Diagonalization of a Canonical System with Nonlinear Actuators

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ABSTRACT

This paper presents a simple but effective method for the diagonalization of canonical systems (where all the interaction occurs in a matrix of constant gains), where the inputs to the system are via rate-limiting actuators. Such situations arise in strip processing problems, including shape control in steel mills.

1. INTRODUCTION

Nonlinear actuators of the form shown in Fig.1 are found in many systems, both hydraulic and electrical. The effect of the relay shown is often manifested as a controller output which has only 'raise' or 'lower' signals, limiting the maximum rate of change of the output, often for safety reasons or due to physical limitations.
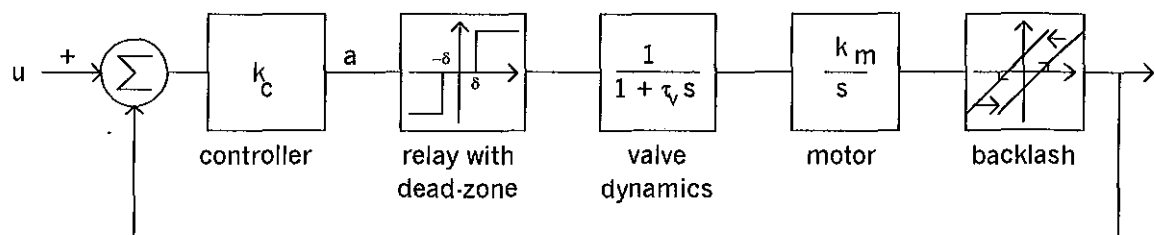

Fig.1: Actuator Block Diagram

 Such actuators give a ramp type output, not normally a problem in scalar systems. However, in multivariable systems, these types of actuators can introduce severe interaction between the signal paths, when an attempt is made to diagonalize the system.

Consider the case of an n by n canonical system, where all the interaction occurs in a matrix of constant gains:

$$y = g(s)G_m [f_1(u_1) \quad f_2(u_2) \quad \cdots \quad f_n(u_n)]^T \qquad (1)$$

$$where \quad G_m \in \Re^{nxn} \quad g(s) \in \Re(s)$$

$$y \in \Re^n \quad , \quad u_1, u_2 ..., u_n \in \Re$$

If compensators $c_1(.)$, $c_2(.)$, ... , $c_n(.)$ can be found so that:

$$f_1(c_1(u')) = f_2(c_2(u')) = ... = f_n(c_n(u')) = q(s)u'$$

the system in equation (1) may be easily diagonalized by the application of G_m^{-1} (where G_m^{-1} exists) as shown in Fig.2, to give:

$$y = q(s)g(s)G_m u^* = g(s)q(s)I_n w$$

$$where \quad u, \; u^*, \; w \in \Re^n$$

and single loop compensation may be applied as appropriate to give suitable dynamic performance. See [1] and [2] respectively for solutions where G_m is singular or non-square.
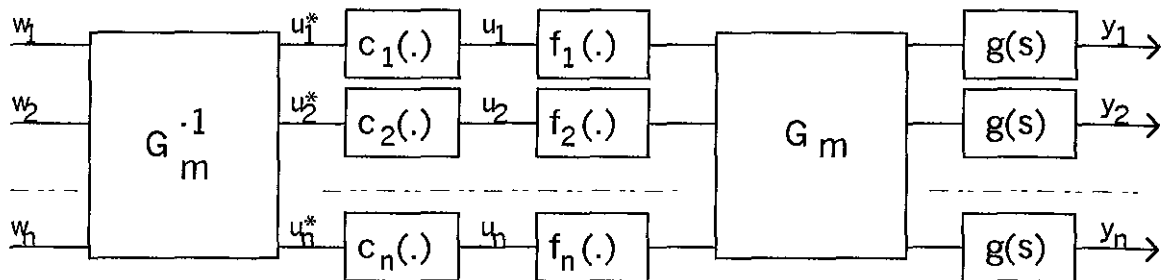

Fig.2: **Diagonalization of Nonlinear System**

Section 2 demonstrates a simple method for the determination of a compensator set $c_1(.)$, $c_2(.)$, ... , $c_n(.)$ which linearizes the actuators, with a 2 x 2 example provided in Section 3, indicating the decoupling performance of the technique. Conclusions are presented in Section 4.

2. ACTUATOR LINEARIZATION

The actuator system under examination is shown in Fig.1. The dynamics of the hydraulic valve (not present for an electrical actuator) are normally assumed to be considerably faster than the motor. The backlash on the position output is due to gearing or a rack and

pinion arrangement on the actuator output. To prevent hunting (small oscillation limit cycling) due to the presence of backlash [3], a small amount of dead-zone is usually introduced into the relay. For analysis purposes, the dominant components of the actuator system are taken to be the controller gain, relay and motor. It will be shown that the omission of the backlash, dead-zone and valve dynamics from the model will not result in any significant errors, while maintaining computational simplicity in the linearizing compensator.

Obtaining the describing function for the relay gives the well-known result [4]:

$$N_r(a) \quad = \quad 4/(\pi a) \tag{2}$$

The extension for a relay with dead zone is [4]:

$$N_{rdz} \quad = \quad \frac{4\delta}{\pi a}\sqrt{1 - (\frac{\delta}{2a})} \tag{}$$

Evaluating the closed loop transfer function for the actuator system of Fig.1, using the describing function approximation of equation (2) for the relay, yields:

$$T(s,a) \quad = \quad \frac{1}{1 + \dfrac{\pi a}{4 k_c k_m} s} \tag{3}$$

Equation (3) represents a first order system with an amplitude dependent (equivalent) time constant of:

$$\tau_e \quad = \quad \frac{\pi a}{4 k_c k_m} \tag{4}$$

If a compensator c(.) is now placed in cascade with the actuator, where

$$c(s,a) \quad = \quad \frac{1 + \tau_e(a)s}{1 + \tau_n s} \tag{5}$$

then the cascade combination of the compensator and actuator reduces to a first order LTI system with time constant τ_n. τ_n may be chosen by the designer subject to the obvious limitation that the compensated system cannot move faster than the rate limit imposed by k_m. Note that the signal input to the relay characteristic is normally available as an electrical signal. As an illustration of the performance of such a compensator, consider the following two actuator systems:

	Cont. gain k_c	Motor gain k_m	Dead-zone δ	Backlash b	Time const τ_v
Actuator A	0.7	3.14	0.25	0.5	0.1

| Actuator B | 0.3 | 8.0 | 0.25 | 0.3 | 0.05 |

Fig.3 shows the step response of both actuators with their associated precompensators along with the step response of an LTI first order system with a time constant of 5 secs. τ_n was chosen to be 5 secs. also.
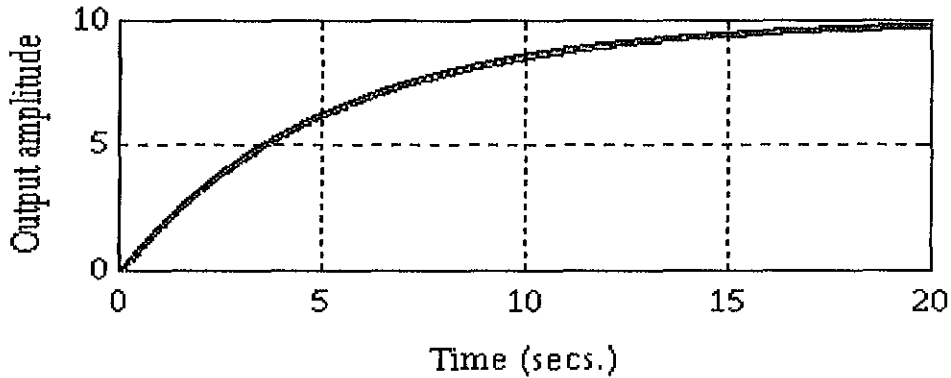

Fig.3: Actuator Equalization

3. A DIAGONALIZATION EXAMPLE

A 2x2 example will be taken here to illustrate the use of the linearizing precompensator in the diagonalization as shown in Section 1. The actuator parameters are those given in the table in Section 2. The scalar dynamics, g(s) will be set to unity, facilitating clearer observation of the output responses. For this example, τ_n was chosen to be 3 seconds, with:

$$G_m = \begin{bmatrix} 2 & 1 \\ 3 & 3 \end{bmatrix} \quad , \quad giving \quad G_m^{-1} = \begin{bmatrix} 1 & -\frac{1}{3} \\ -1 & \frac{2}{3} \end{bmatrix}$$

Fig.4 shows the responses of the two outputs with a step in path 1 and a zero input on the other for the uncompensated nonlinear system. Fig.5 shows the corresponding response for the compensated system. Although the uncompensated system is diagonal at zero frequency (steady-state response), significant interactions occur in the transient response. In certain applications, this may be unacceptable, with the possibility of large undesirable excursions in the outputs leading to critical operating conditions, for example tearing of the strip in steel processing.
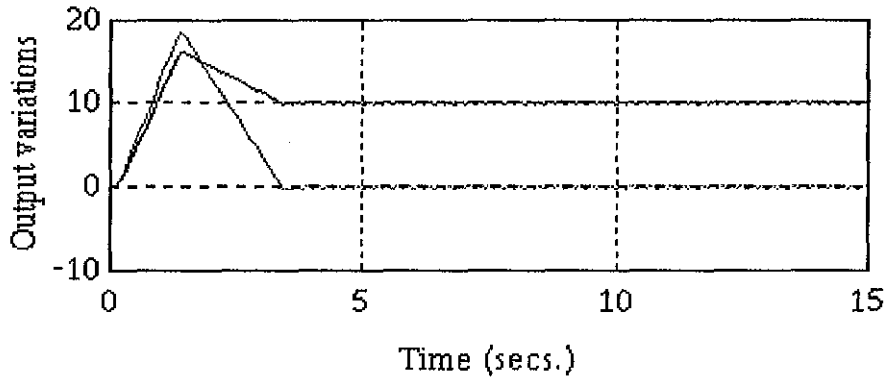
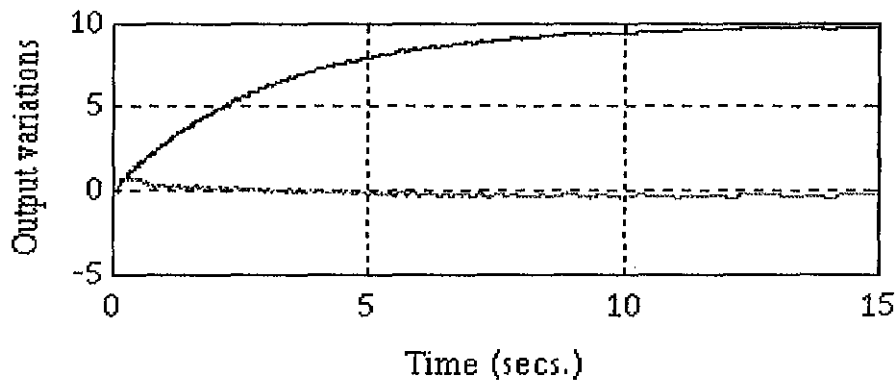**Fig.4: Multivariable Response for Uncompensated System**

Fig.5: Multivariable Response for Compensated System

4. CONCLUSIONS

The paper presents a simple but effective technique for actuator linearization. The price paid for achieving a smooth linear response is degredation in the rise time, but this may be acceptable when considered with possibly large undesirable excursions in outputs for an uncompensated system. For the actuator configuration shown in Fig.1, the describing function provides a good representation, since output harmonics are attenuated heavily by the motor dynamics. This is evidenced by the linearization example in Section 2 and by frequency response measures which have also been performed. The results of Sections 2 and 3 also validate the approximations made in neglecting the effects of backlash, dead-zone and valve dynamics in the construction of the linearizing precompensator. Further computational simplicity is achieved if the compensators are realized in state-space controllable form, with the result that the discrete time A and B matrices are fixed, the amplitude-dependent terms entering only into the C and D matrices, which remain invariant under the transformation from continuous to discrete time.

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