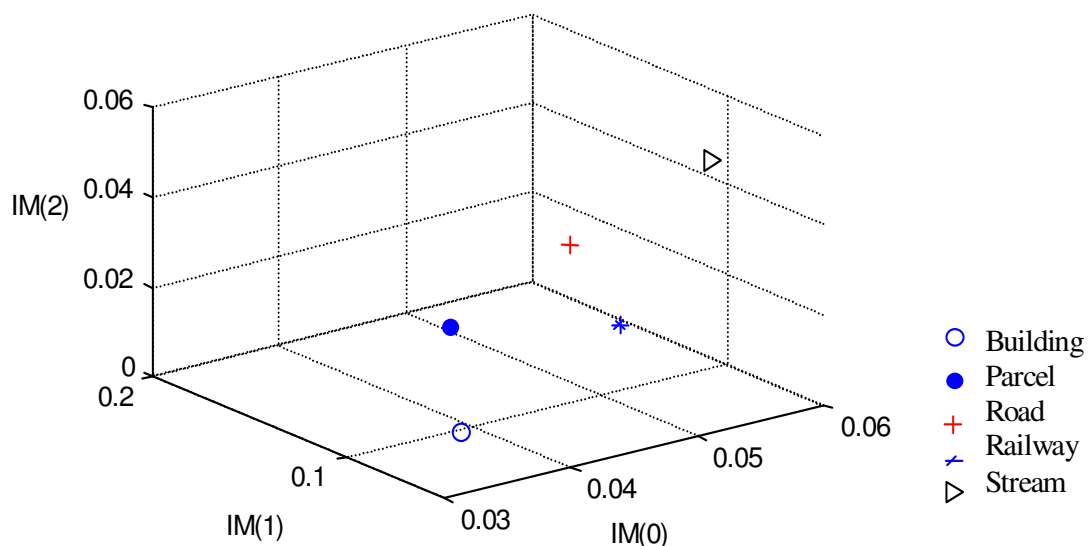
**Table 2:** Summary of performance of fusion of descriptors on all features in Plymouth data set showing number and percentage correctly classified.

The remainder of this chapter consists of the following sections:

1. An evaluation of each individual description methods (i.e. FDs, MIs and scalars);
2. A comparison between methods;
3. An evaluation of Fusion methods;
4. Conclusions;
5. Suggestions for future research derived from the conclusions.

### 5.1 Performance of individual descriptors

In this section a sample of the results produced by the application of the Fourier descriptor, moment invariants and scalar descriptor techniques are presented to evaluate and compare their usefulness in shape discrimination of general topographic features. Figure 3 plots the average values, obtained for five categories of objects from a sample data set (using the Moment Invariant method in this example). This shows some separation between classes in the n-descriptor space. However, in order to classify shapes with any degree of certainty, the variation within classes must in general be less than that between classes.



**Figure 3.** Average descriptor values of five sample shapes (Moment Invariants using Isle of Man data).

To evaluate each of the methods as shape recognition techniques, several shapes from the map (buildings, parcels and roads) were used as test shapes. For the Fourier descriptor and moment invariants methods, the descriptor values used to describe the objects are computed from the equally spaced (x,y) points along the boundary of each of the test shapes using the formulae derived in chapter 2. The scalar descriptors are calculated from the boundary of the objects also, using the scalar shape recognition aspects described in chapter 2. The aspects used are: area; perimeter length; elongation; and number of points. Table 3 is an example of the first 16 low-order Fourier descriptors obtained for a house shape, FD(0) represents the first descriptor value.

0	1.0000	0.0440	0.0415	0.0461	0.0283	0.0095	0.0050
0.0153	0.0013	0.0013	0.0067	0.0048	0.0006	0.0019	0.0043

**Table 3: Fourier descriptor values calculated for a house shape.**

From inspection of the values produced for each polygon, most of the shape information is described by the first few descriptors and so only the first 16 terms were used for comparison, remembering that due to the normalization procedures, FD(0) and FD(1) are redundant. Table 4 is an example of a set of seven invariant moments (IM) obtained for a house, road and parcel shape (starting a index IM(0)).

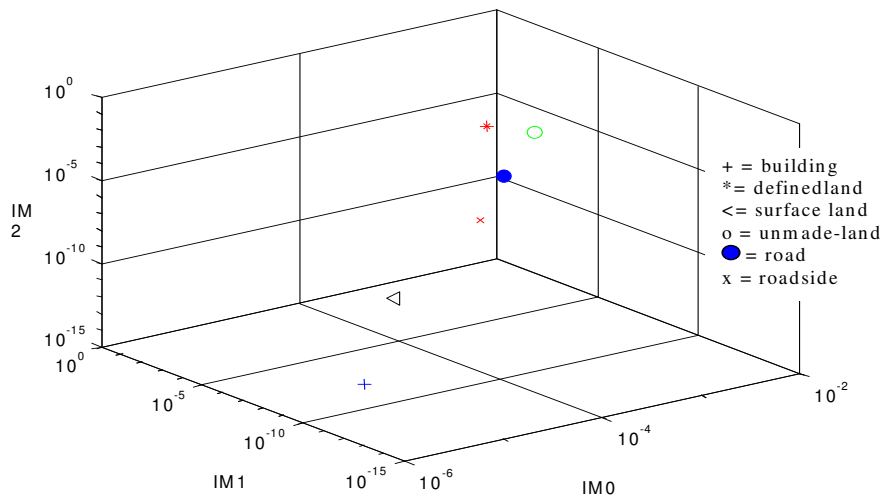
	<b>Buildings</b>	<b>Roads</b>	<b>Parcels</b>
<b>IM(0)</b>	0.00021913563	0.0191903068	0.19419031
<b>IM(1)</b>	1.4175713e-08	0.0028776518	0.0093515524
<b>IM(2)</b>	3.3163274e-12	0.0000022101	0.00055687797
<b>IM(3)</b>	7.332081e-14	0.0000002565	1.0685037e-05
<b>IM(4)</b>	2.4223892e-14	0.0000001930	5.696268e-05
<b>IM(5)</b>	-7.51903311e-18	-3.7718e-08	-6.2343667e-07
<b>IM(6)</b>	2.12921403e-26	-1.5393e-14	3.212549e-11

**Table 4: Moment invariant values calculated for house, road and parcel shapes.**

In this paper each of the shape description techniques, Fourier descriptors, moment invariants and scalar descriptors, were computed for three types of feature, namely buildings, parcels and roads in six different sub-categories used in Ordnance Survey large-scale data-sets (Table 1).

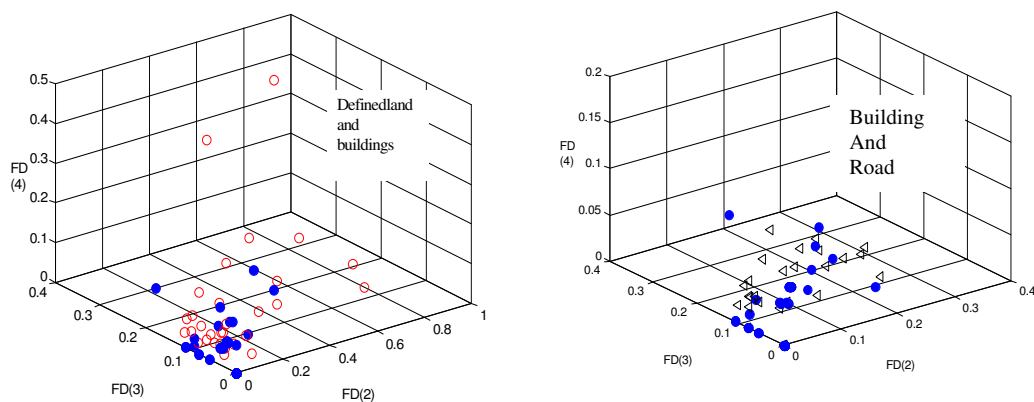


Figure 4, shows a plot of the mean values for each of these categories in three-dimensional space (using the moments invariants method in this example).



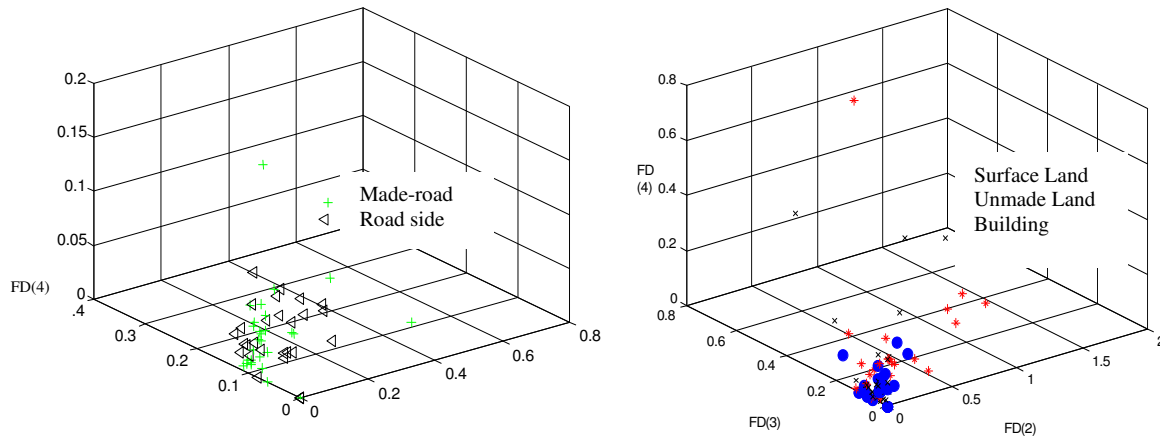
**Figure 4: Average moment invariants (IM) of six shape categories (Purbeck data)**

A sample of the results produced by the application of the Fourier descriptors is presented to evaluate their usefulness in the shape discrimination. These results obtained for each data set were plotted using the Fourier descriptors (FD(2), FD(3), FD(4)) to observe how well the formed separate groups. Figure 5 (a) and (b) and Figure 6 (a) and (b) below show the degree to which these data set cluster in FD(2), FD(3), FD(4) space. Note, that due to normalisation the first two terms obtained in the Fourier descriptors set, FD(0) = 0 and FD(1) = 1 are redundant.



**Figure 5 (a): Clustering of the polygon shapes, buildings and defined natural land cover in three-dimensional space of the features FD(2), FD(3) and FD(4), (b):**

**Cluster of the polygon shapes, *buildings and made-road* in three-dimensional space FD(2), FD(3) and FD(4)**



**Figure 6 (a): Clustering of the polygon shapes, *made-road and roadside* in three-dimensional space of the features FD(2), FD(3) and FD(4), (b): Clustering of the polygon shapes, *surface land, unmade-land and buildings* in three-dimensional space FD(2), FD(3) and FD(4).**

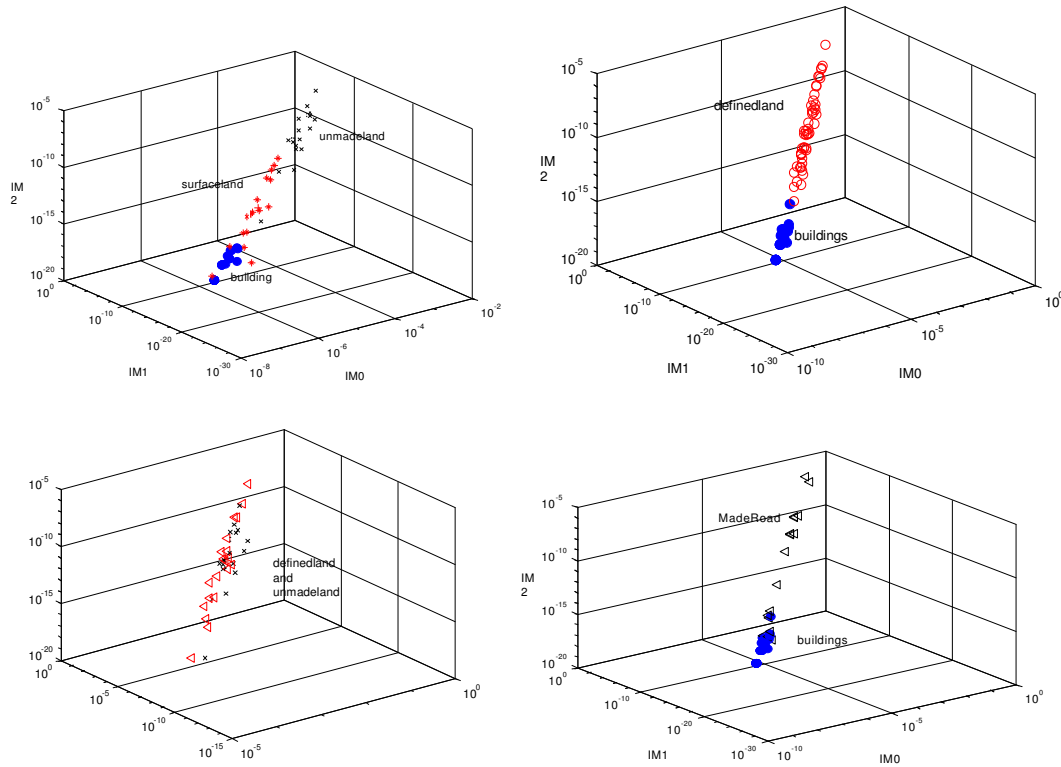
As these plots show, often no two feature classes are completely distinct from each other. This evidence therefore indicates that Fourier descriptors are not very good for use in shape description where the data sets are of a very general shape. To show this mathematically the repeatability function was computed for each of the six map categories. Table 5 shows these measurements in FD(2) as it is the most significant descriptor value. The repeatability of the measurements of each class is represented as three times the standard deviation and can be seen in the shaded diagonal column of the table. The repeatability of each class is sizeably larger than the distance between the mean values for all the six classes which shows that the classes are not distinct enough to conclude any significant positive results.

	<b>Buildings</b>	<b>Definedland</b>	<b>Surfaceland</b>	<b>Unmade-land</b>	<b>MadeRoad</b>	<b>Roadside</b>
<b>No. polygons</b>	7976	3147	3003	1251	487	458
<b>Buildings</b>	0.5166	0.0890	0.3590	0.1095	0.0495	0.0343
<b>Definedland</b>		1.1877	0.2700	0.0205	0.0395	0.0547
<b>Surfaceland</b>			1.7972	0.2495	0.3095	0.3247
<b>Unmade-land</b>				1.3156	0.0600	0.0752
<b>MadeRoad</b>					1.1323	0.0152
<b>Roadside</b>						0.7112

**Table 5: Comparison of repeatability within feature classes and distance between classes for the Fourier descriptor technique in FD(1).**

A sample of the results produced by the application of the moment invariants technique was also evaluated. The Figure 7 shows plots obtained for the moment

invariants technique for a sample of each feature type, each plot showing the degree to which each set of objects cluster in their three-dimensional space.



**Figure 7. Clustering of the polygon shapes, buildings and made-roads, in three-dimensional space of the features IM(0),IM(1) and IM(2).**

Figure 7 shows the degree to which the data sets, building and defined land cover cluster and also in a cluster plot of the data sets, defined land cover and unmade-land. In contrast, it can be seen how the features buildings and roads separate when plotted.

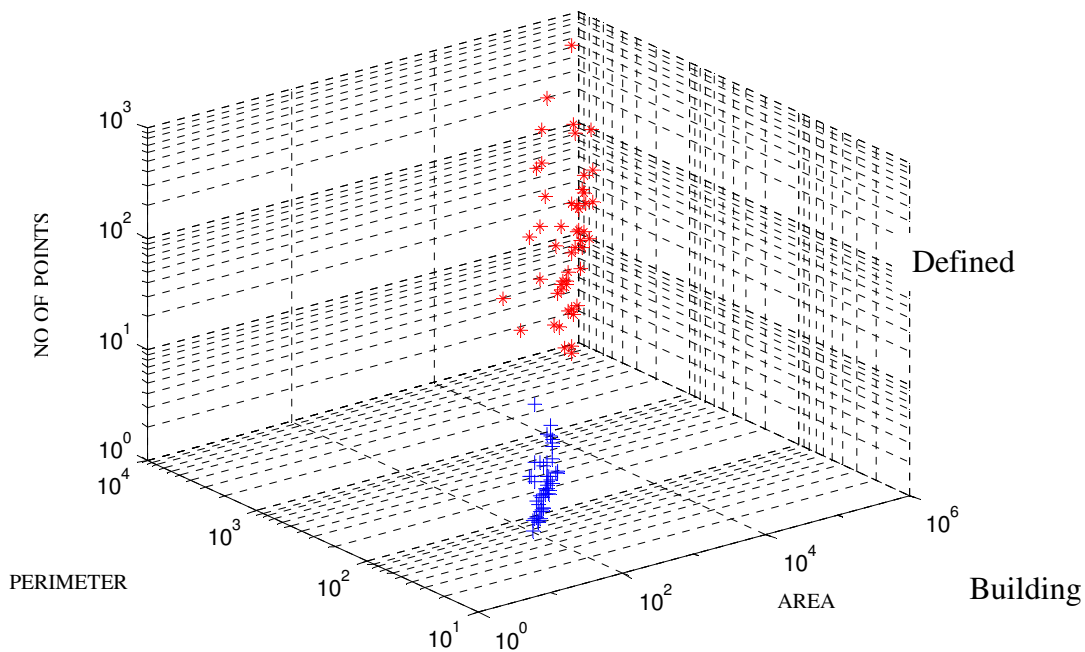
To measure the clustering obtained, the repeatability function and mean value measurements were computed for each set or the sample shapes. The results can be seen in table 6. Only the first moment invariants measure, IM(0) is used here to make it easier to read the table as it is the most significant moment result.

	<b>Buildings</b>	<b>Definedland</b>	<b>Surfaceland</b>	<b>Unmade-land</b>	<b>MadeRoad</b>	<b>Roadside</b>
<b>No. polygons</b>	7976	3147	3003	1251	487	458
<b>Buildings</b>	5.2005e-005	8.8572e-004	1.5488e-005	0.0034	0.0014	4.8116e-004
<b>Definedland</b>		0.0138	8.7023e-004	0.0025	5.5596e-004	4.0456e-004
<b>Surfaceland</b>			3.9330e-004	0.0033	0.0014	4.6567e-004
<b>Unmade-land</b>				0.0231	0.0019	0.0029
<b>MadeRoad</b>					0.0188	9.6051e-004
<b>Roadside</b>						0.0048

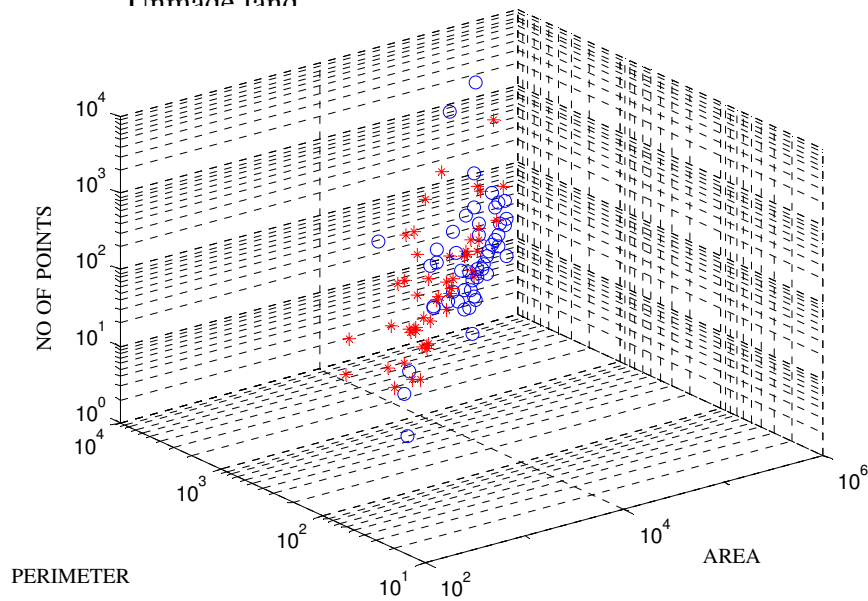
**Table 6: Comparison of repeatability within feature classes and distance between classes for the moment invariants technique in IM(0).**

Each output for the moment invariants method in the shape recognition of general shapes on maps, show that there is a significant separation occurring between most of the classes. Although overlap does exist (also seen by the human eye) good classification occurs. On examining Table 6 more closely it can be seen that the repeatability for the buildings is smaller than the distance between the mean values for all categories except for the surface land data set though these values are close. This is also true for the repeatability measure for the surface land class where the distance between the means values is larger except for buildings. Comparing the figures obtained for the other data sets we see that for many the repeatability measure is larger but still close to the mean distance for most cases.

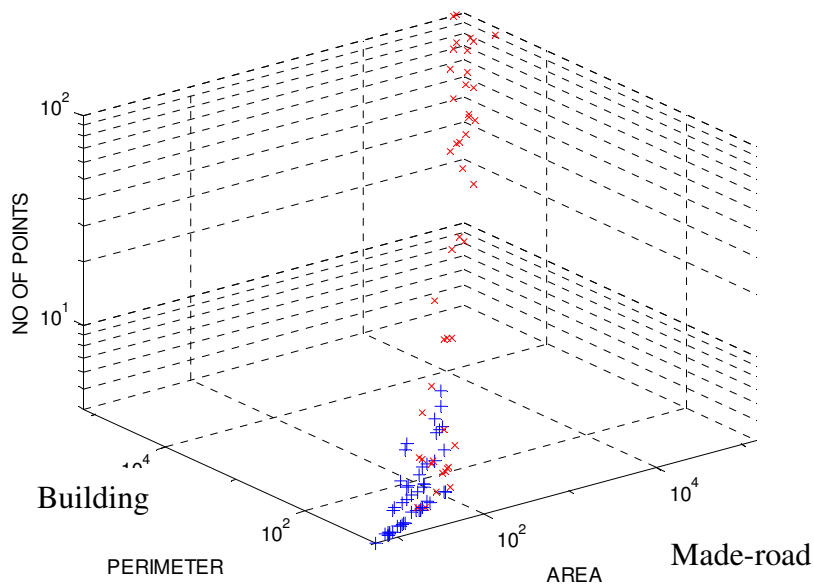
As presented above for the Fourier descriptor and moment invariants methods, a sample of the results produced by applying the scalar descriptor technique to the data set is evaluated also. Figures 8 to 11 show the resulting cluster graphs and the degree to which the features separate in the three-dimensional space of area, perimeter and number of points.



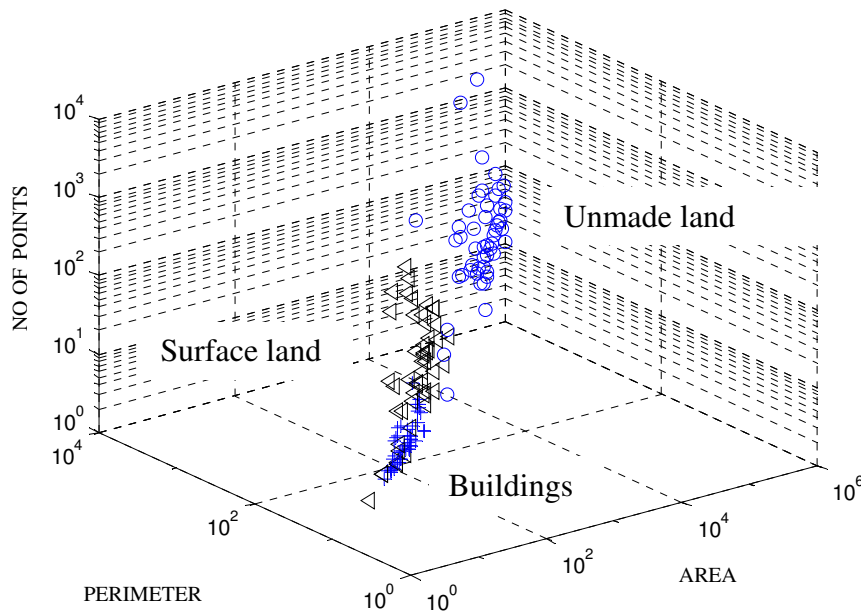
**Figure 8 Clustering of the polygons, buildings and defined land cover, in the three-dimensional space area, perimeter and number of points and Unmade land**



**Figure 9 Clustering of the polygons, defined land cover and unmade-land, in the three-dimensional space area, perimeter and number of points**



**Figure 10 Clustering of the polygons, buildings and made-road, in the three-dimensional space area, perimeter and number of points**



**Figure 11 Clustering of the polygons, *buildings*, *surface land* and *unmade land*, in the three-dimensional space area, perimeter and number of points**

Figure 8 shows the cluster plot of the data sets defined natural land cover and buildings. In Figure 9 a cluster plot of the features defined natural land cover and unmade land. Figure 10 and Figure 11 show the degree to which the data sets buildings and made-roads cluster and the degree to which the data sets buildings, surface land and unmade land cluster.

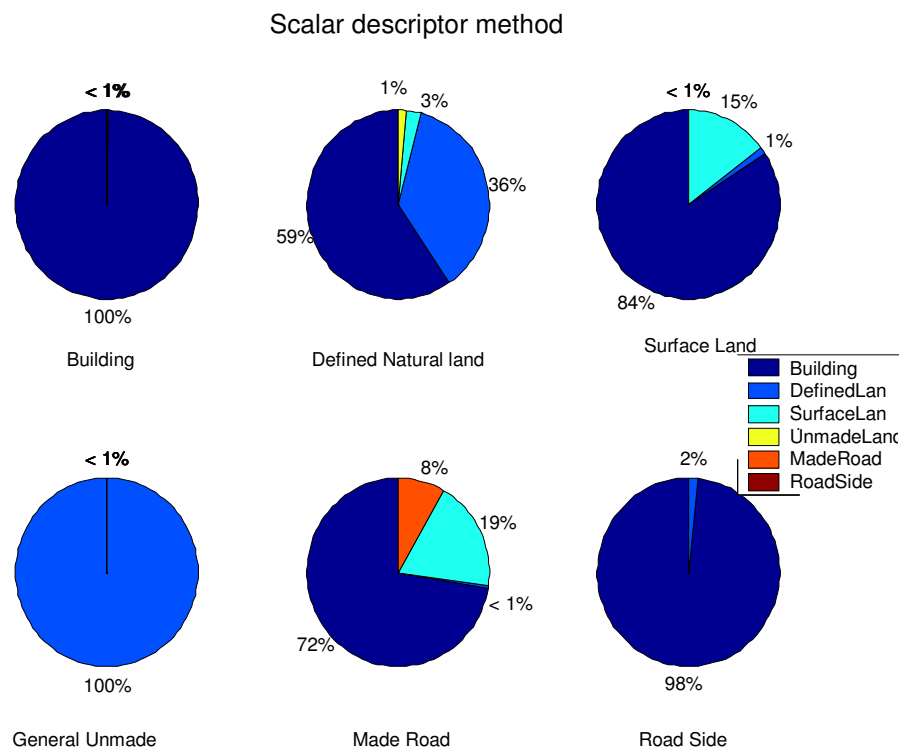
To analysis the results further the results are again represented mathematically, in this case by computing the repeatability function and mean value measurements for the *area*, which is considered the most significant feature descriptor for the scalars. The table for the repeatability and mean values is as follows:

	<b>Buildings</b>	<b>Definedland</b>	<b>Surfaceland</b>	<b>Unmade-land</b>	<b>MadeRoad</b>	<b>Roadside</b>
<b>No. polygons</b>	7976	3147	3003	1251	487	458
<b>Buildings</b>	962.3439	1.2793e+04	250.4747	3.8176e+04	1.0369e+03	255.5874
<b>Definedland</b>		1.0665e+05	1.2543e+04	2.5382e+04	1.1757e+04	1.2538e+04
<b>Surfaceland</b>			1.7478e+03	3.7925e+04	786.3982	5.1127
<b>Unmade-land</b>				1.1575e+05	3.7139e+04	3.7920e+04
<b>MadeRoad</b>					6.7577e+03	781.2856
<b>Roadside</b>						1.7528e+03

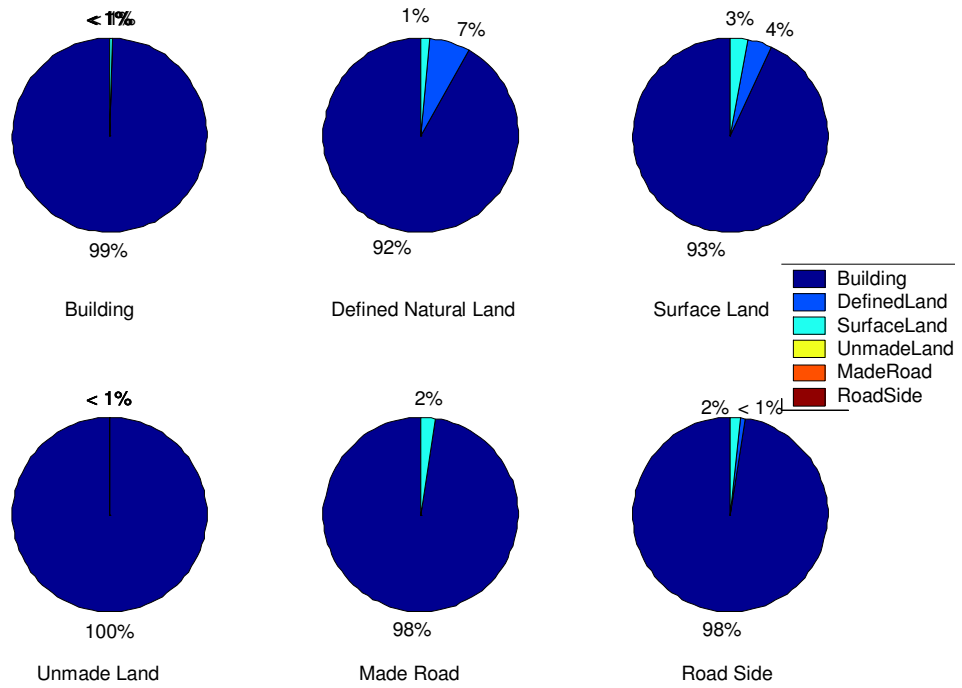
**Table 7: Comparison of repeatability within feature classes and distance between classes for the scalar descriptor technique in area.**

The outputs obtained for the scalar descriptor method of general shapes on maps show that there is a significant distinction between the majority of the classes. Some overlap exists but overall classification is good. On examination, table 7 shows, especially for the building features, that the repeatability is smaller than than the distance between the mean values which indicates good classification performance for the scalar method.

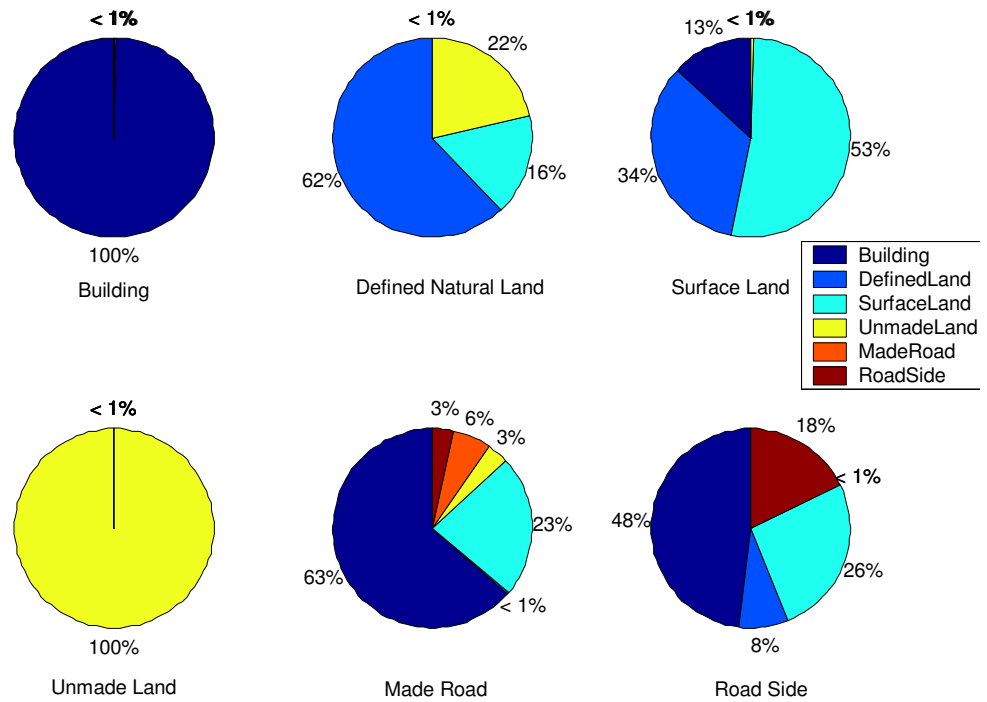
As shape descriptor techniques the evidence published to date is that all three techniques evaluated, Fourier descriptors, moment invariants and scalar descriptors, are very good features to use when dealing with very specific shapes such as a particular aircraft or alphanumeric character. On investigation of their usefulness for the shape description of general shapes on maps, for example houses, roads, parcels etc. the Fourier descriptors do not appear to be very successful. However, the moment invariants technique proved to be significantly more successful in its task and specific scalar measures are also very discriminatory. This is illustrated by the pie charts in Figure 12 derived from the results summary in Appendix 5. Each chart shows the classification results on objects belonging to each of the six feature types considered. For example, scalar descriptors correctly classified almost 100% of buildings.



### Fourier Descriptor method



### Moment Invariants method



**Figure 12 Recognition performance of descriptor methods by feature type.**



## 5.2 Fusion methods

Six methods of data fusion were implemented: majority vote, max rule, min rule, median rule, sum rule and product rule). Two of these (sum and product) had two versions whether they included or excluded the adjustment for normalisation. They were applied to fuse the classification results given by the descriptors obtained from each polygon in three ways:

- Each descriptor (25 in all) treated equally to obtain a global result (Table 8, section 7.3.1)
- Each descriptor fused into its group (3 groups i.e. scalar, FD and MI) to obtain a result for each group (Table 8, section 7.3.2 – 7.3.4)
- Each group result fused to obtain an overall result (Table 8, section 7.3.5).

Table 8 shows that, with notable exceptions, the classification accuracy obtained was fairly consistent no matter which was used. Best performer was the min rule followed by the product rule. Worst performer by far was the normalised sum rule. This confirms the arguments in [Kittler 1998] which questions the theoretical basis of the sum rule.

7.3 Performance of fused descriptors over all selected features									
Number of polygons processed: 8837									
7.3.1 All 25 Descriptors									
ALL		majority	max	min	median	sum	sum adj	product	product adj
	number	5978	5974	7047	5992	5992	186	6938	6603
	percent	68	68	80	68	68	2	79	75
7.3.2 Scalar Descriptors									
SCALARS		majority	max	min	median	sum	sum adj	product	product adj
	number	6195	6010	6592	6223	6185	196	6381	6552
	percent	70	68	75	70	70	2	72	74
7.3.3 Fourier Descriptors									
FOURIERS		majority	max	min	median	sum	sum adj	product	product adj
	number	5940	5958	5789	5921	5949	175	5837	5815
	percent	67	67	66	67	67	2	66	66
7.3.4 Moment Invariants									
MOMENTS		majority	max	min	median	sum	sum adj	product	product adj
	number	6192	6051	7122	6281	6119	6026	7004	6973
	percent	70	68	81	71	69	68	79	79
7.3.5 Majority of 3 methods									
MAJORITY		majority	max	min	median	sum	sum adj	product	product adj
	number	6102	6025	6544	6134	6076	198	6294	6419
	percent	69	68	74	69	69	2	71	73

**Table 8:** Summary of performance of fusion of descriptors on all features in Plymouth data set showing number and percentage correctly classified.

### 5.3 Conclusions and future work

Based on the results presented here, we can claim partial success in demonstrating the application of shape classification to the recognition and feature coding of objects on large-scale topographic maps. We have identified that performance is variable depending on the descriptors used and the object types we are trying to distinguish. However, to further investigate and develop this project we have the following recommendations for the future extension of this research:

#### 1. More data through the current systems

- Perform more trials on more data obtained from Ordnance Survey; the larger data sets available for Basingstoke and Scotland are obvious candidates. These trials will be to:
  - Generate fuller results for supervised learning using the statistical analysis from the Purbeck data set
  - Provide a larger data set for use in supervised learning.
  - Perform trials on a larger set of feature/object types

#### 2. Evaluation of further types of shape descriptor

- Extend scalar descriptors to include more values, for example
  - Sharp corners
  - Corners
  - Parallelism of sides of shape
  - Major/minor axes
  - Rectangularity
  - Topological descriptors (holes etc.)
- Fast Shape Descriptors
- Shape Context Descriptors
- Boundary chain encoding
- Wavelet-based shape description
- Mathematical morphology
- Fractal shape descriptors

#### 3. Improvement with data fusion techniques

- Implementation of other fusion techniques from the fusion literature
  - Cleverer application of fusion to take into account shape types and contexts
4. Combine shape description methods with non-shape recognition techniques, such as
- Context models
  - Structure mapping
  - Markov models

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