

## Quasi-optical analysis of binary optical components at 100GHz

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### Abstract

We report on the use of Gaussian beam mode analysis (GBMA) to study the focusing properties of binary counterparts of lenses, axicons and Gabor zone plates. Optical components at millimetre wavelengths with short focal ratios will inevitably be rather thick. It is therefore advantageous to devise a method that can maximize space in a linear imaging system. This can be achieved by using a binary form of refractive and diffractive focusing elements to localize power from beams subject to a large degree of diffraction as is common in quasi-optics at these wavelengths. In this paper, we present numerical analysis of the aforementioned elements using GBMA as a computationally efficient alternative to Fresnel-Kirchoff diffraction integrals.

### Introduction

GBMA assumes a spatially coherent field  $E(r,z)$  as being represented by a sum of independently propagating modes  $\varphi_n(r,z)$  which are solutions of the Helmholtz wave equation. The magnitude of each mode (mode coefficient) is given by the overlap integral

$$A_n = \int E(r, z_0) \varphi_n(r, z_0, W(z_0), R(z_0))$$

$\varphi_n$  denoting a Laguerre-Gaussian polynomial of order  $n$ . Thus, the field at any subsequent point can be calculated as

$$E(r, z) = \sum_{n=0}^{\infty} A_n [\varphi_n(r, z, W(z), R(z))]$$

where  $R(z)$  and  $W(z)$  have their usual significance<sup>[1]</sup>. We shall use this technique to develop numerical simulations for various binary optical components, namely, Gabor zone plates, axicons, lenses and a grating for the creation of Bessel beams.

### Theory

The phase or amplitude transmission provided by an optical component can be described by a complex exponential term of the form

$$T(r) = A(r) \exp(i\phi(r))$$

where  $\phi(r)$  represents the phase delay introduced by the optical component over some output plane, and  $A(r)$  represents the amplitude transmission.

**Bessel Gratings:** A type of grating to be called a Bessel grating here, has been simulated that can generate a Bessel beam more efficiently than an axicon. Their surfaces are defined by creating zones alternating in phase between 0 and  $\pi$ , with radii  $2.4048\alpha$ ,  $5.5201\alpha$ ,  $8.6537\alpha$ , etc., where  $\alpha$  is a scale length parameter upon which the desired beam radius depends, and the numbers represent the roots of  $J_0(r)$ . Such a Bessel-like field will alternate spatially between positive and negative maxima and minima with well defined zero crossings. By generating these zero crossings in phase using a grating one can create the phase profile held by a Bessel beam. Such a beam will behave as an approximate true Bessel beam exhibiting several of its useful focusing features in the near field<sup>[4]</sup>.

**Gabor Zone Plates:** The Gabor zone plate is defined to be the amplitude hologram of a point source, in other words, the interference pattern generated by the superposition of a plane wave with a spherical beam from a single point source<sup>[2]</sup>. Such numerically generated patterns for millimetre wavelengths can be realised on copper clad fiber glass boards and can be shown to have similar if not better focusing abilities than their analogue counterparts.

**Lenses and Axicons:** The phase variation imposed by a lens on a plane beam can be represented by the parabolic approximation of a circular wavefront<sup>[3]</sup>,

$$\phi(r) = \exp\left(\frac{ikr^2}{2f}\right),$$

where  $k$  is the wavenumber of the radiation, and  $f$  is the desired focal length of the lens.

Any lens that produces this phase can be simplified by rounding off the phase at a point on its surface to the nearest value of  $\pi$ . In an identical fashion, the depth of axicons can be reduced showing no degradation of the "diffraction free" beam<sup>[4], [5]</sup> produced at 100GHz.

### Computational Results

The generation of Bessel beams through the use of a circularly symmetric transmission grating consisting of high density polyethylene with circularly milled grooves is to be discussed first. A model of a grating of  $\alpha=1$  was developed using the GBM theory, simulating the generation of a beam having a central spot size equal to 2.4048mm which is of the order of one wavelength at 100GHz. The beam from a BG exhibits the "scissors" effect like a conventional axicon, but due to the instant generation of the Bessel beam's phase structure, the output pattern shows more high order lobes in the plane of the grating than an axicon would. Figure 1 shows the closeness of fit of the output beam at 5cm from the grating with the desired intensity  $|J_0(r)^2|$ .

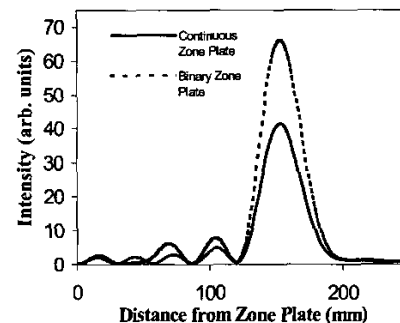


Figure 1 – Comparison of beam from Bessel grating to the ideal  $|J_0(r)^2|$  function.

By reducing the height of an axicon from a tall cone to a thin grating, one can easily generate the profile required to produce non-diffracting beams without any loss of resolution or depth of field. This is done by the transformation of the conical surface to a grating type structure using a modulo  $2\pi$  transformation similar to the method used in the design of Fresnel lenses. This surface is then quantized to two levels having a relative phase difference of  $\pi$  to each other. Because of the two level phase variations of Bessel gratings and Binary axicons, both exhibit similar properties, and can be seen to form a narrow central spot immediately in their output plane (figure. 2).

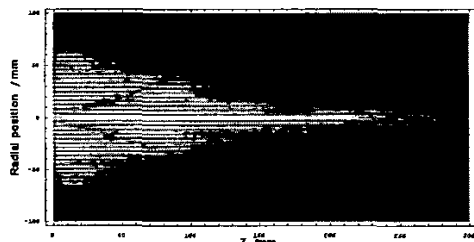


Figure 2 – Logarithmic density plot showing the formation of a Bessel beam using a binary axicon.

Shown in figure 3 is a comparison of the on-axis intensity distribution for the binary axicon with an ideal axicon having a diffraction free range of 160mm.

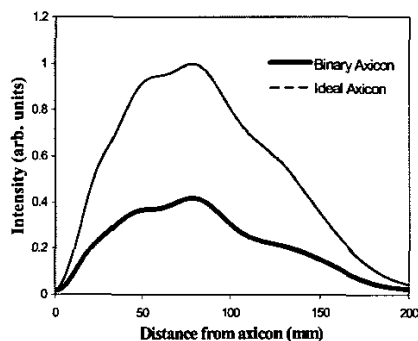


Figure 3 – Comparison of central intensities of binary and conical axicons.

Gabor Zone Plates (GZPs) can be manufactured rather simply for operation at long wavelengths. Using software developed for holographic simulation, one can generate the complete amplitude transmission pattern of an on-axis hologram for the generation of a real point image. Because of their ease of manufacture the GZP can be seen as an alternative to refractive lenses that can be expensive and quite large. We have examined the focus generated by binary GZPs and found the effect to be more pronounced in its binary form, as is evident in figure 4.

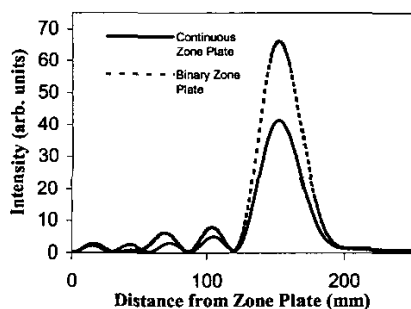


Figure 4 – On-axis intensity for binary and analogue Gabor Zone Plates of focal length 150mm

Binary lenses were modeled in a similar fashion to the two-level axicon. Unlike the conical lens, which has a constant surface gradient, the formation of a binary element from an approximately circular surface seemed to have more noticeable effect on the focus when compared to its analogue counterpart.

We numerically studied the focusing of a “top-hat” plane wave by multi-level lenses of a varying number of levels  $n$ . It was found that for the focusing of such a beam at 100GHz, the effect caused by using a lens of  $n \geq 6$  did not produce a noticeable deviation from the ideal focus for a lens of focal length 300mm.

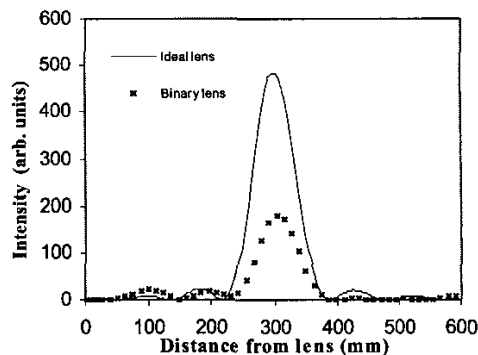


Figure 5 – On-axis intensities of simulated binary lens ( $n=2$ ), and ideal lens of focal lengths 300mm.

### Conclusion

We have theoretically demonstrated the use of binary components for the production of high-resolution foci and diffraction-free beams using Gaussian beam mode analysis. A new method for the production of Bessel beams has been analysed and shown to be more effective in the confocal distance achieved with the more commonly used axicon. Such components are to be examined experimentally in the near future, results of which will be published.

### References

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