ADAPTIVE CONTROL OF BAKER'S YEAST FERMENTATION -THE PROBLEMS AND SOME SOLUTIONS

P. Comerford and J.V. Ringwood

School of Electronic Engineering
Dublin City University
Glasnevin, Dublin 9
Ireland.

ABSTRACT

This paper proposes an adaptive control scheme for a fermentation process. This incorporates identification of the process parameters via a recursive identification in which a non-linear process model is parameterised in terms of a linear identification model. This allows the application of traditional least-squares techniques for on-line identification of system parameters. As the primary states, biomass and substrate, are not directly measurable on-line state estimation via the method of Kalman filtering is proposed. The filter algorithm uses the measurements from an established sensor for gaseous carbon dioxide to estimate process states. Adaptive control of the process using a pole placement state feedback controller is utilized to give the process some desired dynamic response characteristics. Results are presented demonstrating parameter convergence, estimated state trajectories and regulation of the system output and conclusions are drawn which summarise the sucessful approaches to fermentation control and make some recommendations on the choice of algorithms and direction of future work.

1. INTRODUCTION

Fermentation control provides a considerable challenge to the control engineer. Not only is the system non-linear but many of the key process variables are not measurable on line. The dynamics of the process, however, are slow, allowing the designer much leeway in the specification of highly complex controllers, due to the relatively slow sampling rate required (of the order of minutes). Progress in the application of computer control techniques has been hampered by many practical problems. Firstly, instruments for the measurement of biomass and substrate concentrations have not withstood the rigorous sterility requirements and those that pass the above test prove less than durable. Secondly, the actual fermentation process contains non-linearities and time varying parameters. Hence conventional two and three term controllers cannot provide optimal performance over the whole range of the process.

Adaptive control of the process has been suggested as a solution. The adaptive controller consists of two main elements. Firstly, a recursive parameter that computes estimates of plant dynamics in terms of a set of parameters in a structured model and secondly a control design algorithm that uses these identified parameters.. The adaptive control of a bioreactor also include a state estimator as shown in Figure 1.

This paper will be organised as follows: section 2 will develop a mathematical model for the fermentation system. In section 3 a discrete time identification model for the process that is linear in the parameters is presented. Section 4 introduces the Kalman filter state estimator and shows that if the measurement aand disturbance noises can be modelled by a white noise process then the estimate of the state vector can be found as a soultion of the filter equations. The performance of the above components in unison with a control law is presented in the adaptive controller outlined in section 5. Results and conclusions are presented in Section 6 and 7 respectively.

2. PROCESS MODEL

In fermentation, as in other processes, an accurate mathematical model is a prerequisite for the simulation, control and optimisation of the process. Models have been proposed using differential equations describing the microbial kinetics [1]. These require assumptions

that the parameters of the model equations are constant. This does not account for the fact that the cell activity may switch between different metabolic pathways during a fermentation life cycle. The final choice of model must therefore be a calculated comprimise between the degree of simplicity that will allow good control and the equally important desire to represent all the important aspects of the process accurately.

Consideration is given here to a fermentation process described by the Monod model for growth. For a perfectly mixed fed batch fermentor in which the culture is being diluted at a rate D (hr^{-1}) then reactor activity of biomass and substrate is given by,

$$\dot{X} = (\mu - D).X \tag{1}$$

$$\dot{S} = D.(S_i - S) - \frac{\mu X}{Y_{X:S}} \tag{2}$$

$$\mu = \mu_{max}.\frac{S}{k_S+S} \tag{3}$$

with $Y_{X:S}$ the yield of biomass per unit substrate, k_S a saturation constant and μ the growth rate. A discrete model can be obtained by obtaining an Euler approximation of the derivatives process,

$$X(k) = X(k-1) + \frac{h\mu S(k-1)X(k-1)}{k_S+S(k-1)} - hD(k-1)X(k-1) \tag{4}$$

$$S(k) = S(k-1) + hD(k-1)[S_i(k)-S(k)] - \frac{h\mu_m S(k-1)X(k-1)}{Y_{X:S}[k_S+S(k-1)]} \tag{5}$$

where h is the sampling period in hours.

3. PARAMETERIZATION OF THE FERMENTATION PROCESS

The goal in process identification is to infer a model (and identify model parameters) given a process input/output data record. If we consider a dynamical system with input system u(t) and output signal y(t). Suppose that these signals are sampled in discrete time at t=1,2,3.. and that the sampled data values can be related through the following linear difference equation,

$$y(t)+a_1y(t-1)+.+ a_ny(t-n) = b_1u(t-1)+.+ b_mu(t-1) \tag{6}$$

This may be written in the form

$$y(t) = \Theta^T(t).\Phi(t). \tag{7}$$

where $\Phi(t)$ and $\Theta(t)$ are the regressor and parameter vectors respectively,

$$\Theta^T(t) = \{ a_1 \ldots a_n : b_1 \ldots b_m \} \tag{8}$$

$$\Phi^T(t) = \{ -y(t-1), \ldots -y(t-n), u(t-1), \ldots u(t-m) \} \tag{9}$$

One way to obtain estimates of $\Theta(t)$ is to try and minimise the prediction error using the following criterion and minimising J_N with respect to $\Theta(t)$.

$$J_N(\Theta) = \sum_{t=1}^{N} [y(t) - \Theta^T(t)\Phi(t)]^2 \tag{10}$$

Minimisation of J_N w.r.t Θ gives the estimate of Θ as,

$$\Theta(N) = \left[\sum_{t=1}^{N} \Phi(t)\Phi^T(t) \right]^{-1} . \sum_{t=1}^{N} \Phi(t)y(t) \tag{11}$$

The RLS algorithm can be modified to maintain its sensitivity to process parameter variations. This may be done by introducing an exponential weighting factor, called a forgetting factor in the performance index,

$$J_N(\Theta) = \sum_{t=1}^{N} \lambda^{N-t} [y(t) - \Theta^T(t)\Phi(t)]^2 \qquad (12)$$

The following modified RLS for time varying systems results,

$$\Theta(t) = \Theta(t-1) + L(t)[y(t) - \Theta^T(t-1)\Phi(t)] \qquad (13)$$

$$L(t) = \frac{P(t-1)\varphi(t)}{\lambda + \Phi(t)P(t-1)\Phi(t)} \qquad (14)$$

$$P(t) = \frac{1}{\lambda}\left[P(t-1) - \frac{P(t-1)\varphi(t)\varphi^T(t)P(t-1)}{1 + \Phi^T(t)P(t-1)\Phi(t)} \right] \qquad (15)$$

To use recursive least squares type identification algorithms we must first obtain expressions for the system which are linear in the parameters. This set of equations will consist of a known measurement vectors y(t), a known regressor $\Phi(t)$ and an unknown parameter vector $\Theta(t)$. It is the function of the RLS routine to identify the elements of the parameter vector given a set of process input and output measurments.

The recursive least squares structure for (4) is,

$$y(t) = \Theta^T(t).\Phi(t) \qquad (16)$$

where

$$y(t) = \{ X(t)-X(t-1)+hD(t-1)X(t-1) \}S(t-1) \qquad (17)$$

$$\Phi^T(t) = \{ hS(t-1)x(t-1),\ X(t-1)-X(t)-hD(t-1)X(t-1) \} \qquad (18)$$

$$\Theta^T(t) = \{ \mu_{max},\ k_s \} \qquad (19)$$

Similiarly by discretization of eqt. (2) the parameter $Y_{X:S}$ may may be identified,

$$Y_{X:S}[S(t-1)-S(t)-hD(t-1)S_i(t-1)-hD(t-1)S(t-1)] = h\mu(t-1)X(t-1) \qquad (20)$$

Substitution of eqt. (4) into the rhs of eqt. (20) gives a direct estimate of the yield coefficient $Y_{X:S}$.

$$Y_{X:S} = \frac{[X(t)- X(t-1)-hD(t-1)X(t-1)]}{[S(t-1)-S(t)-hD(t-1)S_i(t-1)-hD(t-1)S(t-1)]} \qquad (21)$$

4. STATE ESTIMATION

In a biochemical reactor system, as is often the case with state estimation problems, the state of the system is not directly measurable but is observable through the measurement of outputs of the system, such as exhaust gas concentrations. If the state x of a dynamical system satisfies the linear equation of (22) then the state estimation problem can generally be stated as follows [4],

$$\dot{x} = Ax + Bu + \zeta(t) \qquad (22)$$

The linear equation is forced by the non–random control input u and the random disturbance $\zeta(t)$. One must develop an algorithm for determining the state x(t) at time t from the observations of an output y(t) of the system, contaminated by the random errors $\xi(t)$ and related to the state x by

$$y = Hx + \xi(t) \qquad (23)$$

Because of the errors ξ in the measurement, x, the true state cannot be found - only an estimate for x is possible. Therefore in the presence of the random noises $\zeta(t)$ and $\xi(t)$ the estimation problem is understood in the sense of finding an estimate of the state x such that the uncertainity, or varience, of the estimation error is minimised. If the noises $\zeta(t)$ and $\xi(t)$ can be modelled by a white noise process then the estimate x can be found as the solution of the following filter equation;

$$\dot{x} = Ax + Bu + K[y - Hx] \qquad (24)$$

with K, the filter gain given by,

$$K = PH^T R^{-1} \qquad (25)$$

and the varience of the estimation error (a measure of the uncertainity in the estimate of x), $P = E[(x-x)(x-x)^T]$, given by

$$\dot{P} = AP + PA^T + Q - PH^T R^{-1} HP \qquad (26)$$

The matrices Q and R (positive semi-definite) are measures of the intensity of the noises ξ and ζ, respectively with $Q\delta(t) = E[\zeta(t)\zeta^T(t)]$ and $R\delta(t) = E[\xi(t)\xi^T(t)]$

If we consider a continuous non–linear stochastic model of micro-organism growth and substrate consumption dynamics in a batch fermentation process and output model that consists of measurement of the *carbon dioxide evolution rate*,

$$\dot{x}(t) = F(x(t)) + \zeta(t) \qquad (27)$$
$$y(t) = H(x(t)) + \xi(t) \qquad (28)$$

where x(t) is the state vector consisting of biomass (x_1) and substrate concentrations (x_2) and the components of F(x) are,

$$f_1(x) = \frac{\mu_m . x_1 . x_2}{k_S + x_2} - Dx_1 \qquad f_2(x) = \frac{\mu_m . x_1 . x_2}{Y_{X:S}(k_S + x_2)} + D(S_i - x_2) \qquad (29,30)$$

y(t) is the *carbon dioxide evolution rate* which is related to the state vector by the non-linear function H(x(t)) where,

$$H(x(t)) = \frac{1}{Y_{X:C}} . \frac{\mu_{max} . x_1 . x_2}{k_S + x_2} \qquad (31)$$

where $Y_{X:C}$ is yield of carbon dioxide on biomass.

The initial state of the model is to be assumed normally distributed stochastic variable with mean $E[x(0)] = x(0)$ and covariance,

$$E[x(0) - x(0))(x(0) - x(0))^T] = P \qquad (32)$$

Since the state vector x(t) is unmeasurable on–line and in the presence of state and observation noise the problem posed is how to find an on–line estimation algorithm based on available measurement of carbon dioxide evolution rate. The extended Kalman filter application is presented as a technique for on–line estimation of state variables based on the model process equations (27-32).

Applying a simple Euler approximation to the above continuous model, we can derive the following discrete non–linear stochastic model of micro–organism growth and substrate consumption for the batch fermentation process as

$$x(k+1) = g(x(k),h) + \zeta(k) \qquad (33)$$

$$y(k) = H(x(k)) + \xi(k) \quad (34)$$

where g(x(k),h) = x(k) + h.F(x(k)) and h is the sampling interval. The extended Kalman filter equations may be written as follows.

$$x(k+1/k+1) = x(k+1/k) + K(k+1)[y(k+1) - H.x(k+1/k) \quad (35)$$

$$x(k+1/k) = g(x(k/k),h) \quad (36)$$

$$K(k+1) = P^*(k+1).H^T.[HP^*(k+1)H^T + R]^{-1} \quad (37)$$

$$P^*(k+1) = \Phi(k)P(k)\Phi(k) + Q \quad (38)$$

$$P(k+1) = [I - K(k)H]P^*(k) \quad (39)$$

where

x(k+1/k+1) is the optimal state estimate at time k+1.
x(k+1/k) is the state estimate at time k+1 based on estimate x(k/k) at time k (prediction).
y(k+1) is the output observation vector.
H is the linearized observation matrix.
K(k+1) is the Kalman gain matrix.
P(k) is a symmetric error filtering matrix.
$P^*(k+1)$ is a symmetric error prediction covarience matrix.
$\Phi(k)$ is the transition matrix of the linearised model evaluated at time k for estimate x(k/k) where $\Phi(k) = I + h.J|_{x(k)}$ and J is the Jacobian matrix of the model equations.
Q is the state noise covarience.
R is the output noise covarience.

5. CONTROL LAW FORMULATION

A non-linear system may be adequately approximated by a linear system near some operating conditions. Significant advances in the linear control theory permit the synthesis and design of very effective controllers even for non-linear processes. Fundamental, therefore, is the concept of linearization and the procedure for approximating non-linear systems by linear systems.

Using Taylor series to expand the model equations about the equilibrium point (x_0,u_0), the linear terms of a Taylor series expansion we obtain the following multi-input multi-output linear variational model for the plant,

$$\delta x(k+1) = A^* \delta x(k) + B^* \delta u(k) \quad (40)$$

where

$$\delta x(k) = [x(k)-x_0,\ s(k)-s_0]^T \quad (41)$$

$$\delta u(k) = [u_1(k)-u_{10}(k),\ u_2(k)-u_{20}(k)]^T \quad (42)$$

with $u_1(k) = D(k)$ (42)

and $u_2(k) = D(k)S_i(k)$ (44)

and the components of the A^* and B^* are respectively,

$$a^*_{11} = 1 + \frac{h\mu_m S_0}{[k_S+S_0]} - hu_{10}; \quad a^*_{12} = \frac{hk_S\mu_m X_0}{[k_S+S_0]^2} \quad (45,46)$$

$$a^*_{21} = \frac{-h\mu_m S_0}{Y_{X:S}[k_S+S_0]}; \quad a^*_{22} = 1 - \frac{h\mu_m k_S}{Y_{X:S}(k_S+S_0)^2} - hu_{10} \quad (47,48)$$

and

$$b^*_{11} = -hX_0; \quad b_{12} = 0 \quad (49,50)$$

$$b^*_{12} = h(S_i(k)-S_0) \quad ; \quad b_{22} = h \qquad (51,52)$$

The following *equilibrium points* for biomass and substrate of the system result,

$$S_0 = \frac{k_S u_{10}}{\mu_{max}-u_{10}} \quad ; \quad X_0 = Y_{X:S}[(u_{20}/u_{10}) - S_0] \qquad (53,54)$$

Regulation based on pole placement via state feedback is now discussed. In general representing a linear system as [5],

$$x(k) = A(k-1)x(k-1) + B(k-1)u(k-1) \qquad (55)$$

With the folowing linear feedback law

$$u(k) = L.x(k) = -l_1X(k)-l_2S(k) \qquad (56)$$

the closed loop system becomes

$$x(k+1) = [A(k)-B(k)L(k)]x(k) = A_C(k)x(k) \qquad (57)$$

where

$$A_C = \begin{bmatrix} a_{11}-b_{11}l_1 & a_{12}-b_{11}l_2 \\ a_{21}-b_{21}l_1 & a_{22}-b_{21}l_2 \end{bmatrix} \qquad (58)$$

Equating the characteristic equation of $A_C(k)$ to the following desired characterstic equation

$$z^2 + p_1z + p_2 = 0 \qquad (59)$$

where p_1 and p_2 are the desired pole locations and solving for the two state feedback gains l_1 and l_2 we get [6],

$$l_1 = (t + l_2u)/v \qquad (60)$$
$$l_2 = (vp_1p_2 - vq - rt)/(ru + v) \qquad (61)$$

where

$$q = a_{11}a_{22} - a_{12}a_{21} \qquad (62)$$
$$r = a_{12}b_{21} + b_{22}a_{12} - a_{22}b_{11} - a_{22}b_{12} \qquad (63)$$
$$s = a_{21}b_{11} + a_{21}b_{12} - a_{11}b_{21} - a_{11}b_{22} \qquad (64)$$
$$t = p_1 + p_2 - a_{11} - a_{22} \qquad (65)$$
$$u = b_{21} + b_{22} \qquad (66)$$
$$v = -(b_{11} + b_{12}) \qquad (67)$$

The pole placement problem has been solved explicitly and in the following the section simulation results will be presented for a self tuning state feedback controller incorporating the Kalman filter.

6. RESULTS

Any adaptive controller for a fermentation system relies on a good state estimator. Figures 2-6 investigate the tracking properties of the Extended Kalman Filter (EKF) for a fermenter simulation. Figure 2 depicts the carbon dioxide evolution rate and additive white noise. The EKF's good tracking capabilities for correct initial estimates of states is shown in Figures 3 and 4. Unfortunately, the EKF is sensitive to incorrect initial state estimates as is shown in Figures 5 and 6 and the filter fails to recover from the initial bias. Correct initial state estimates are given in the controller results that follow.

The non-linear plant was simulated with a Monod growth structure and parameter values of μ_{max}=0.8, k_S=0.22 and $Y_{X:S}$=0.5. The plant outputs under self tuning control are shown in Figure 7. State trajectories of (2,3) -> (2.4,3.8) and closed loop poles $p_1=p_2$=0.8 apply. Figure 8 shows the convergence of the plant parameters for P(0)=10^5 and λ=0.95

while Figure 9 shows the control inputs. The initial equilibrium inputs u_{10} and u_{20} are calculated using equation (53) and (54) with initial parameters of μ_{max}=0.1, k_S=0.05 and $Y_{X:S}$=0.1 and also initial states X(0)=2.0 and S(0)=3.0. This model is then updated at each sampling interval.

The performance of the self tuning is now investigated for a changing plant parameter. At t=0.7hr μ_{max} is changed 10% about its nominal value to μmax=0.88. P(0)=10^5 and λ=0.9 to aid parameter tracking after the disturbance. Results for plant outputs, parameter convergence and control inputs are shown in Figures 10-12.

7. CONCLUSIONS

Studies on fermentation control over a range of conditions has been outlined. The main merit of a self tuner is its ability to track time varying parameters and coupled with the process of linearization may improve control of an actual fermentation process. Future work includes replacing the fixed parameter Kalman filter with a self tuning EKF and perhaps use of an iterative EKF to reduce output error and the bias of bad initial estimates.

As shown in some of the simulaion studies the initial control signal based on bad initial estimates is sometimes unacceptable. In a real system care would be needed in setting limits on control signal and parameter variations and perhaps a self tuner as one part of an overall control system would be more beneficial.

REFERENCES

1. Scheiding W. and Hecht V., "Aspects of Modelling of Microbial Growth Processes". *IFAC Conference on Modelling and Control of Biotechnial Processes, Helsinki, Finland (1982).*

2. Ljung L. and Soderstrom T., "Theory and Practice of Recurive Identification". *MIT Press (1985).*

3. Lozano R. and Carillo J., "Adaptive Control of a Fermentation Process". *IFAC 9th Triennial World Congress, Budapest (1984).*

4. Leigh J.R. et al, "Development of Strategies for On-Line Estimation of Biomass and Secondary Product Formation in Growth Limitied Batch fermentations". *IEEE Proc., Vol 133, Pt D, No. 5, pp 235-239, Sept 1986.*

5. Astrom K.J., and Wittenmark B., "Computer Controlled Systems". *Prentice-Hall (1984).*

6. Comerford, P., "Self Tuning Control of Fermentation Processes", *M. Eng Thesis, Electronic Eng. Dept., DCU, Sept 1990.*

Fig. 1 Self Tuning Controller for Fermentation Process

Fig. 2 Carbon Dioxide Evolution Rate

Fig. 3 Biomass Estimate - Good Initial Estimates

Fig. 4 Substrate Estimate - Good Initial Estimates

Fig. 5 Biomass Estimate - Bad Initial Estimates

Fig. 6 Substrate Estimate - Bad Initial Estimates

Fig. 7 Plant Outputs

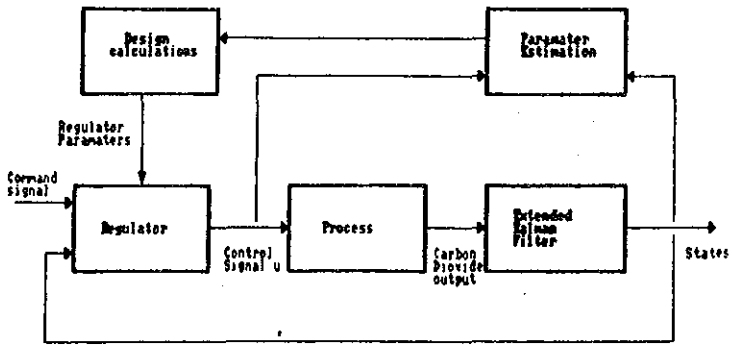
Fig. 8 Parameter Convergence

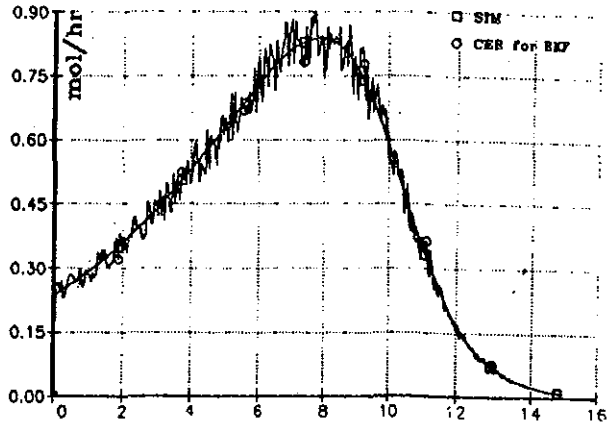
Fig. 9 Control Inputs

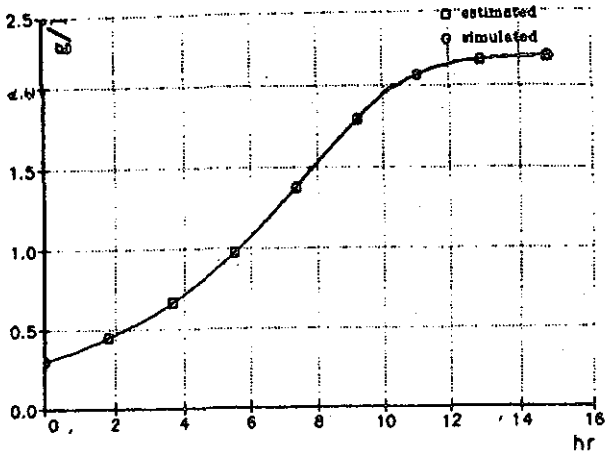
Fig. 10 Plant Outputs (with Parameter Disturbance)

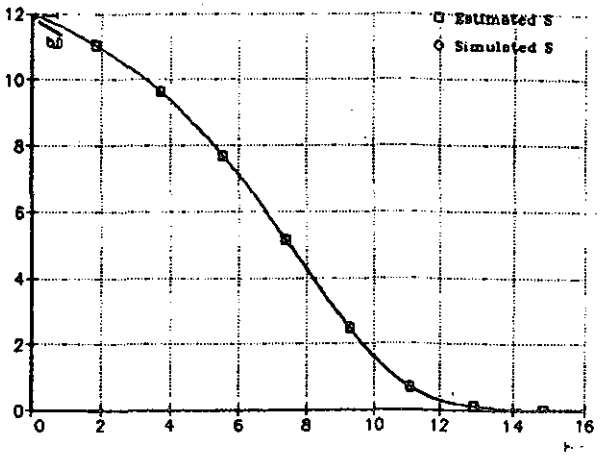
Fig. 11 Parameter Convergence (with Parameter Disturbance)

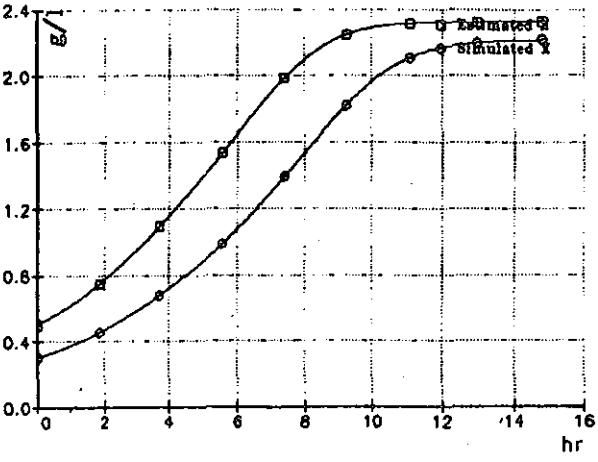
Fig. 12 Control Inputs (with Parameter Disturbance)

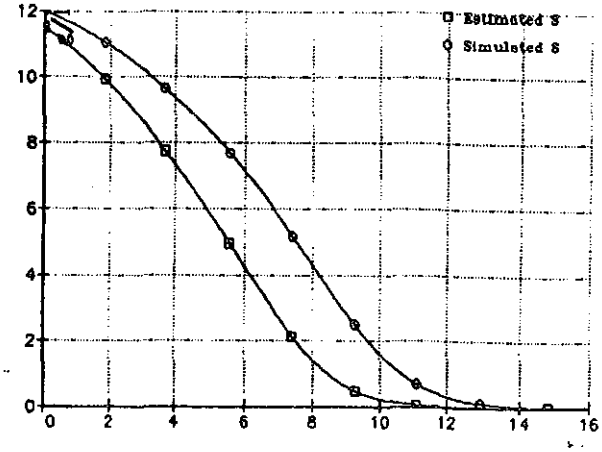

Design calculations
Parameter Estimation
Regulator Parameters
Command signal
Regulator
Process
Extended Kalman Filter
Control Signal u
Carbon Dioxide output
States

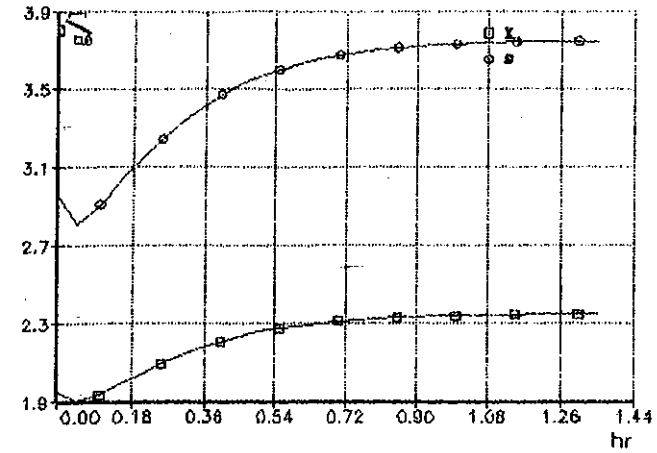

mol/hr
SIM
CER for EKF

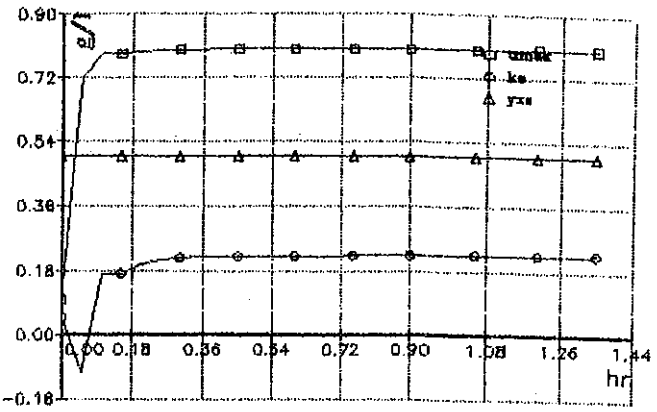

g/l
estimated
simulated
hr

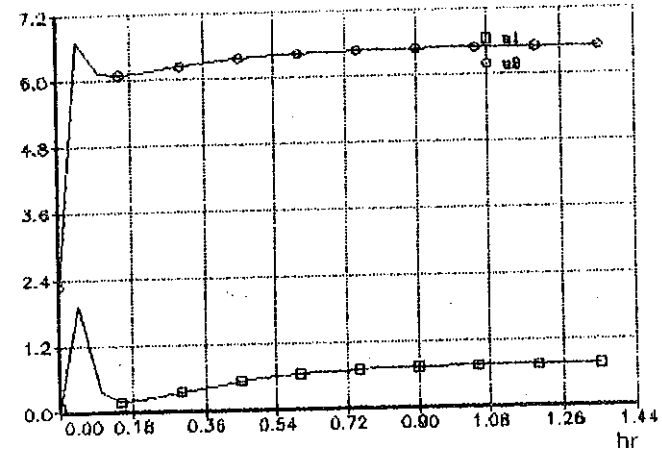

g/l
Estimated S
Simulated S

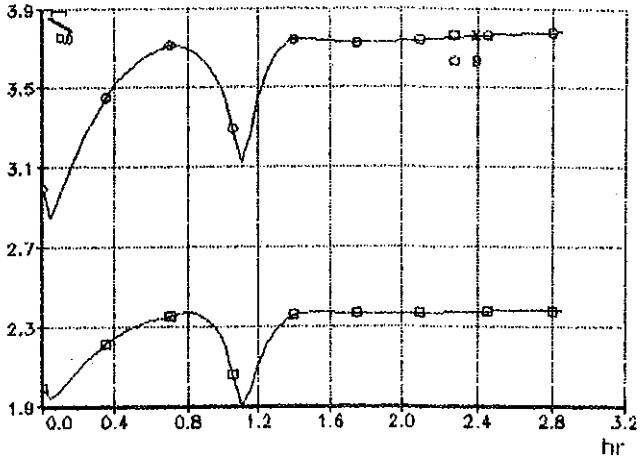

g/l
Estimated X
Simulated X
hr

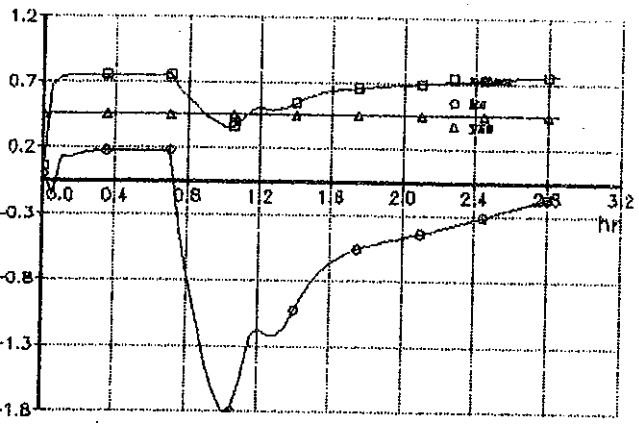

g/l
Estimated S
Simulated S

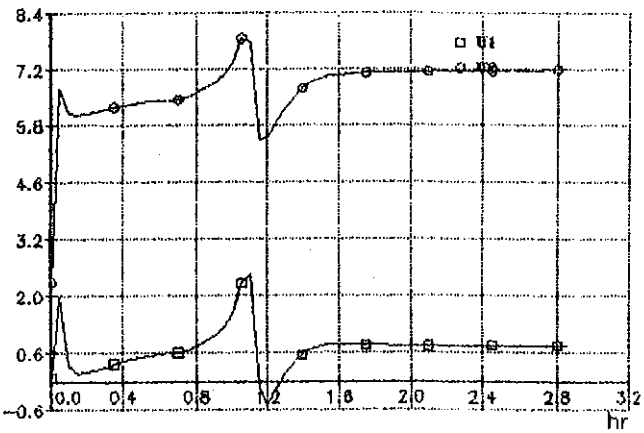

3.9
3.5
3.1
2.7
2.3
1.9
0.00 0.18 0.36 0.54 0.72 0.90 1.08 1.26 1.44
hr


0.90
0.72
0.54
0.36
0.18
0.00
-0.18
0.00 0.18 0.36 0.54 0.72 0.90 1.08 1.26 1.44
hr


7.2
6.0
4.8
3.6
2.4
1.2
0.0
0.00 0.18 0.36 0.54 0.72 0.90 1.08 1.26 1.44
hr


3.9
3.5
3.1
2.7
2.3
1.9
0.0 0.4 0.8 1.2 1.6 2.0 2.4 2.8 3.2
hr


1.2
0.7
0.2
-0.3
-0.8
-1.3
-1.8
0.0 0.4 0.8 1.2 1.6 2.0 2.4 2.8 3.2
hr


8.4
7.2
5.8
4.6
3.2
2.0
0.6
-0.6
0.0 0.4 0.8 1.2 1.6 2.0 2.4 2.8 3.2
hr