Quasi-Optical Phase Retrieval of Radiation Patterns of Non-Standard Horn Antennas at Millimetre and Submillimetre Wavelengths

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Abstract— The location of the phase centres of antenna feeds is critical for optimised sensitivity and resolution on reflector antennas and telescopes. While the measurement of the far-field intensity patterns of such feeds is relatively straightforward, the direct recovery of their phase patterns requires access to expensive phase sensitive instrumentation such as a vector network analyzer. We present an inexpensive alternative quasi-optical technique, analogous to off-axis holography at visible wavelengths, that allows for the phase curvature of the feed pattern, and thus the phase centre, to be recovered with sufficient accuracy for optimizing aperture efficiency and resolution on a reflector antenna. We discuss the accuracy of the technique and compare results for the case of a specialized horn antenna for CMB polarization operating at 100GHz, using both the quasi-optical method and a vector network analyzer as a bench mark measurement tool for verification of the approach. We also include some measurements made of a lens antenna fed by a bare waveguide radiator.

Index Terms— horn antenna, millimeter wave, measurement, phase centre.

I. INTRODUCTION

The quasi-optical phase-centre retrieval technique relies on recording the interference pattern in intensity produced by a beam from the horn antenna-under-test with a well defined reference beam. Such a reference beam can be derived from a standard horn antenna, such as a corrugated conical horn, as illustrated in Fig. 1 [1],[2]. Both beams need to be coherent with respect to each other and can be derived from the same oscillator source making use of, for example, a waveguide beam splitter in a similar way to a typical off-axis holographic set-up at visible wavelengths. Then, by comparing the interference pattern for the test horn antenna, whose phase centre we wish to locate, with that of a waveguide probe (a point-like source), we can calibrate out any phase curvature of the reference beam. Thus, we can extract the position of the test horn phase centre with respect to that of the waveguide probe with useful accuracy (a fraction of the depth of focus of the horn). Similar interferometric applications are reported in the literature for both microwave set-ups (e.g. [4],[5],[6]) and terahertz holography (e.g. [7],[8]), as well as other millimeter-wave applications (e.g. [9])

We summarize our results here for an example case of a smooth walled profiled horn specially designed for Cosmic Microwave Background (CMB) polarization experiments and operating at 100 GHz. In this case access to a vector network analyzer (VNA) allowed us to verify the quasi-optical technique. We also present results for a waveguide fed lens antenna configuration, whose phase centre is more difficult to predict theoretically due to the difficulty in applying physical optics to the small volume of the lens. Clearly, the quasi-optical technique would be particularly useful at short millimeter and submillimetre wavelengths, where the cost of a VNA for locating the phase centre could be prohibitive. It is also possible in theory to recover the full phase pattern in the far field if required using this phase retrieval approach.
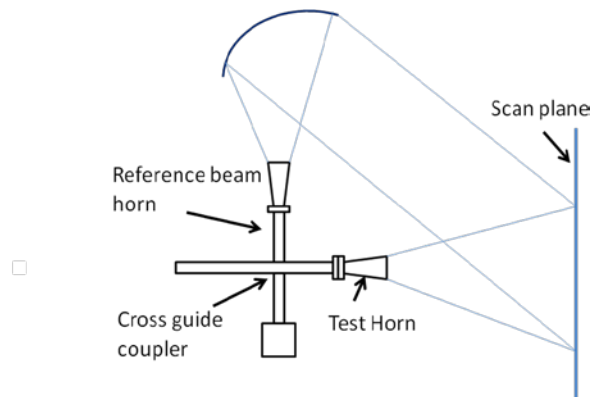

Fig. 1. Quasi-optical holographic set-up for recording interference patterns on the scan plane.

II. QUASI-OPTICAL TECHNIQUE

The proposed set-up for the quasi-optical phase retrieval is shown in Fig. 1. This is essentially the same set-up as is used in classical off-axis holographic experiments at visible wavelengths [10]. The interference fringe pattern between the reference horn antenna beam and the horn antenna-under-test (equivalent to the hologram) is recorded using a detector which scans over a precisely defined plane at the location of the waist of the reference beam (and thus quasi-collimated).

If the system has been aligned so that the optical axes of the two horn beams intersect at a point in the centre of the scan plane, which itself lies orthogonal to the optical axis of the

beam from the horn-under-test, then it is straightforward to derive an expression for the interference pattern across the scan plane. Thus, assuming a pseudo-Gaussian beam derived from a conical corrugated horn antenna the interference pattern due to the two propagating beams is given by:

$$I(x,y) = \left| \mathbf{E}_{ref} + \mathbf{E}_{test} \right|^2 \approx \left| E_0 \exp\left(-\frac{x^2 \cos^2\theta + y^2}{W_0^2} \right) \right.$$

$$\left. \exp(-jk\sin\theta) + A_0(x,y)\exp\left(-jk\frac{x^2 + y^2}{2R} + j\phi_0 \right) \right|^2$$

(1)

where the x-y plane is the scan plane, the z-axis is aligned with the optical axis of the beam of the horn-under-test and the two beams lie in the x-z plane (we can take $z=0$ to be the scan plane). R is the radius of curvature of the phase front of the antenna-under-test beam (of amplitude $A(x,y)$) and W_0 is the beam width of the pseudo-Gaussian beam of amplitude E_0. Here it is assumed that the reference beam has a waist at the scan plane so that its phase curvature $1/R_{ref}$ is zero.

If both beams are similar in intensity and extent at the scan plane (and W_0 is many wavelengths), then near the centre of the scan plane the interference fringe pattern along a direction parallel to the y - axis (so that x is a constant) is determined by the two phase terms, so that:

$$I(y) \, \alpha \, \left| 1 + r\exp(j\phi_x)\exp\left(-jk\frac{y^2}{2R} \right) \right|^2$$

(2)

Thus, by fitting a plot of a measurement of $I(y)$ for various values of x with this relationship, R in theory can be recovered as illustrated in Fig.2. However, this requires perfect alignment of the beams and also that a waist for the reference beam coincides exactly with the scan plane.

If the reference beam is not quite at a waist so that it also has a phase curvature $1/R_{ref}$ then there will be an error in the recovered curvature if the equation 2 is used directly. If such is the case then in fact the $1/R$ in equation (2) should be replaced with the reduced curvature in the y direction ($1/R_y = 1/R_0 - 1/R_{ref}$) , where $1/R_0$ is the curvature of the beam of the feed antenna-under-test at the scan plane. The scanning detector feed antenna should also clearly have a well defined phase centre in order to precisely recover the location of the phase centre of the feed-under-test. The system therefore needs to be calibrated using a source with a well defined phase centre position, such as a bare waveguide, to take these effects into account.

We thus also measure the interference pattern for a second calibration test horn (with the required precisely defined phase centre, such as shown in the set-ups in Fig. 3). Then the difference of the $1/R_y$ recovered in that case with the $1/R_y$ recovered for the antenna-under-test case is given by $\Delta(1/R_y) = 1/R_0 - 1/R_C$ (where C denotes calibrator), where now the reference beam curvature has been eliminated. This difference can further usefully be re-expressed as $\Delta(1/R_y) = (R_C - R_0)/R_0 R_C$, so that the distance of the unknown phase centre from that of the calibrator can be written as $(R_C - R_0) \approx (R_C)^2 \times\Delta(1/R_y)$, which is useful if the distance between the two phase

centres is small compared to the distance to the scan plane (equivalent to $(R_C - R_0)/R_C \,\square\, 1$) and the approximation becomes usefully accurate. It is also true in this case that to achieve high accuracy it is important to able to determine the difference $\Delta(1/R_y)$ as precisely as possible for the relatively small magnitude of $(R_C - R_0)$. However, we do not therefore need to know the distance R_C from the detector phase centre on the scan plane to the horn -under-test to such high precision. The errors in the estimation will be dominated by the determination of the minima in the fringe patter in the y-direction, as illustrated in Fig. 2 (b).

Furthermore, a more detailed analysis shows that we can also include possible tolerance offsets in the x and y directions for the reference and antenna-under-test beams (as well as the reference beam having curvature), in a similar way (as in [7], for example).
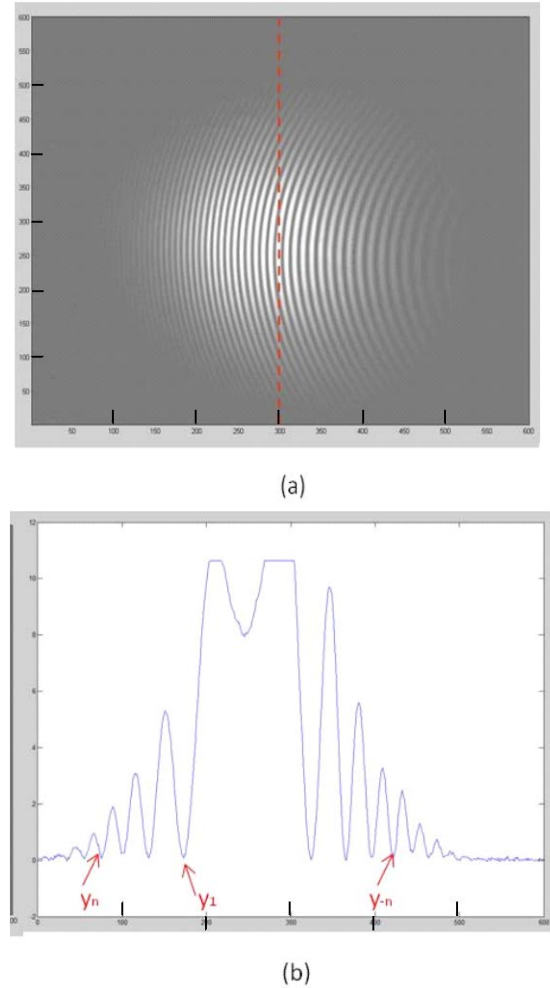

(a)

(b)

Fig. 2. Example showing interference pattern (a) and a plot of a 1-D scan in the y-direction for typical horn described in paper (b). The ticks show displacements of 100 stop sizes on the scan plane (1 step = 0.5 mm).
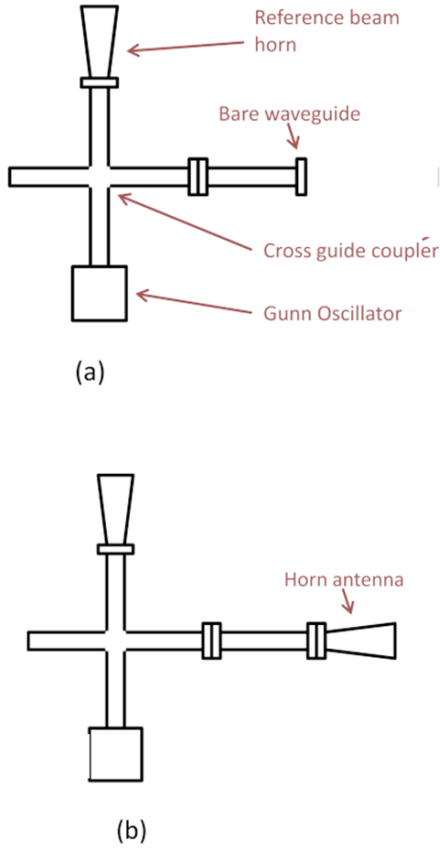
Fig. 3. Setup for determining relative location of the phase centres between a bare waveguide (a) and waveguide with a horn fitted to the flange.

Any off-set tolerance errors in the *y*-direction between the two beams result in an asymmetric fringe pattern in a cut along the *y*-axis (such as can be seen in Fig. 2, for example). In this case we require that at least two minima in the *y*-direction either side of the *x*-axis ($y = 0$) be identifiable (minima are more precisely defined than maxima) for the cases of interest. It can be shown that in this case $1/R_y = 2(n-1)\lambda/(y_n- y_1) (y_1- y_{-n})$, (see Fig.2).

III. EXAMPLE HORN AND LENS FEED ANTENNAS

The quasi-optical phase retrieval technique was applied to the case of a specially designed horn for use on CMB polarization experiments (see Fig. 4, [10]). The horn is smooth walled and made up of spline-fit conical sections. The beam pattern has a well behaved quasi-Gaussian shape with very low sidelobe levels and low cross polarization at the operation frequency of 100GHz. The phase centre of this horn was measured both using the quasi-optical technique described here and also by a VNA which was available for this work as a verification tool. The results are summarized in Table 1.

The beams were aligned experimentally by measuring the beam patterns of the two horns separately to ensure the two beam centers (optical axes) coincided at the measurement plane. The measurement probe on the scan plane was essentially a bare waveguide which could be assumed to behave like a point source radiator. The physical dimension of the mouth of the probe was about 0.5 wavelengths and this determines the error in the location of the minima in Fig. 2. Any alignment errors can be included in the analysis from precise measurements of the positions of the off-axis minima in the *y*-direction as in Fig. 2.

As can be seen in Table 1 the uncertainty in the two techniques (using VNA or the QO phase retrieval technique) is about the same magnitude. In the case of the spline fit horn the two measurements of the phase centre locations agree well to within the measurement accuracy and also to within the depth of field of a such a horn antenna.

We also measured the phase centre of a standard corrugated horn with this technique to check for consistent results. The errors for the quasi-optical technique are mostly associated with precisely measuring the positions of the minima along a y-scan such as illustrated in Fig. 2. The depth of focus is approximately 6.0 mm in the case of the corrugated horn with the phase center located 6.0 mm behind the aperture.
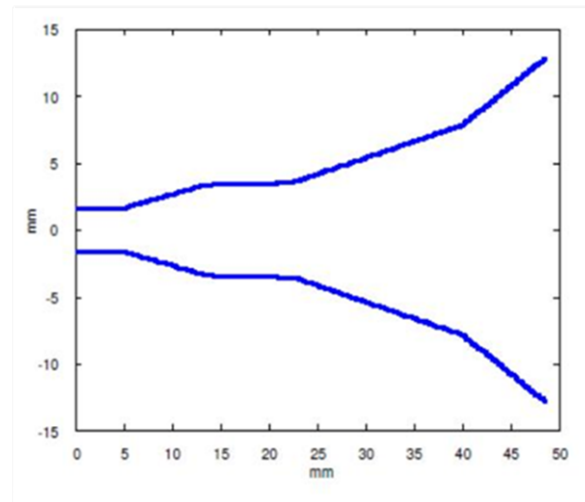

Fig. 4. Spline fit smooth walled horn for CMB polarisation.

TABLE I. COMPARISON OF PHASE CENTER POSITIONS WITH RESPECT TO HORN APERTURES

Horn type	QO technique (mm)	VNA (mm)
Spline fit profiled horn	24.7 ± 3.0	24.2 ± 2.0
Corrugated conical horn	12.8 ± 8.0	6.0 ± 3.0
Waveguide-fed Lens Antenna	12.5 ± 3.5	18.6 ± 2.5

It is intended to repeat this measurement to investigate the reason for the difference between to two phase centre positions predicted for the corrugated horn example, even if within the phase centre depth of focus. It is also possible to predict the location of the phase centre theoretically which will be pursued as part of this investigation. One of the issues actually is the precise definition of the centre of phase position and whether this corresponds to the phase curvature on-axis or an average phase curvature over the entire beam extent weighted appropriately by the beam intensity, or indeed where the coupling to some ideal Gaussian is optimized. Also the temperature stability was not monitored but will be included in future experimental campaigns.

Another type of feed antennas investigated was a lens antenna [11],[12]. The term lens antenna is in general used to describe a range of antenna designs which feature the combination of a planar antenna and a collimating lens which is placed in contact with the lens to produce a wide equivalent waist for the radiated beam. Unlike waveguides and most single mode horn antennas, the phase centers of lens antennas are often ill-defined as they are difficult to model and thus it is important to be able to measure them by experimental methods.

An example of a lens antenna was manufactured and tested. Rather than being fed by a planar antenna the high density polyethylene (HDPE) lens used was designed to mechanically fit on the flange of a standard WR10 rectangular waveguide, with the waveguide aperture feeding the signal to the lens at the focus of the lens (Fig. 5), [11]. For the antenna investigated the phase retrieval method put the phase centre at a position of 12.5 ± 3.5 mm behind the front apex of the lens. This particular lens antenna was also measured with a VNA which yielded a result of 18.6 ± 2.5 mm, also behind the front apex of the lens. However the actual depth of field of the lens beam is very long (as was clear form an image reconstruction of the beam using the hologram recorded by the scanning detector, using the technique described in [1]). This is due to the wide beam waist resulting from the refocusing effect of the lens on the beam. Although the high degree of collimation of the beam resulting thus appears to introduce a large uncertainty in determining the location of the phase centre because of the large depth of field, however, the coupling to any beam of the same width would be very high even for large waist misalignments.

Fig. 5. A schematic diagram of the dielectric lens milled from HDPE (left). Photograph of the lens attached to the waveguide (right)

IV.   CONCLUSION

We have presented a technique for phase centre retrieval using a holographic interference approach that is applicable to antenna feeds in the millimeter- and submillimeter-wave parts of the EM spectrum. The technique is inexpensive in comparison to the use of vector network analyzers. Furthermore as well as recovering the phase centre positions, the holographic inference patterns recorded can also be used to predict the far field phase and amplitude patterns of the horns, which would be useful for high efficiency coupling to reflector antennas or for injection into quasi-optical wave-guides.

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